Deadlock Avoidance of Flexible Manufacturing Systems by Colored Resource-Oriented Petri Nets With Novel Colored Capacity

Xiang Zhaoyu

September 25, 2020

Deadlock Avoidance of Flexible Manufacturing Systems by Colored Resource-Oriented Petri Nets With Novel Colored Capacity*

ZhaoYu Xiang[1[0000000151550190]]

Institute of System Engineering, Macau University of Science and Technology, Taipa 999078, Macau SAR China 1909853gmi30002@student.must.edu.mo

Abstract. A variety of colored resource-oriented Petri nets (CROPN)-based control method to forbid deadlock in flexible manufacturing systems (FMS) are to add control places to the original net, which makes the net being complex. This paper proposes a novel concept in colored resource-oriented Petri nets (CROPN) called colored capacity. Firstly, the foraml definition of colored capacity in a CROPN is given. Based on this concept, the new execution rule of the transitions is proposed. Then, a procedure is developed such that the colored capacity function of each place in a CROPN can be obtained. By colored capacity function, all control places that are used to forbid illegal markings in CROPN are displaced by the colored capacity and the deadlock can be avoided by the new execution rule, which makes the structru of the net much simpler than the net with control places. Finally, an FMS example is used to illustrate the proposed method.

Keywords: Deadlock avoidance· discrete event systems· flexibility manufacturing systems· Petri nets· colored resource-oriented Petri net.

1 Introduction

Flexible manufacturing systems (FMS) have been widely used in industrial fields [1]-[7]. However, since a large number of jobs have to share same resource in an FMS, deadlocks may arise, which leads to serious consequences. To deal with deadlock issue in FMSs, many control policies based on Petri net models have been established.

In these control policies, a deadlock avoidance strategy to prevent the FMS modeled by Petri nets from being deadlock is to add some constraints to the targeted FMS such that the system is deadlock-free. The work in [9] proposes a colored resource-oriented Petri net (CROPN) model to analyze the deadlock problem in FMS. The CROPN is considered to be more powerful than other Petri nets such as the resource place-based Petri nets[5].

* This work was supported in part by the Science and Technology Development Fund, Macau SAR (File Nos. 0012/2019/A1).

The work in [5] establishes the deadlock-free operations in CROPN. However, to use the deadlock avoidance policy proposed in [5], we need to add control places to the original net, which makes the net being complex.

In this paper, a novel CROPN concept called colored capacity is proposed. We define the colored capacity of the place, which represents the biggest number of tokens with the same color that the place can retain simultaneously. It is shown that a place in CROPN may have different colored capacity corresponding to different color. Based on the concept of colored capacity, the new transitions firing rule for CROPN is presented. Then, after we obtain all deadlock and impending deadlock markings by using the method proposed in [5], a method is presented to determine the colored capacity for each place in CROPN by simple calculation. It is shown that the colored capacity of a place in CROPN will change along with the marking change. Based on the dynamically changing color capacity, combined with the new transitions firing rule proposed in this paper, all deadlock markings and impending deadlock markings in a flexible manufacturing systems (FMS) modeled by CROPN are forbidden, therefore, this FMS is deadlock-free and we do not need to add control places to the net.

The contributions in this paper are as follows:

1) In this paper, the colored concept for FMS modeled by CROPN is proposed, which is not considered in [5].

2) Based on the dynamically changing color capacity, combined with the new transition firing rule proposed in this paper, all deadlock markings and impending deadlock markings in a flexible manufacturing systems (FMS) modeled by CROPN are forbidden without the need to add control places to the net.

This paper is organized as follows. Section 1 is the introduction. Section 2 defines the concept of colored capacity for CROPN. Section 3 proposes a method to determine the colored capacity for each place in all interactive subnets in CROPN by simple calculation. Section 4 presents a FMS example to explain the application of the proposed method. We conclude in Section 5.

2 Realization of Control Policies for Interactive Subnets

In this section, a method is developed to determine the colored capacity for each place in CROPN such that the deadlock-avoidance policy $U(v^k)$ is realized by using the the new transitions firing rule.

Consider CROPN in Fig. 2 [5], there are three part types A, B, and C, where $p_0 \rightarrow t_2 \rightarrow p_1 \rightarrow t_4 \rightarrow p_2 \rightarrow t_5 \rightarrow p_3 \rightarrow t_7 \rightarrow p_4 \rightarrow t_9 \rightarrow p_0$ for the operating of part-A and $p_0 \rightarrow t_{10} \rightarrow p_4 \rightarrow t_8 \rightarrow p_3 \rightarrow t_6 \rightarrow p_2 \rightarrow t_1 \rightarrow p_0$ for the operating of part-B and $p_0 \rightarrow t_2 \rightarrow p_1 \rightarrow t_3 \rightarrow p_5 \rightarrow t_{11} \rightarrow p_0$ for the operating of part-C with $K(p_1) = K(p_3) = K(p_5) = 1$ and $K(p_2) = K(p_4) = 2$. Hence, we have $C(p_0) = \{b_2, b_{10}\}, C(p_1) = \{b_3, b_4\}, C(p_2) = \{b_1, b_5\}, C(p_3) = \{b_6, b_7\}, C(p_4) = \{b_8, b_9\}, C(p_5) = \{b_{11}\}$. In this net, there is only one interactive subnet $v^2 = \{p_2, t_5, t_6, p_3, t_7, t_8, p_4\}$ formed by two PPCs:

$$v_1 = \{p_2, t_5, p_3, t_6\}, v_2 = \{p_3, t_7, p_4, t_8\} \tag{1}$$

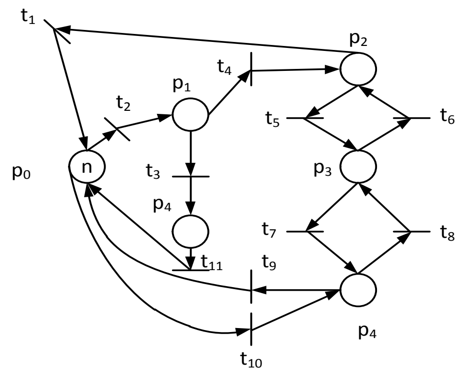

Fig. 1. A CROPN with one interactive subnet formed by two PPCs[5]

$$\text{Let}\ \ \Theta(v^2) = (M(p_2)(b_5),\ M(p_2)(b_1),\ M(p_3)(b_7),\ M(p_3)(b_6) \tag{2}$$

Then, by the algorithm proposed in [5], we have $M_{FBM} **(v^2) = \{M_1, M_2, M_3\}$ such that

$$\begin{aligned} M_1(v^2) &= \Theta_1(v^2) = (0,0,1,0,0,2)^T \\ M_2(v^2) &= \Theta_2(v^2) = (2,0,0,1,0,0)^T \\ M_3(v^2) &= \Theta_3(v^2) = (2,0,0,0,0,2)^T \end{aligned} \tag{3}$$

Hence, to prevent deadlock in this net, we have three control policies in $U(v^2)$ used to prohibit $M_{FBM} **(v^2)$ as follows:

$$\begin{aligned} C(M_1) : M(p_3)(b_7) + M(p_4)(b_8) \leq M_1(p_3)(b_7) \\ +M_1(p_4)(b_8) - 1 = 2; \end{aligned} \tag{4}$$

$$\begin{aligned} C(M_2) : M(p_2)(b_5) + M(p_3)(b_6) \leq M_2(p_2)(b_5) \\ +M_2(p_3)(b_6) - 1 = 2; \end{aligned} \tag{5}$$

$$\begin{aligned} C(M_3)) : M(p_2)(b_5) + M(p_4)(b_8) \leq M_3(p_2)(b_5) \\ +M_3(p_4)(b_8) - 1 = 3. \end{aligned} \tag{6}$$

where $M \notin M_B(v^2)$.

Next, we discuss how to realize $U(v^2)$ by using colored capacity and its new execution rule. First, let us present some notations as follows:

1. In a CROPN, given a marking M. For a place $p \in P$, let $RM(M(p)(b_i)) = K(p)(b_i)(M) - M(p)(b_i)$ denote the number of free room available to tokens with color b_i in p at marking M, where $b_i \in C(p)$.
2. Given a CROPN, a subnet v^k in the net with $M_i \in M_{FBM} **(v^k)$, let $S_i(v^k) = \Lambda_i(v^k) - 1$, where $\Lambda_i(v^k) = \sum_{D(v^k)(i,l,j)} M_i(p_l)(b_j)$.

3. In a CROPN, given a mrking M, for a place $p \in P$, we use $C(p, z)$ to denote a set of colores of $b_z \in C(p)$. Hence, let $\sum_{C(p,z)} M(p)(b_z)$ denote the number of tokens corresponding to $C(p, z)$ for marking M.
4. In a CROPN, we use $A(L, J)$ to denote the set of combinations of $p_L \in P$ and $b_J \in C(P)$, where $C(P) = \{C(p_u)|u = 0, 1, ..., |P| - 1\}$.
5. Given a CROPN, a subnet v^k in the net, we use $C(v^k)(L, J)$ to denote the set of combinations of $p_L \in P(v^k)$ and $b_J \in C(v^k)$, where $C(v^k) = \{C(p)|p \in P(v^k)\}$.
6. Given a CROPN, a subnet v^k in the net. When $M_{FBM} * *(v^k)$ have been found, let us rearrange the markings in $M_{FBM} * *(v^k)$ as $M_1, M_2, ...,$. Then, define $D(v^k)(A, l, j) = \{D(v^k)(1, l, j) \cup D(v^k)(2, l, j) \cup, ...\}$
7. Given a CROPN, a subnet v^k in the net. Define $ND(v^k)(L, J) = C(v^k)(L, J) - D(v^k)(A, l, j)$ such that for any combination $(p_L, b_J) \in ND(v^k)(L, J)$, we have $(p_L, b_J) \in C(v^k)(L, J)$ and $(p_L, b_J) \notin D(v^k)(A, l, j)$.

In view of the above-mentioned definitions, given an interactive subnet v^k in a CROPN, we classify the colored capacity $K(p)(b_i)(M), p \in P(v^k), b_i \in C(p), M \notin M_B(v^k)$ into two categories:(1) $K(p)(b_i)(M)$ for $(p, b_i) \in D(v^k)(A, l, j)$. (2) $K(p)(b_i)(M)$ for $(p, b_i) \in ND(v^k)(L, J)$.

To realize $U(v^2)$ for subnet v^2 in the net shown in Fig.2, we need to consider all items in $D(v^2)(A, l, j)$, where $D(v^2)(A, l, j) = D(v^2)(1, l, j) \cup D(v^2)(2, l, j) \cup D(v^2)(3, l, j)$ and $U(v^2) = \{C(M_1), C(M_2), C(M_3)\}$.

We first determine $K(p)(b_i)(M), (p, b_i) \in D(v^2)(1, j, l)$ for realizing $C(M_1)$ by using the new execution rule. Consider two combinations (p_3, b_7) and (p_4, b_8) from $D(v^2)(1, l, j)$. Suppose

$$RM(M(p_3)(b_7)) = \left\{ S_1(v^2) - \sum\nolimits_{D(v^2)(1,l,j)} M(p_l)(b_j) \right\} \tag{7}$$

$$RM(M(p_4)(b_8)) = \left\{ S_1(v^2) - \sum\nolimits_{D(v^2)(1,l,j)} M(p_l)(b_j) \right\} \tag{8}$$

where $M \notin M_B(v^2)$, $S_1(v^2) = 2$, $D(v^2)(1, l, j) = \{(p_3, b_7), (p_4, b_8)\}$. Then, we have $K(p_3)(b_7)(M) = 2 - M(p_4)(b_8)$ and $K(p_4)(b_8)(M) = 2 - M(p_3)(b_7)$. It is clear that for any $M \notin M_B(v^2)$, the constraint is realized by using the colored capacity and new transitions firing rule. For example, assume that $M(p_3)(b_7) = 0, M(p_4)(b_8) = 2$, we have $K(p_3)(b_7)(M) = 0 < M(p_3)(b_7) - I(p_3, t_5)(b_5) + O(p_3, t_5)(b_5) = 1$. Hence, transition t_5 can not fire at marking M. Similarly, the transition t_{10} can not fire at marking M. Hence, $\nexists t \in T, M[t > M'$ such that $M'(p_3)(b_7) + M'(p_4)(b_8) > 2$. Therefore, with $K(p_3)(b_7)(M)$ and $k(p_4)(b_8)(M)$ being determined for any reachable marking $M \notin M_B(v^2)$, we have $M(p_3)(b_7) + M(p_4)(b_8) \leq 2$, imply that $C(M_1)$ is realized.

Note that, for any marking $M \notin F_B(v^2)$, we have $K(p_3)(b_7)(M) \leq K(p_3) = 1$ and $K(p_4)(b_8)(M) \leq K(p_4) = 2$. But, assume that $M(p_4)(b_8) = M(p_4)(b_9) = 0$ and $M(p_3)(b_7) = M(p_3)(b_6) = 0$, by Eqs.(11) and (12) we have $K(p_3)(b_7)(M) =$

2, which is a contradiction. Hence, Eq.(11) and (12) must be rewritten as follows:

$$RM(M(p_3)(b_7)) = \min\Big\{S_1(v^2) - \sum\nolimits_{D(v^2)(1,l,j)} M(p_l)(b_j) \\ , K(p_3) - \sum\nolimits_{C(p_3,z)} M(p_3)(b_z)\Big\} \tag{9}$$

$$RM(M(p_4)(b_8)) = \min\Big\{S_1(v^2) - \sum\nolimits_{D(v^2)(1,l,j)} M(p_l)(b_j) \\ , K(p_4) - \sum\nolimits_{C(p_4,z)} M(p_4)(b_z)\Big\} \tag{10}$$

where $C(p3, z) = \{b_7, b_6\}$ and $C(p4, z) = \{b_9, b_8\}$. For example, assume that $M(p_4)(b_8) = M(p_4)(b_9) = 0$ and $M(p_3)(b_7) = M(p_3)(b_6) = 0$, by Eq.(10) and Eq.(11), we have $RM(M(p_3)(b_7)) = 1$ and $RM(M(p_4)(b_8)) = 2$. That is $K(p_3)(b_7)(M) = 1$ and $K(p_4)(b_8)(M) = 2$ do not contradict $K(p_3)(b_7)(M) \leq K(p_3) = 1$ and $K(p_4)(b_8)(M) \leq K(p_4) = 2$.

Furthermore, assume that $M(p_2)(b_5) = 2$, $M(p_3)(b_7) = 0, M(p_3)(b_6) = 0$ and $M(p_4)(b_8) = 1, M(p_4)(b_9) = 0$, by Eq.(13) and Eq.(14), we have $K(p_3)(b_7)(M) = 1$ and $K(p_4)(b_8)(M) = 2$. By new transitions firing rule, the transition t_{10} can fire, i.e., $M[t_{10} > M'$ such that $M'(p_4)(b_8) = 2$,$M'(p_2)(b_5) = 2$, and $M'(p_3)(b_7) = 0$. Note that, at marking M', constraint is satisfied. But, some constraint is not satisfied, since $M'(p_4)(b_8)+M'(p_2)(b_5) = 4 > 3$. Hence, Eq.(13) and (14) must be rewritten as follows:

$$RM(M(p_3)(b_7)) = \min\Big\{S_1(v^2) - \sum\nolimits_{D(v^2)(1,l,j)} M(p_l)(b_j) \\ , K(p_3) - \sum\nolimits_{C(p_3,z)} M(p_3)(b_z)\Big\} \tag{11}$$

$$RM(M(p_4)(b_8)) = \min\Big\{S_1(v^2) - \sum\nolimits_{D(v^2)(1,l,j)} M(p_l)(b_j) \\ , S_3(v^2) - \sum\nolimits_{D(v^2)(3,l,j)} M(p_l)(b_j) \\ , K(p_4) - \sum\nolimits_{C(p_4,z)} M(p_4)(b_z)\Big\} \tag{12}$$

where $S_3(v^2) = 3$, $D(v^2)(3, l, j) = \{(p_2, b_5), (p_4, b_8)\}$. Then, assume that

$$\begin{aligned} &M(p_2)(b_5) = 2, \\ &M(p_3)(b_7) = 0, M(p_3)(b_6) = 0, \\ &M(p_4)(b_8) = 1, M(p_4)(b_9) = 0. \end{aligned} \tag{13}$$

By Eq.(15) and Eq.(16), we have $K(p_3)(b_7)(M) = 1$ and $K(p_4)(b_8)(M) = 1$. Hence, by new transition firing rule, the transition t_{10} can not fire at marking M.

Similarly, to realize constraint, we select two combinations (p_2, b_5) and (p_3, b_6) from $D(v^2)(2, l, j)$ and determine the corresponding colored capacity as follows:

$$\begin{aligned} RM(M(p_2)(b_5)) = \min \Big\{ & S_2(v^2) - \sum\nolimits_{D(v^2)(2,l,j)} M(p_l)(b_j) \\ & , S_3(v^2) - \sum\nolimits_{D(v^2)(3,l,j)} M(p_l)(b_j) \\ & , K(p_2) - \sum\nolimits_{C(p_2,z)} M(p_2)(b_z) \Big\} \end{aligned} \tag{14}$$

$$\begin{aligned} RM(M(p_3)(b_6)) = \min \Big\{ & S_2(v^2) - \sum\nolimits_{D(v^2)(2,l,j)} M(p_l)(b_j) \\ & , K(p_3) - \sum\nolimits_{C(p_3,z)} M(p_3)(b_z) \Big\} \end{aligned} \tag{15}$$

where

$$\begin{aligned} & M \notin M_B(V^2) \\ & D(v^2)(2, l, j) = \{(p_2, b_5), (p_3, b_6) \\ & C(p2, z) = \{b_1, b_5\} \end{aligned} \tag{16}$$

Thus $RM(M(p_2)(b_5)) + M(p_2)(b_5) = K(p_2)(b_5)(M)$, and $RM(M(p_3)(b_6)) + M(p_3)(b_6) = K(p_3)(b_6)(M)$.

Note that, with $K(p_3)(b_7)(M), K(p_4)(b_8)(M), K(p_2)(b_5)$, and $K(p_3)(b_6)(M)$ being determined, where $M \notin M_B(v^2)$, all items in $D(1, l, j), D(2, l, j)$, and $D(3, l, j)$ are considered, the constraint set $U(v^2)$ is realized by using the excuation rule.

The next step is to determine $K(p_L)(b_J)(M)$ for $(p_L, b_J) \in ND(v^2)(L, J)$. Since every item in $ND(v^2)(L, J)$ has no contribution to realize $U(v^2)$, we do not need to think over $S_1(v^2), S_2(v^2), S_3(v^2)$ for $K(p_L)(b_J)(M)$. Therefore, we have

$$RM(M(p_0)(b_2)) = RM(M(p_0)(b_{10})) = \infty \tag{17}$$

$$\begin{aligned} RM(M(p_1)(b_3)) = RM(M(p_1)(b_4)) = K(p_1)- \\ \sum\nolimits_{C(p_1,z)} M(p_1)(b_z) \end{aligned} \tag{18}$$

$$RM(M(p_2)(b_1)) = K(p_2) - \sum\nolimits_{C(p_2,z)} M(p_2)(b_z) \tag{19}$$

$$RM(M(p_4)(b_9)) = K(p_4) - \sum\nolimits_{C(p_4,z)} M(p_4)(b_z) \tag{20}$$

$$RM(M(p_5)(b_{11})) = K(p_5) - \sum\nolimits_{C(p_5,z)} M(p_5)(b_z) \tag{21}$$

where $M \notin M_B(v^2)$ and $K(p_0)(b_2)(M) = K(p_0)(b_{10})(M) = \infty$, $K(p_1)(b_3)(M) = K(p_1) - M(p_1)(b_4)$, $K(p_1)(b_4)(M) = K(p_1) - M(p_1)(b_3)$, $K(p_2)(b_1)(M) = K(p_2) - M(p_2)(b_5)$, $K(p_4)(b_9)(M) = K(p_4) - M(p_4)(b_8)$, $K(p_5)(b_{11})(M) = K(p_5)$.

Next, we transform the above analysis into Algorithm 1 for determining the colored capacity of each place in an interactive subnet in a CROPN.

Algorithm 1 *Dtermine the colored capacity of every place in an interactive subnt in a CROPN.*

Given a CROPN, an interactive subnet v^k *in the net. With* $M_{FBM}**(v^k)$ *being found by Algorithm presented in [5], we rearrange the markings in* $M_{FBM}**(v^k)$ *as* $M_1, M_2, ..., M_{|M_{FBM}**(v^k)|}$.

Input: $D(v^k)(i,l,j)$, $ND(v^k)(L,J)$, $S_i(v^k)$, $i \in \{1, 2, ..., |M_{FBM}**(v^k)|\}$. $K(p)$, *and* $C(p,z), p \in P(v^k)$;

Output: colored capacity $K(p_L)(b_J)(M)$, *where* $M \notin M_B(v^k)$, *and* $(p_L, b_J) \in C(v^k)(L,J)$.

1. *Step 1:*
   For I=1 to $|M_{FBM}**(v^k)|$;
   Relay(I) $\leftarrow D(v^k)(I,l,j)$;
   End for;
   Let $RM_t = \emptyset$;
2. *Step 2:*
   For q=1 to $|M_{FBM}**(v^k)|$;
   While $D(v^k)(q,l,j) \neq \emptyset$;
   Select a combination (p_l, b_j) *from* $D(v^k)(q,l,j)$;
   $RM_t \leftarrow S_q(v^k) - \sum_{Relay(q)} M(p_l)(b_j)$;
   For d= q+1 to $|M_{FBM}**(v^k)|$;
   If (p_l, b_j) *is an item in Relay(d);*
   Delete the item (p_l, c_l) *from* $D(v^k)(d,l,j)$;
   $RM_t \leftarrow RM_t \cup \left\{S_d(v^k) - \sum_{Relay(d)} M(p_l)(b_j)\right\}$;
   End if;
   End for;
   $RM_t \leftarrow RM_t \cup K(p_l) - \sum_{C(p_l,z)} M(p_l)(b_z)$;
   $K(p_l)(b_j)(M) = min(RM_t) + M(p_l)(b_j)$;
   Delete the item (p_l, c_l) *from* $D(v^k)(q,l,j)$;
   $RM_t \leftarrow \emptyset$;
   End while;
   End for;
3. *Step 3:*
   While $ND(v^k)(L,J) \neq \emptyset$;
   Select a combination (p_L, b_J) *from* $ND(v^k)(L,J)$;
   $K(p_L)(b_J)(M) = k(p_L) - \sum_{C(p_L,z)} M(p_L)(b_z)$
   $+M(p_L)(b_J)$;
   Delete the item (p_L, b_J) *from* $ND(v^k)(L,J)$;
   End while;

In algorithm 1, we preprocess a set in step 1. Next, in step 2, we determine $K(p_l)(b_j)(M)$ for any $(p_l, b_j) \in D(v^k)(A,l,j)$, where $M \notin M_B(v^k(x))$. Finally, in step 3, we determine $K(p_L)(b_J)(M)$ for any $(p_L, b_J) \in ND(v^k)(L,J)$.

The algorithm 1, combined with the analysis in this section, we have the following theorems.

Theorem 1. *Given a CROPN, an interactive subnet v^k in the net, if for any combination $(p_L, b_J) \in C(v^k)(L, J)$, $K(p_L)(b_J)(M)$ is determined by algorithm 1, where $M \notin M_B(v^k)$, then the subnet v^k is deadlock-free.*

Proof: According to the analysis in this section, if for every place in subnet v^k, the colored capacity is determined by algorithm 1, then $U(v^k)$ is realized by using the new execuation rule. Furthermore, by algorithm 1, no control places and arcs are added such that no new bad marking will produce in that subnet. Therefore, the subnet v^k is deadlock-free. The conclusion holds. ■

Theorem 2. *Given a CROPN, an interactive subnet v^k in the net, and a set $M_{FBM}{**}(v^k)$, if $U(v^k)$ is realized by the colored capacity and the new execuation rule, such that for any reachable marking M, $\sum_{D(v^k)(i,l,j)} M(p_l)(b_j) \leq \Lambda_i(v^k) - 1$, where $M_i \in M_{FFM}**(v^k)$, then, the control policy set $U(v^k)$ is maximally permissive.*

Proof: By Theorem 1, with $U(v^k)$ being realized by using the colored capacity and the new execuation rule, the subnet v^k is deadlock-free, imply that for any marking $M_b \in M_{FBM}$, M_b is prohibited. Suppose the control policy set $U(v^k)$ is not maximally permissive. Then, $\exists M_1 \in M_L$, M_1 is prohibited, which implies that there exists $M_i \in M_{FBM}{**}(v^k)$ such that $\sum_{D(v^k)(i,l,j)} M_1(p_l)(b_j) > \Lambda_i(v^k) - 1$, which is a contradiction. The conclusion holds. ■

By Theorem 1, with $K(p_L)(b_J)(M), (p_L, b_J) \in C(v^k)(L, J)$ being determined by Algorithm 1, the net is deadlock-free. The remaining problem is how to determine $K(p)(b_J)(M)$ for $(p, b_j) \in A(L, J) - C(v^k)(L, J)$. On the one hand, for any place $p \notin P(v^k)$, p has no influence to deadlock. On the other hand, we need to deliver as many products as possible into system modeled by CROPN. Therefore, we have following Algorithm.

Algorithm 2 *Dtermine the colored capacity of place that is not in interactive subnts in a CROPN.*

Given a CROPN formed by l subnet $v^k(1), v^k(2), \ldots,$and $v^k(l)$.

Input: $A(L, J)$, and $C(v^k(1))(L, J)$,...,$C(v^k(l))(L, J)$.

Output: colored capicity $K(p_L)(b_J)(M)$, where $M \in R(M_0)$, and $(p_L, b_J) \in A(L, J) - C(v^k(1))(L, J) \cup C(v^k(2))(L, J), \ldots, C(v^k(l))(L, J)$, and $K(p)$, $p \in P(v^k)$.

1. *Step 1:*
   Relay $= \emptyset$;
   Relay1 $= \emptyset$;
   For I=1 to l;
   Relay $\leftarrow C(v^k(I))(L, J) \cup$ Relay;
   End for;
   Relay1 $= A(L, J) -$ Relay;
2. *Step 2:*
   While Relay1 $\neq \emptyset$;
   Select a combination (p_L, b_J) from Relay1;

$K(p_L)(b_J)(M) = k(p_L) - \sum_{C(p_L,z)} M(p_L)(b_z)$
$+M(p_L)(b_J)$;
Delete the item (p_L, b_J) from Relay1;
End while;

In summary, we develop a deadlock avoidance procedure as follows such that for every interactive subnet v^k in a CROPN, $U(v^k)$ is realized and the net is made deadlock-free.

Procedure 1 *Deadlock avoidance procedure for FMSs modeled by CROPN.*

- *Step 1:Given a FMS, let us construct its corresponding CROPN model by using the Procedure proposed in [5].*
- *Step 2:Find all interactive subnets in the CROPN [5].*
- *Step 3:For every interactive subnet v^k in the net, let us determine the colored capacity $K(p_L)(b_J)(M)$ for $(p_L, b_J) \in C(v^k)(L, J)$ by using Algorithm 1.*
- *Step 4:Determine the colored capacity $K(p_L)(b_J)(M)$ for $(p_L, b_J) \in A(L, J) - C(v^k(1))(L, J) \cup C(v^k(2))(L, J), ..., C(v^k(l))(L, J)$ by using Algorithm 2.*

3 FMS Example

Consider a flexible manufacturing system (FMS) cell with three machines m_1, m_2, and m_3 processing two part types A and B. A-part has three operations: $m_1 \rightarrow m_2 \rightarrow m_3$. B-part has three operations: $m_3 \rightarrow m_2 \rightarrow m_1$. The capacity of resource m_1, m_2 and m_3 is 1, 1 and 1, respectively.

To make the FMS cell deadlock-free, by Procedure 1, the first step is to construct its corresponding CROPN model as shown in Fig. 3, where the place p_0 represents the central storage with $K(p_0) = \infty$, the place p_x represents the machine m_x with $K(p_x) = 1$ for $x \in \{1, 2, 3\}$. With the modele, we have $C(t_i) = b_i$, where $i \in \{1, 2, ..., 8\}$. Hnece, we have $C(p_0) = \{b_2, b_8\}$, $C(p_1) = \{b_1, b_3\}$, $C(p_2) = \{b_4, b_5\}$, and $C(p_3) = \{b_6, b_7\}$.

The second step is to find all interactive subnets in the net. It is clear that there is only one interactive subnet $v^2 = \{p_1, t_3, p_2, t_5, p_3, t_6, t_4\}$ formed by two PPCs: $v_1 = \{p_1, t_3, p_2, t_4\}, v_2 = \{p_2, t_5, p_3, t_6\}$. Hence, we have the following result:

$$\sum_{C(p_1,z)} M(p_1)(b_z) = M(p_1)(b_3) + M(p_1)(b_1)$$

$$\sum_{C(p_2,z)} M(p_2)(b_z) = M(p_2)(b_4) + M(p_2)(b_5)$$

$$\sum_{C(p_3,z)} M(p_3)(b_z) = M(p_3)(b_6) + M(p_3)(b_7)$$

$$A(L, J) = \{(p_0, b_2), (p_0, b_8), (p_1, b_1), (p_1, b_3), (p_2, b_4), (p_2, b_5), (p_3, b_6), (p_3, b_7)\}$$

$$C(v^2)(L, J) = \{(p_1, b_1), (p_1, b_3), (p_2, b_4), (p_2, b_5), (p_3, b_6), (p_3, b_7)\}$$

In the third step, we first need to do the reachability analysis for the net by using the traditional execution rules Eq(1) and Eq(2). The reachability graph and reachable markings of the net are shown in Fig.4 and Table I, respectively. Note that, in Table I, only markings $M(p)(b_i) > 0$ are presented. It is clear that markings $M_0 - M_3$, $M_5 - M_7$, $M_{10} - M_{12}$, and $M_{15} - M_{19}$ are legal markings, while markings M_4, M_8, and M_9 are impending deadlock markings, markings M_{13}, M_{14} are deadlock markings. Hence, we have $M_{FBM} = \{M_4, M_8, M_9, M_{13}, M_{14}\}$. Further, we have $M_{FBM} **(v^2) = \{M_4(v^2), M_8(v^2), M_9(v^2)\}$, where $M_4(v^2) = (0,1,0,0,1,0)^T$, $M_8(v^2) = (0,0,0,1,1,0)^T$, and $M_9(v^2) = (0,1,1,0,0,0)^T$. Let us rearrange the markings in $M_{FBM}**(v^2)$ as M_1, M_2, M_3, where $M_1 = M_4(v^2)$, $M_2 = M_8(v^2)$, and $M_3 = M_9(v^2)$. Hence, we have the following result:

$$D(v^2)(1,l,j) = \{(p_1,b_3),(p_3,b_6)\}$$

$$D(v^2)(2,l,j) = \{(p_2,b_5),(p_3,b_6)\}$$

$$D(v^2)(3,l,j) = \{(p_1,b_3),(p_2,b_4)\}$$

$$S_1(v^2) = S_2(v^2) = S_3(v^2) = 1$$

$$C(M_1) : M(p_1)(b_3) + M(p_3)(b_6) \leq 1$$

$$C(M_2) : M(p_2)(b_5) + M(p_3)(b_6) \leq 1$$

$$C(M_3) : M(p_1)(b_3) + M(p_2)(b_4) \leq 1$$

$$D(v^2)(A,l,j) = \{(p_1,b_3),(p_2,b_4),(p_2,b_5),(p_3,b_6)\}$$

$$ND(v^2)(L,J) = \{(p_1,b_1),(p_3,b_7)\}$$

With these results, we then determine the colored capacity function of each place in interactive subnets in the net by using the Algorithm 1 as follows:

$$K(p_1)(b_1)(M) = K(p_1) - M(p_1)(b_3) \tag{22}$$

$$\begin{aligned} K(p_1)(b_3)(M) = \min\{&1 - M(p_1)(b_3) - M(p_3)(b_6),\\ &1 - M(p_1)(b_3) - M(p_2)(b_4),\\ &K(p_1) - M(p_1)(b_3) - M(p_1)(b_1)\} + M(p_1)(b_3) \end{aligned} \tag{23}$$

$$\begin{aligned} K(p_2)(b_4)(M) = \min\{&1 - M(p_1)(b_3) - M(p_2)(b_4),\\ &K(p_2) - M(p_2)(b_4) - M(p_2)(b_5)\} + M(p_2)(b_4) \end{aligned} \tag{24}$$

$$\begin{aligned} K(p_2)(b_5)(M) = \min\{&1 - M(p_2)(b_5) - M(p_3)(b_6),\\ &K(p_2) - M(p_2)(b_4) - M(p_2)(b_5)\} + M(p_2)(b_5) \end{aligned} \tag{25}$$

$$\begin{aligned} K(p_3)(b_6)(M) = \min\{&1 - M(p_1)(b_3) - M(p_3)(b_6),\\ &1 - M(p_2)(b_5) - M(p_3)(b_6),\\ &K(p_3) - M(p_3)(b_6) - M(p_3)(b_7)\} + M(p_3)(b_6) \end{aligned} \tag{26}$$

$$K(p_3)(b_7)(M) = K(p_3) - M(p_3)(b_6) \tag{27}$$

Finally, by step 4 in Procedure 1, we have

$$K(p_0)(b_2)(M) = \infty \tag{28}$$

$$K(p_0)(b_8)(M) = \infty \tag{29}$$

With the colored capacity function of each place in the net determined by Procedure 1, the non-colored capacity CROPN net becomes the colored capacity CROPN net. Let us do the reachability analysis for it by using the new execution rule for verifying whether the FMS cell modeled by colored capacity net is made deadlock-free. The result is shown in Fig.5, where markings $\{M_4, M_8, M_9, M_{13}, M_{14}\}$ in Fig.4 and Table I are forbidden and no legal marking is forbidden, imply that the FMS cell modeled by colored capacity CROPN net is made deadlock-free.

Let us analyze the mechanism of making the colored capacity net deadlock-free by using the new execution rule. We consider the following three cases.

1. If we use the non-colored capacity net (traditional CROPN) and the traditional execution rule, then By Fig.4, the transitions t_8 and t_2 can fire at marking M_1 and M_2, respectively. This means that the FMS cell modeled by non-colored capacity net can enter bad state M_4 from state M_1 or M_2. If the colored capacity are introduced into the net, then the net becomes colored capacity net. At marking M_1(Table I), by Eq. (25), we have $K(p_3)(b_6)(M_1) = 0$. This implies that $K(p_3)(b_6)(M_1) = 0 < M(p_3)(b_6) - I(p_3, t_8)(b_8) + o(p_3, t_8)(b_8) = 1$. Hence, by Definition **??**, the transition t_8 can not fire at marking M_1. Similarly, at marking M_2, we have $K(p_1)(b_3)(M_2) = 0$ and $K(p_1)(b_3)(M_2) = 0 < M(p_1)(b_3) - I(p_1, t_2)(b_2) + o(p_1, t_2)(b_2) = 1$. Hence, the transition t_2 can not fire at marking M_2. This means that the FMS cell modeled by colored capacity net cannot enter bad marking M_4 from state M_1 or M_2.
2. At marking M_3, we have $K(p_3)(b_6)(M_3) = 0$ and $K(p_3)(b_6)(M_3) = 0 < M(p_3)(b_6) - I(p_3, t_8)(b_8) + o(p_3, t_8)(b_8) = 1$. Hence, the transition t_8 can not fire at marking M_3. At marking M_5, we have $K(p_1)(b_3)(M_5) = 0$ and $K(p_1)(b_3)(M_5) = 0 < M(p_1)(b_3) - I(p_1, t_2)(b_2) + o(p_1, t_2)(b_2) = 1$. Hence, the transition t_2 can not fire at marking M_5. Therefore, the FMS cell modeled by the colored capacity net cannot enter bad markings M_8 and M_9 from state M_3 and M_5 while the traditional non-colored capacity net can as shown in Fig.4.
3. At marking M_6, we have $K(p_3)(b_6)(M_6) = 0$ and $K(p_3)(b_6)(M_6) = 0 < M(p_3)(b_6) - I(p_3, t_8)(b_8) + o(p_3, t_8)(b_8) = 1$. Hence, the transition t_8 can not fire at marking M_6. At marking M_{11}, we have $K(p_1)(b_3)(M_{11}) = 0$ and $K(p_1)(b_3)(M_{11}) = 0 < M(p_1)(b_3) - I(p_1, t_2)(b_2) + o(p_1, t_2)(b_2) = 1$. Hence, the transition t_2 can not fire at marking M_{11}. Therefore, the FMS cell modeled by colored capacity net cannot enter markings M_{13} and M_{14} from markings M_6 and M_{11}, while the traditional non-colored capacity net can as shown in Fig.4.

In terms of deadlock prevention, to make the FMS cell deadlock-free, compared the work in [5], the control policies of both proposed methods are equal. However, as shown in Table II, 1 control place and 8 arcs are added to the non-colored capacity CROPN net for this example in [5], while no control place and arc needed to be added by using the method proposed in this paper. Thus, the method proposed in this paper is structurally simpler than the work in [5].

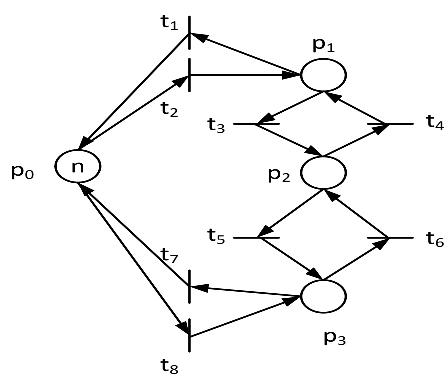

Fig. 2. The CROPN for FMS Example

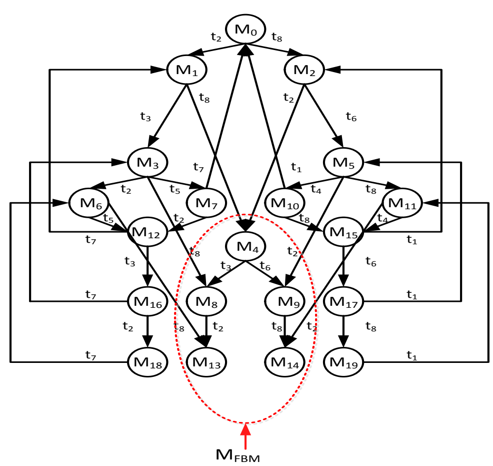

Fig. 3. Reachability graph of the non-colored capacity CROPN in Fig.3

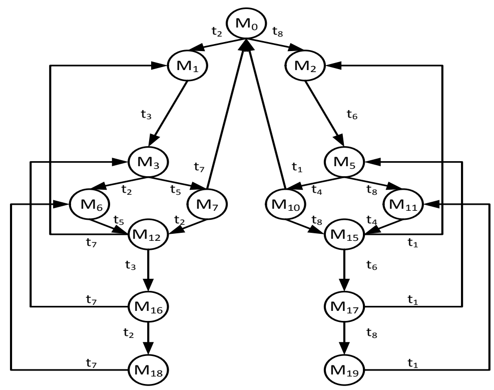

Fig. 4. Reachability graph of the colored capacity CROPN in Fig.3

Table 1. Reachable Markings of the CROPN in Fig. 3

Marking	
M_0	$M_0(p_0)(b_2) = n$ and $M_0(p_0)(b_8) = n$
M_1	$M_1(p_0)(b_2) = n$, $M_1(p_0)(b_8) = n$ and $M_1(p_1)(b_3) = 1$
M_2	$M_2(p_0)(b_2) = n$, $M_2(p_0)(b_8) = n$ and $M_2(p_3)(b_6) = 1$
M_3	$M_3(p_0)(b_2) = n$, $M_3(p_0)(b_8) = n$ and $M_3(p_2)(b_5) = 1$
M_4	$M_4(p_0)(b_2) = n$, $M_4(p_0)(b_8) = n$, $M_4(p_1)(b_3) = 1$ and $M_4(p_3)(b_6) = 1$
M_5	$M_5(p_0)(b_2) = n$, $M_5(p_0)(b_8) = n$ and $M_5(p_2)(b_4) = 1$
M_6	$M_6(p_0)(b_2) = n$, $M_6(p_0)(b_8) = n$, $M_6(p_1)(b_3) = 1$ and $M_6(p_2)(b_5) = 1$
M_7	$M_7(p_0)(b_2) = n$, $M_7(p_0)(b_8) = n$ and $M_7(p_3)(b_7) = 1$
M_8	$M_8(p_0)(b_2) = n$, $M_8(p_0)(b_8) = n$, $M_8(p_2)(b_5) = 1$ and $M_8(p_3)(b_6) = 1$
M_9	$M_9(p_0)(b_2) = n$, $M_9(p_0)(b_8) = n$, $M_9(p_1)(b_3) = 1$ and $M_9(p_2)(b_4) = 1$
M_{10}	$M_{10}(p_0)(b_2) = n$, $M_{10}(p_0)(b_8) = n$ and $M_{10}(p_1)(b_1) = 1$
M_{11}	$M_{11}(p_0)(b_2) = n$, $M_{11}(p_0)(b_8) = n$, $M_{11}(p_2)(b_4) = 1$ and $M_{11}(p_3)(b_6) = 1$
M_{12}	$M_{12}(p_0)(b_2) = n$, $M_{12}(p_0)(b_8) = n$, $M_{12}(p_1)(b_3) = 1$ and $M_{12}(p_3)(b_7) = 1$
M_{13}	$M_{13}(p_0)(b_2) = n$, $M_{13}(p_0)(b_8) = n$, $M_{13}(p_1)(b_3) = 1$, $M_{13}(p_2)(b_5) = 1$ and $M_{13}(p_3)(b_6) = 1$
M_{14}	$M_{14}(p_0)(b_2) = n$, $M_{14}(p_0)(b_8) = n$, $M_{14}(p_1)(b_3) = 1$, $M_{14}(p_2)(b_4) = 1$ and $M_{14}(p_3)(b_6) = 1$
M_{15}	$M_{15}(p_0)(b_2) = n$, $M_{15}(p_0)(b_8) = n$, $M_{15}(p_1)(b_1) = 1$ and $M_{15}(p_3)(b_6) = 1$
M_{16}	$M_{16}(p_0)(b_2) = n$, $M_{16}(p_0)(b_8) = n$, $M_{16}(p_2)(b_5) = 1$ and $M_{16}(p_3)(b_7) = 1$
M_{17}	$M_{17}(p_0)(b_2) = n$, $M_{17}(p_0)(b_8) = n$, $M_{17}(p_1)(b_1) = 1$ and $M_{17}(p_2)(b_4) = 1$
M_{18}	$M_{18}(p_0)(b_2) = n$, $M_{18}(p_0)(b_8) = n$, $M_{18}(p_1)(b_3) = 1$, $M_{18}(p_2)(b_5) = 1$ and $M_{18}(p_3)(b_7) = 1$
M_{19}	$M_{19}(p_0)(b_2) = n$, $M_{19}(p_0)(b_8) = n$, $M_{19}(p_1)(b_1) = 1$, $M_{19}(p_2)(b_4) = 1$ and $M_{19}(p_3)(b_6) = 1$

Table 2. Comparison between the Supervisors Obtained based on CROPN and Colored Capacity CPOPN.

The number of control places of the net	The number of arcs of the net
CROPN: 1	8
Colored Capacity CROPN:0	0

4 Conclusion

In this paper, a novel CROPN concept called colored capacity is proposed. We define the colored capacity of the place, which represents the biggest number of tokens with the same color that the place can retain simultaneously. A method is presented to determine the colored capacity for each place in CROPN by simple calculation. Then all deadlock markings and impending deadlock markings in a flexible manufacturing systems (FMS) modeled by CROPN are forbidden, therefore, this FMS is deadlock-free and we do not need to add control places to the net.

References

1. Yufeng Chen, Gaiyun Liu.: Computation of Minimal Siphons in Petri Nets by Using Binary Decision Diagrams. ACM 12(1), 1-15 (2013)
2. X Li, G Liu, Z Li, N Wu, K Barkaoui.: Elementary Siphon-Based Robust Control for Automated Manufacturing Systems With Multiple Unreliable Resources. IEEE ACCESS 12(1), 1-15 (2013)
3. Liu, Gai, Yun, Chao, Daniel, Yuh, Uzam, Murat.: A merging method for the siphon-based FMS maximally permissive controllers with simpler structures. IMA Journal of Mathematical Control and Information 31(4), 551-573 (2014)
4. G Liu, D Chao, F Yu, Murat.: Control policy for a subclass of Petri nets without reachability analysis. IET Control Theory Applications 7(8), 1131-1141 (2013)
5. H. Chen, N. Q. Wu, and M. C. Zhou.: A novel method for deadlock prevention of ams by using resource-oriented petri nets. Information Sciences 363 178-189 (2016)
6. Y. Feng, K. Xing, M. Zhou, and H. Liu, Liveness analysis and deadlock control for automated manufacturing systems with multiple resource requirements. IEEE Transactions on Systems, Man, and Cybernetics: Systems 1–14 (2017).
7. Z. W. Li, N. Q. Wu, and M. C. Zhou, Deadlock control of automated manufacturing systems based on petri nets—a literature review. IEEE Transactions on Systems Man Cybernetics Part C 42(4), 437–462, (2012).