Landau–Ginzburg/Calabi–Yau Correspondence for FJRW–Potential Taking Gromov–Witten Connection

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Abstract

Any Frobenius manifold associated with a cohomological field theory is analogous to Gromov–Witten connection for Fan–Jarvis–Ruan–Witten Theory where A–model is better termed as Landau–Ginzburg A-model while its mirror symmetry relates to the B-model through a degenerate critical point of Landau–Ginzburg theory with Calabi–Yau manifolds for N=2 as concerned over sigma models relating the two as the same theory.

Keywords – String Theory, Calabi-Yau manifold, Superconformal Algebra, Sigma Model

I. FJRW Potential

For every compact Kahler with vanishing Ricci curvatures the Calabi-Yau manifold has always been associated with string theory and Enumerative geometry where the presence of CY–manifolds are established through threefolds, fourfolds with other higher degree generalizations of both Quartic and quintic verities through multi–homogeneous polynomials.

For any super Lie Algebra of a supersymmetric superconformal N=2 2D category a Wess–Zumino–Witten model can be discovered from a non – linear sigma model for a topological skyrmion for a induced torsion to the former in quartic terms. Thus, this can be defined over the equation for a Riemann manifold $\mathcal{M}$ equipped with a Lie algebra $\mathcal{L}$ over a 3D boundary $\partial_3 \simeq \mathcal{M}$ the action $\mathcal{S}^{(f)}$ can be defined through the field $f : \mathcal{M} \longrightarrow \mathcal{L}$ at level ℓ through ∇ killing form,

$$\mathcal{S}^{(f)} = -\frac{\ell}{8\pi}\int_{\mathcal{M}} 2\pi\ell\mathcal{S}^{(\mathcal{L})}(f) + \nabla\left(f^{-1}\partial^i f, f^{-1}\partial_i f\right) d^2x$$

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The Calabi–Yau/Landau-Ginzburg correspondence defined for the quotient action $\mathcal{Q}$ for the hypersurface $\mathcal{H}$ over a symmetry operator N for Lie group $\mathcal{L}_g$ (where g denotes group) gives the Gromov–Witten for orbifold in quotient form,

$$\mathcal{Q} = [\mathcal{H}_N/\mathcal{L}_g]$$

Partially through the defined action over for a one parameter group that makes a symplectic transformation of $(N, \mathcal{L}_g)$ in group action,

$$\mathcal{S}_g = \mathcal{L}_g/\langle \mathcal{L}_g \cap \delta^\star \rangle$$

The FJRW potential ρ^{FJRW} for the pair $(N, \mathcal{L}_g)$ can be defined via genus $\hat{g} \geq 0$ (for $\hat{g}$ denoting genus) and for the concerned killing vector form ∇; the associated level ℓ for the field $f : \mathcal{M} \longrightarrow \mathcal{L}$ for $\mathcal{L}$ Lie superalgebra; a modified relation $\widetilde{\mathcal{M}}_{\hat{g},\ell}$ for the moduli parameter; for the cohomological field theory suffice,

$$\mathcal{J}^N_{g,\ell} : (\mathcal{H}_N)^{\otimes \ell} \longrightarrow H^*(\widetilde{\mathcal{M}}_{\hat{g},\ell})$$

For each class $\varphi \in \mathcal{H}_f$ where the pre–potential can be given for the Frobenius manifold $\mathcal{H}_f$ through FJRW potential,

$$\rho_{0,\widetilde{N}} = \sum_{\ell \geq 3} \sum_{(\varphi_1, \ldots\ldots, \varphi_\ell) \in \partial_3{}^{(\ell)}} \langle \varphi_1, \ldots \ldots, \varphi_\ell \rangle^N_{\hat{g}} \frac{\alpha_{\varphi_1}, \ldots \ldots, \alpha_{\varphi_\ell}}{\ell!}$$

Suffice the relation,

$$\rho_{\widetilde{N}} = exp\left(\sum_{\hat{g} \geq 0} \hbar^{\hat{g}-1} \sum_{\ell \geq 0} \langle \varphi_1, \ldots \ldots, \varphi_\ell \rangle^{\mathcal{H}}_{g} \frac{\alpha_{i_1\beta_1}, \ldots \ldots, \alpha_{i_\ell \beta_\ell}}{\ell!} \right)$$

II. CALABI – YAU Manifold

For the 5D Quintic 3-fold the Barth – Nieto theorem satisfies the $\mathbb{P}^5 -$ space where the CY–manifold can be established over a 0 – Kodiara Dimensions frameworks. Level – n of a symplectic group having the polarized abelian group with Dim_g can have a complex analytic structures with the preserved norms of Siegel modular variety $A_g(n)$ with $g \geq 7$ having the general type A_g where birationally equivalence can be prescribed by taking $g = 1,3$ and $n = 2$ yielding the compactified structure of CY – manifold with Kodiara Dim-0. Another interesting case that appears is the equivalence yielding the CY-manifold asserts[1,2,3,4],

$$Consani - Scholton\ quintic\ \equiv Non - singular\ quintic\ 3 - fold\ \Longrightarrow CY - manifold$$

Being the algebraic hypersurface with the multiple variable polynomials, the Consani – Scholton defined the $Projectivezed - norms$, CY – manifold comes in 2 – forms,

[1] In the representation theory the G – module $\mathbb{Q}$ over the ring satisfied a $Rigid\ CY\ 3-fold$ where the Dirichlet series via analytic continuation gives rise to the topological group $\mathcal{G}$ under the under the automorphisms via discrete subgroup Γ_{Sub} over the relation,

$$\Gamma_{Sub} \subset \mathcal{G}$$

[2] A$Non-Rigid\ CY\ 3-fold\ of\ Dim_2$ which also satisfied the Dirichlet series (L-form) with the G – module $\mathbb{Q}$ as per the modular form of its 2D representations.

In case of the $Rigid\ CY\ 3-fold$ $\acute{e}$tale cohomology groups can take the $\ell = p$ being the $\ell - adic$ $\mathbf{Q}_{\boldsymbol{\ell}}$ − sheafs with $prime\ \ell \neq 2, 3, 5$ as,

$$H^i(V, \mathbf{Q}_{\boldsymbol{\ell}}) \otimes \mathbf{Q}_{\boldsymbol{\ell}}$$

Being the part of the complex elliptic curve with an algebraic variety, CY – manifold falls under the complex holonomy group of $SU(1)$. CY – being a satisfying property of a Ricci – flat metric, can in essence be justified under 4 – categories (first 3 being non-compact and last being compact):

A. $Enriques\ surfaces$: The real cohomology group associated with this type of surfaces are the result of a vanishing 1st Chern class thus being occupied with non – trivial canonical bundle K equipped with the genus – 0 elliptic surface, with a self-dual lattice of $II_{1,9}$ with Dim_{10} with 3 – sub-characteristics making it as a quasi elliptic surface. Although the Ricci flat metric attached with these types of surface falls under the CY – category of Yau's theorem, but not all the time they behave or rather considered as a proper CY – manifolds. The 2nd cohomology group indicates its norm as,

$$\left(H^2(X, \mathbf{Z}) \xrightarrow{isomorphic\ to} \text{self} - \text{dual lattice of } II_{1,9}\right) \neq SU(2) of\ String\ Landscape$$

B. $Bi-Elliptic\ surfaces$: They are also not considered as proper CY – surface falling under the Enriques – Kodaira classifications of Kodaira 0D – surfaces. Being a compact surface with 10 – classes, the 0D Kodaira setted the equation,

$$8h^{0,1} + 2(2h^{0,1} - b_1) + b_2 = \begin{cases} 22 & K = 0 \\ 10 & otherwise \end{cases}$$

$$\exists\ for\ K\ddot{a}hler\ surface\ 2h^{0,1} = b_1\ iff\ 2h^{0,1} - b_1 = 0\ where\ h^{0,1} \equiv h^{1,0}$$

C. $Abelian\ surfaces$: $\mathbf{Z}^4$ being the fundamental group, this type of surfaces are the non-compact algebraic surface obeying the Riemann bilinear relations where any extensions from $a_{\mathbf{R}}$ satisfies through **R** with the norms (with 4 – Tori structures $\mathbb{T}^4 = S^1 \times S^1 \times S^1 \times S^1$),

$$\frac{a_{\boldsymbol{R}}}{a_{\boldsymbol{R}}} = \frac{(m,n)}{(im,in)}\ \forall\ a_{\boldsymbol{R}}: \boldsymbol{C}^g \times \boldsymbol{C}^g \longrightarrow \boldsymbol{R}\ \exists (m,n) \subseteq\ \boldsymbol{C}^g \times \boldsymbol{C}^g$$

D. K – 3 $surfaces$: The quartic surface of vanishing locus satisfying complex algebraic varaity having the Koidaira 0D in Enriques – Kodaira classifications of $4-classes$ minimal surface existing in complex projective 3 – space as,

$$\mathbb{P}^3 \equiv x^4 + y^4 + z^4 + u^4$$

Satisfying the Kac – Moody infinite dimensional Lie algebras, they represents the Generalized form Cartan Matrix G_2 as,

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{3}{2} & -1 \\ -1 & 21 \end{bmatrix} \equiv A = BC \; where \; if \; a_{ii} > 0 \; and \; C = Positive \; Def.$$

$\Longrightarrow$ a proper Cartan algebra thereby further reproducing the concept of Mirror symmetry in string theory.

Compactified dimensions of the string theory as and when applied over the CY – manifold, then the necessity is to count the number of solutions related to that compact K$\ddot{a}$hler Ricci flat metric. Thus arises the applications of Enumerative geometry which according to the various conjectures are stated as,

1. $Clemens\ conjecture$: In case of the isomorphisms of two algebraic structures, the unicursal curve when taken through a projective line via that isomorphisms being birationally equivalent, then the rational functions can be observed is a 0 – genus, algebraically closed structures being deterrent over the indeterminate functions denoting the polynomial parameterization $n + 1$ through the projective space over any affine parameterizations. Here, monomials are observed over the nonnegative integers through repetitions. Thus for any quintic – 3-fold, when the rational curves are observed over degree d over X, mirror symmetry takes place for any X having the degree $d > 0$ satisfying the Clemens conjecture as,

$$X_{d>0} \subset P^4$$

For the counting of the boundary values of this topological variety, for any vector space V_{n-dim} satisfies the Chow ring $A^*\big(G(k,V)\big)$ with Grassmannian $G_{k-plane}$ gives a finite dimensional vector space $\mathcal{U}$ over a sequence of subspaces $\mathcal{U}_1$, $\mathcal{U}_2$, … $\mathcal{U}_{k-1}$, $\mathcal{U}_k$ having a structure,

$$\{0\} \longmapsto \mathcal{U}_1 \subset \mathcal{U}_2, \ldots \mathcal{U}_{k-1} \subset \mathcal{U}_k = \mathcal{U}$$

Which helps is $Flag$ though the coherent sequence of subspaces through the enumerative counting over the boundary of a cohomology space.

2. $Strominger - Yau - Zaslow\ conjecture$: By splitting a 6D CY – manifold X into pieces and then transformed to $\bar{X}$ can provide a mirror symmetry which in essence relates the Type II-A and Type II-B in terms of T – Duality. Then for every X when there exists a $\bar{X}$, then the result of decomposition of the torus via auxiliary circle ζ_c then the correspondence arises as,

$$\begin{gathered} Parametrization\ of\ the\ circles\ of\ decomposition \\ \Longleftrightarrow \\ Auxiliary\ points\ \zeta\ through\ \zeta_c \end{gathered}$$

Thus, the conjecture takes two points over surjections, $\pi: X \longrightarrow \zeta_c\ and\ \bar{\pi}: \bar{X} \longrightarrow \bar{\zeta}_c$ to a compact manifold of Dim_3 as,

$$Dual\ abelian\ variety\ A^l\ with\ the\ Poincaré\ bundle\ P \longrightarrow A \times A^l:$$

A dense open subset $\zeta_{c_{reg}} \subset \zeta_c$ for every maps $\pi, \bar{\pi}$satisfying the 3 – tori via a special $n - form$ Holomorphic construction over a Lagrangian submanifold $\mathcal{L}_{\otimes}^{M}$ with the CY manifold satisfying the real part δ_{CY^1} and the imaginary part $i\delta_{CY^2}$ as,

$$\delta_{\mathcal{L}}^{CY}\left(\mathcal{L}_{\otimes}^{M}\right): \delta_{CY^1} + i\delta_{CY^2}$$

$\forall\, points\ \zeta \in \zeta_{c_{reg}}$ there exists a dual torus fibres between,

$$\frac{1}{\pi}(\zeta) \Leftrightarrow \frac{1}{\overline{\pi}}(\zeta)$$

$3D\ Singular\ special\ Lagrangian\ submanifolds\ of\ X\ and\ \bar{X}$: For each $\zeta_c \backslash \zeta_{c_{reg}}$ fibres $\frac{1}{\pi}(\zeta)\ and\ \frac{1}{\overline{\pi}}(\zeta)$ exists.

1. $D - Branes \longrightarrow A - Model, B - Model$: The maps with structure preserving properties occupied with Floer – chain groups gives the $A_{\infty} - category$ when the submanifolds of a symplectic manifold say, $\mathcal{C}_0,\ \mathcal{C}_{00}$ over the $Hom\ functor$ over the intersections through $\mathcal{C}_0\ \cap\ \mathcal{C}_{00}$ with the cochain complex preserving the Floer chain groups $FC^*(\mathcal{C}_0,\ \mathcal{C}_{00})$ when morphisms from $\mathcal{C}_0$ to $\mathcal{C}_{00}$ takes the structure, $Hom(\mathcal{C}_0,\ \mathcal{C}_{00})$ in Floer chain groups. Thus, A – Model and B – Model of CY – manifold can be described as,

1.1. $A - Model$: CY – manifolds having the submanifolds containing the D – Branes over special Lagrangian submanifolds having the ½ properties over the spatial dimensions over which they are residing as l, a, v minimizers.

1.2. $B - Model$: Coherent sheaves of the CY – manifolds having complex embedded structures D – Branes that are noted for the Chan – Paton factors or concentrated charges at string endpoints on D – Branes.

Morphisms are established on the open string end points over 2 – Branes satisfying Dirichlet Boundary conditions. The $Homological\ Mirror\ symmetry$ states that,

$$Coherent\ sheves\ of\ CY - Manifold\ \Leftrightarrow Fukaya\ Category\ of\ its\ Mirror$$

III. Gromov – Witten Connection

Novikov ring which has been used as a coefficient over the closed symplectic manifold that when encounters through an extension, Big one, from the ordinary cohomology to the quantum cohomology, then the 'fuzzy' quantum nature could be more precisely described by the 'quantum cup product' inducing the variations from the Riemann sphere connecting two points as analogous to the J – Holomorphic curves or Pseudoholomorphic curves between two points as in symplectic manifolds. Poincare duality as interpreted over the curves makes the associativity over two bubble – manifolds through Gromov connections which later makes a non – local invariance over the Gromov – Witten Invariants. Complex graded Novikov ring being associated over Poincare duality finds its way through the Riemann surface over genus – 0 and marked points – K, through the perturbed Cauchy – Riemann Equation. For the n – point Gromov – Witten Invariants, $n = 3$ is taken for small quantum cohomology and $n \geq 4$ for big quantum cohomological models. This proves essential in establishing the duality (Topological) between Heterotic SO(32), Heterotic E8×E8 with Type II-A supersymmetric strings in M – Theory. Algebraic Topology and symplectic geometry being a vast domain of mathematical studies, when it's a subset called Quantum Cohomology and Gromov – Witten Invariants are considered thereupon [1,4,5,6,7].

The special unitary group $SU(N)$ comes under the gauge groups sufficing the Yang – Mills Theory associated with Lie algebras. The non – abelian Lie groups which portrays the relation between electromagnetic, strong nuclear, weak nuclear forces are reasoned by this Yang – Mills notion to describe the elementary particles behaviours which further got into smaller sets as,

$$U(1) \otimes\ SU(2) \otimes SU(3) \subseteq SU(5) \subseteq SO(10) \subseteq E(8)$$

The simple non – abelian symmetry is given by the Lagrangian where the field strength is denoted by F and Trace – Tr having the form,

$$\mathcal{L}_{gf} = -\frac{1}{2}\mathrm{Tr}(F^2) = -\frac{1}{4}F^{amn}F^{a}_{mn}$$

Using this Yang – Mill functional, the 3 – dimensional closed manifolds that comes under homology groups could be associated with the Floer homology which is also used to study low – dimensional manifolds in symplectic topology and algebraic geometry with its relation to Morse Theory for the infinite dimensional manifolds. A real valued function is preserved on an infinite dimensional manifold when its associated with Floer homology where the quantum field theory of the topological invariants are associated with the Schwarz – type, then the action $\mathcal{S}$ when computed over the manifold M as a form of integration of the 2 – form σ_f and scalar ϑ_{Aux} (where *Aux* is the auxiliary) then it satisfies the basic notions of Chern – Simon theory. The action computed as,

$$\mathcal{S} = \int_M (\sigma_f)(\vartheta_{Aux})$$

The J – holomorphic curve or Pseudoholomorphic curve when satisfies the Cauchy – Riemann equation as a smooth map Δ^{SM} satisfies,

$$\Delta^{SM} \xrightarrow{maps} \begin{array}{c} Riemann\ surface\ to\ almost\ complex \\ masnifold \end{array}$$

The 3 – Tuple relation of the holomorphic curve (j, J, ρ) where the inhomogeneous term ρ satisfied the perturbed Cauchy – Riemann equation as,

$$\bar{\partial}_{j,J} f = \rho$$

The original Cauchy – Riemann equation satisfies with the J – holomorphic curve J_c mapped from the Riemann surface C gives $f : C \longrightarrow J_c$ suffice the result,

$$\begin{aligned} \bar{\partial}_{j,J} f &\coloneqq \frac{1}{2}(df + J \circ df \circ j) \\ &= 0 \qquad where\ df\ is\ complex - linear\ and\ J\ maps\ tangent\ space\ T_X X \end{aligned}$$

This is immensely helpful in studying the Gromov – Witten Invariants with closed domain C the element of Deligne – Mumford moduli space of curves, C with genus – g (0 taken in this paper) and marked points – $n \geq 3$ proved useful in quantum theories regarding the path integral formulations and Supersymmetric string theories with the implementation of the $A - Twist$ over J – holomorphic curves that makes the reducible path integrals of finite dimensional stable maps arises in the Type – IIA Strings in M – Theoty that incorporates the Type – IIB strings which formed F – Theory. Another important element is the 11 – dimensional supersymmetric gravity (SUGRA). All theories comprising M – Theory with Topological (T) and Strong – Weak (S) duality are – Type I, Type IIA, Type IIB, Heterotic SO (32), Heterotic E8×E8, 11-D SUGRA.

For the cohomology groups of manifolds M that are compact without boundary, the isomorphisms over n – dimensions with k – cohomology groups, the canonically defined results over a fundamental class $[M]$ the affine element ϵ_H being in the k^{th} homology group over oriented M makes the mapping as,

$$Isomorphic\ mapping: \ \epsilon_H \in H^k(M) \ \longrightarrow [M] \frown \epsilon_H$$

Thus, dimension being n and cohomology group being k, when this isomorphic mapping over manifold M is computed to $(n-k)^{th}$ homology group then the fundamental results of this duality can be expressed as,

$$H^k(M, \mathbb{Z}) \xrightarrow{isomorphed\ via\ canonically\ defined} H_{n-k}(M, \mathbb{Z})$$

Poincar$\acute{e}$ Duality indded concreted by the proof of the existence of isomorphisms over chain complexes where the image of a homomorphism is mapped to the kernel of the next, thus giving a chain structures. Thus the homomorphism induced connected modules $(\theta_0, \theta_1, \theta_2, \dots \dots \dots, \theta_8 \dots)$ as defined over d_n in the form (θ_*, d_*) the mapping satisfied through,

$$[\theta_n \longrightarrow \theta_{n-1}]: \overset{d_0}{\leftarrow} \theta_0 \overset{d_1}{\leftarrow} \theta_1 \overset{d_2}{\leftarrow} \theta_2 \overset{d_3}{\leftarrow} \theta_3 \overset{d_4}{\leftarrow} \theta_0 \dots \dots$$

Therefore, the cellular homology $(C_{n-i}M)$ and the cohomology $(C^{n-i}M)$ could in principle be defined for the i^{th} cell of the CW – decomposition of the M through two relations,

$$C_iM \bigotimes C_{n-i}M \longrightarrow \mathbb{Z}$$

$$C_iM \longrightarrow C^{n-1}M$$

For the distinguished element of the fundamental class the Poincar$\acute{e}$ complex acts over the homology groups satisfying the duality between homology and cohomology classes over the chain complex for the k^{th} group over n – dimensions as given by,

$$\left(\frown \mu \in H_n(C)\right): H^k(C) \longrightarrow H_{n-k}C \qquad \forall\ 0 \le k \le n$$

Thus, a close relation between the Poincar$\acute{e}$ duality and Thom isomorphism could be given by the n – ranked vector bundle over a restricted μ for any fibre F then the isomorphism is defined over the vector map $\sqcap^0$ to $\sqcap^1$ with the associated ring Ψ suffices,

$$Isomorphism \begin{cases} H^k\left(\sqcap^0; \Psi\right) \longrightarrow H^{k+n}\left(\sqcap^0, \sqcap^0 \setminus \sqcap^1; \Psi\right) \\ I: \ o \xrightarrow{isomorphed\ over\ constructions} o \smile \mu \end{cases}$$

Apart from this, several dualities exist in mathematics and mathematical physics that connects different subgroups within groups where in case of a few specific dualities, one is dual to another and among them in the M – Theory, the T (Topological) duality depicts the winding numbers and S (Strong – Weak) depicts the charges being one dual to the other. The other forms are Hodge duality, Alexander duality, Lefschetz duality. Coming to the T and S duality in M – theory the connections are as follows,

- Type I and Heterotic SO (32) : S - duality
- Type II-A and Type II-B : T - duality ($\mathbb{R}^{8,1} \times S^1$)
- Type II-B : T - duality and S - duality upon itself
- Heterotic SO(32) and Heterotic E8×E8 : T – duality ($\mathbb{R}^{8,1} \times S^1$)

The vibrations of the fundamental strings through the 3 – genus of the complex K$\ddot{a}$hler that is Calabi – Yau manifold with a vanishing Ricci curvature suffices the origin of the particles of nature.

Earlier versions of this theory is Bosonic, however the Polchinsky action incorporated the SUSY over the Bosons with fermions thus enabled a theory of supersymmetric strings which was in 5 – categories, that got glued by Edward Witten via M – They by the addition of 11 – dimensional SUGRA. Topological and Strong – Weak duality interconnects the 5 distinct theories in a unified schemes. Predominately the Type II-B constituting both open and closed strings being also Topological and Strong – Weak dual to its own made up the F – Theory as proposed by Cumrum Vafa.

In supersymmetric gauge theories, the prepotentials and superpotentials over $N = 2$ and $N = 1$constitutes the topological A – Model and B – Model respectively in four and five dimensions. Their combinations relates the dimensional reducibility in topological M – Theory, however, the A – Model and B – Model are related by mirror symmetries over mirror manifolds that asserts the S – Duality in between them through a dimensional extension of NS5 – Brane. Holomorphic duality being a important aspects of B – Models comprises the low – energy effective actions in the quantum background.

The 2 – cycles over a Brane warping defines a deformed conifold that when gets holomorphic to Chern – simons theory makes arrangements over the Kodaira – Spencer gravity via B – Models. The nonvanishing nature of this model always take up complex parts when regarding the resolving of the conifold in B – fields. The quite different approach taken by the A – Model are,

- $U(N)$ Chern –Simons theory makes up the open strings.
- K$\ddot{a}$hler gravity describes the closed strings.

The A and B – Models of the topological string theories contain holomorphic quantities being in a supersymmetric origins of particles are related with Gromov – Witten invariants, mirror symmetry and Chern – Simons theory. Ordinary worldsheet string backgrounds are not topological but when Witten applied the topological twist by mixing rotations over two $U(1)$ symmetries as, Lorentz and R $(symmetries\ with\ N = (2,2)\ with\ U(1) \times U(1))$. This instead makes up an exact BRST quantization where the theory is no more dynamic with all the configurations localized over the precise configurations as topological strings.

Calabi – Yau consists of the 6 – dimensional complex or generalized K$\ddot{a}$hler manifolds. These are the defining source of the scattering amplitudes over the $N = (2,2)$ SUSY sigma models having holomorphic curves over 2 – real dimensions making the correlation functional attachments to the cohomology rings as best described by the Gromov – Witten Invariants. For this A – Model, the N – D2 Branes when stacked up then the Lagrangian submanifolds of the worldsheet supersymmetry establishes over *U(N)* Chern – Simon Theory. The String – Brane interaction can be best described by,

$$K\ddot{a}hler \wedge 3 - Form\ Holomorphic = 0\ for\ ensuring\ configurational\ string\ stability$$

B – Model however takes over warp of submanifolds that coexists with different Brane configurations where low – dimensional Branes are reduced by the reducing dimensions suffices,

- $D(-1) \xrightarrow{warps}$ Holomorphic 0 – Submanifolds
- $D(1) \xrightarrow{warps}$ Holomorphic 2 – Submanifolds
- $D(3) \xrightarrow{warps}$ Holomorphic 4 – Submanifolds
- $D(5) \xrightarrow{warps}$ Holomorphic 6 – Submanifolds

Regarding the dependability of the superpotentials of the Models, the supercoordinates of A – Models are integrals over θ^1 or $\bar{\theta}^+$ while the B – Model takes the conjugate forms of $\bar{\theta}^{\pm}$ where A – Models are not dependent on superpotentials but on twisted superpotentials in a holomorphic ways while the reverse is true for B – Model.

For a nondegenerate 2 – form, closed differentiable manifolds, equipped with the algebraic geometry and differential topology, the symplectic geometry makes sense over this particular form as analogous to Riemannian geometry which relates the angles, lengths of the concerned metric space, while symplectic connects the relations or measures the oriented areas. Thus, over the symplectic norms ω when its integrated over the region S then the required area produced given by,

$$A = \int_S \omega$$

The non-trivial de Rham cohomology group (2nd) relates the symplectic manifold M as the realtion $H^2(M)$. The Riemann geometry when considered spheres between two marked points, the symplectic geometry considers two curves or geodesics as analogous to that spheres called the Pseudoholomorphic / J – Holomorphic curves with the surface area of a minimal magnitude.

K$\ddot{a}$hler manifold is an important tool, more precisely the generalized one, or the complex one with vanishing Ricci curvatures as the Calabi – Yau Manifold, which the symplectic geometry doesn't takes into account, but Mikhail Gromov pointed out that there are an abundance of complex structures permitted within this geometry where the holomorphic transition maps are the requirements to satisfy. Those Pseudoholomorphic curves, that are subjected to a class of invariants called the Gromov – Witten (GW) Invariants which war further deduced by Floer where the homology of a symplectomorphism makes a nondegenerate form as in Floer Homology.

When the cup product of the cohomology theory that too being quantum when preserved over a space, then the enumerative counting results in the formation of a Pseudoholomorphic curve over a symplectic topology. The strong theory that have been trying to make the impossible, almost difficult in making a scale invariant approach between General Relativity and Quantum Theory where out of 10 dimensions the Calabi – Yau contains 6 – compactified dimensions over the regime of the symplected manifold which makes the finite dimensional moduli spaces associated with J – Holomorphic curves incorporates the genus g with the invariants as GW. The Deligne – Mumford moduli spaces with the curve as over genus g and marked points n through the computation of a 4-tuple relation $(X, A, g, n)$where g, n are non-negative integers associated with $2k$ closed with 2D homology class in X, the relations that can be given by the GW invariants as,

$$\bar{M}_{g,n}(X, A) \; with\; stable\; maps\; of\; symplectic\; forms\; into\; X\; of\; class\; A$$

The rational numbers of GW invariants could be calculated from α_i cycles where marked points n are mapped. If the Deligne– Mumford curve have a subset of domain called $\mathbb{D}$, then its homology class could be $\bar{M}_{g,n}(X,A)\big(Or\ much\ better\ to\ write\ \bar{M}_{g,n}\big)$ with homology class $\sum_{n=1}^{n}\alpha_n$, then the rational number suffice over the connected homology of two topology spaces making the Gromov Connections making each bubble attaches via the GW invariants. If we think such 2 – manifolds as one then, naming the manifolds as on with their cross product acan be defined – Let the one manifold be $\cup^1$ and the other be $\cup^2$, the relations are,

$$Connections\ \longrightarrow \cup^1 \times \cup^2$$

$$GW_{g,n}^{X,A}\left(\mathbb{D}, \sum_{n=1}^{n}\alpha_n\right) := GW_{g,n}^{X,A} \times \mathbb{D}$$

Thus if $\cup^1$ in denoted as ∇^{x} and $\cup^2$ as $\nabla^{\mathrm{Y}} \times \nabla^{\mathrm{Z}}$, or if $\cup^1$ in denoted as $\nabla^{\mathrm{Y}} \times \nabla^{\mathrm{Z}}$ and $\cup^2$ as ∇^{x}, the associative property of GW Invariant is expressed as,

$$\nabla^{\mathrm{x}} \times (\nabla^{\mathrm{Y}} \times \nabla^{\mathrm{Z}}) = (\nabla^{\mathrm{x}} \times \nabla^{\mathrm{Y}}) \times \nabla^{\mathrm{Z}} \Longrightarrow Thus\ being\ connected\ and\ invariant$$

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