Optimizing the Investments in Mobile Networks and Subscriber Migrations for a Telecommunication Operator

Adrien Cambier, Matthieu Chardy, Rosa Figueiredo,
Adam Ouorou and Michael Poss

October 5, 2020

Optimizing the investments in mobile networks and subscriber migrations for a telecommunication operator

Adrien Cambier[1,2] ,Matthieu Chardy[1],Rosa Figueiredo[2], Adam Ouorou[1],Michael Poss[3]

[1] Orange Labs, Châtillon, France
adrien.cambier@orange.com
[2] LIA, Avignon Université, Avignon, France.
[3] UMR CNRS 5506 LIRMM, Université de Montpellier, Montpellier, France.

Keywords : *OR in telecommunications, Capacity Expansion, Investments Optimization, Subscriber Dynamic, Bass model, Mixed-Integer Linear Programming*

1 Introduction

New bandwidth-consuming usages and the increase in the number of users induce an exponential growth of mobile traffic [4]. Facing this traffic growth, telecommunication companies are hence pushed to expand their network through important investments (several billion € to improve the mobile network in the last six years, see [6]). This network design problem has two specific features. First, a fast roll-out between mobile networks generation and a highly competitive environment that encourage operators to invest in the newest technology available. Second, telecommunication companies are often both infrastructure operators, planning their network expansion, and service providers, designing the offers proposed to the subscribers.

Through marketing investments, the operator is able to control the demand on its different technologies and avoid over-dimensioning, hence reducing its investments in network design. Reversely, the efficiency of such investments over a given year is also dependent on the network deployment performed in the previous years: subscribers will be more reluctant to shift towards the newest technology if no investments on it have been performed. This trade-off between network and subscribers dynamic can be financially more interesting than a separate optimization of the two problems. Moreover, an operator fixes strategical guidelines on its network for remaining competitive. Some of these indicators (for example, throughput) depend on both network and subscriber performances. Investments in subscribers and networks should hence be jointly optimized over the whole time-horizon of a strategical planning, which is typically 5 years for a telecommunication operator.

This enlightens that subscriber and network dynamics are intertwined. However, to the best of our knowledge, these dynamics have been until now studied separately. To represent the subscriber dynamic, we consider in this work discrete subsidies that represent different possible marketing savings on a new phone required to access the new service. In order to model how the subscribers react to such subsidies, we choose in this article to use the well-studied Bass model (see [1] for the original paper, [5] for a model with the notion of generations, and [2] for applications to telecommunication context). Adapting this model for our context (see the extended version of the article [3] for more details), we assume that subscribers react according to both the subsidy amount proposed and an indicator of the network deployment.

Our contribution is two-fold. First, we provide a modeling of the subscribers' behavior and incorporate it in a linear Mixed-Integer Programming (MIP) formulation. Second, we improve the performance of the model through (i) the strengthening of the MIP with several families of valid inequalities, and (ii) a heuristic algorithm that assigns fixed values of the decision variables (subsidies and coverage) and solves the resulting problem as a knapsack problem.

The remainder of this article is organized as follows. Section 2 introduces our Mobile Master Plan problem (MMP) for two technologies, for which a mixed-integer formulation is provided and linearized. Section 3 introduces the aforementioned valid inequalities and heuristic algorithm. Numerical experiments assess these methods in Section 4. Concluding remarks are given in Section 5.

2 Mathematical model and formulations

In this work, we focus on a framework in which the operator has two network technologies, the current one CG and the newest one NG ($g \in \mathcal{G}$) that the operator aims to deploy. The MMP problem for these two technologies consists in finding the subsidies decisions (amount of subsidy given at each period) and networks decisions (installing NG technology and adding modules for both technologies) for each site (denoted by $s \in \mathcal{S}$), while satisfying load-balancing and capacity constraint at each time-period ($t \in \mathcal{T}$), and strategical guidelines at the end of the time horizon. The amount of subsidy is denoted by $\sigma \in \mathcal{K}$ and the coverage range by $c \in \mathcal{C}$. The reaction of the subscribers is assumed to depend on these two parameters. Parameters and variables used are stored respectively in Table 1 and 2.

Parameters:	
CA_{NG}	cost of adding NG technology on each site
CM_g	cost of adding a module of a technology $g \in \mathcal{G}$ on each site
$\underline{M}^0_{s,g}$	initial number of modules of technology $g \in \mathcal{G}$ on site $s \in \mathcal{S}$
$\overline{M}_g$	technical upper bound on the number of modules of technology $g \in \mathcal{G}$
$Z^0_{s,NG}$	initial presence (yes/no) of NG technology on site $s \in \mathcal{S}$
$U^0_{s,o}$	initial number of subscribers on site $s \in \mathcal{S}$ to technology $o \in \mathcal{G}$
D^t_g	unitary demand of a subscriber served by technology $g \in \mathcal{G}$ at time period $t \in \mathcal{T}$
CAP_g	capacity of adding a module of a technology $g \in \mathcal{G}$
$f_{\sigma,c}$	reaction to the subsidy offered $\sigma \in \mathcal{K}$ under range of coverage interval $c \in \mathcal{C}$
L_c	the lower bound of coverage range $c \in \mathcal{C}$
U_c	the upper bound of coverage range $c \in \mathcal{C}$
$\overline{U}^t_{s,o}$	an upper bound on the total number of subscribers to technology $o \in \mathcal{G}$ on site $s \in \mathcal{S}$ at the end of time period $t \in \mathcal{T}$
α^0	the sites coverage at the beginning of the time horizon
$\underline{\alpha}$	threshold for the coverage (strategic guideline)
$\underline{QoE}$	threshold for the quality of service (strategic guideline)

TAB. 1: Model parameters

Variables:	
$z^t_{s,NG}$	binary variable indicating if NG technology is deployed on site $s \in \mathcal{S}$ at time-period $t \in \mathcal{T} \cup \{0\}$
$m^t_{s,g}$	the total number of modules of technology $g \in \mathcal{G}$ deployed on site $s \in \mathcal{S}$ at the end of time period $t \in \mathcal{T} \cup \{0\}$
$u^t_{s,o}$	the total number of subscribers to technology $o \in \mathcal{G}$ on site $s \in \mathcal{S}$ at the end of time period $t \in \mathcal{T} \cup \{0\}$
$u^t_{s,o,g}$	the total number of subscribers to technology $o \in \mathcal{G}$ on site $s \in \mathcal{S}$ served by technology $g \in \mathcal{G}$ at the end of time period $t \in \mathcal{T} \cup \{0\}$
α^t	redundant variable that denotes the NG sites coverage (fraction of sites where NG technology is deployed) at the end of the time period $t \in \mathcal{T}$, which is equal to $\frac{\sum_{s \in S} z^t_{s,NG}}{N_S}$
$\delta^t_{\sigma,c}$	binary variable indicating if α^t belongs to range $c \in \mathcal{C}$ and subsidy offered at time period $t \in \mathcal{T}$ is $\sigma \in \mathcal{K}$

TAB. 2: Model variables

Consequently, the MMP problem can be modeled as follows:

$$\begin{aligned} \min \sum_{\sigma \in \mathcal{K}} \sum_{c \in \mathcal{C}} \sum_{s \in S} \sigma f_{\sigma,c} \delta^t_{\sigma,c} u^{t-1}_{s,CG} + \sum_{s \in S} \sum_{g \in \mathcal{G}} CM_g (m^{\bar{t}}_{s,g} - M^0_{s,g}) \\ + \sum_{s \in S} CA_{NG} (z^{\bar{t}}_{s,NG} - Z^0_{s,NG}) \end{aligned} \quad (1)$$

$$\text{s.t. } m^t_{s,CG} \leq \overline{M}_{CG} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T}, \quad (2)$$

$$m^t_{s,NG} \leq \overline{M}_{NG} z^t_{s,NG} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T}, \quad (3)$$

$$m^{t-1}_{s,g} \leq m^t_{s,g} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T},\ \forall\, g \in \mathcal{G}, \quad (4)$$

$$u^t_{s,NG} = u^t_{s,NG,CG} + u^t_{s,NG,NG} \qquad \forall\, s \in \mathcal{S}\ \forall\, t \in \mathcal{T}, \quad (5)$$

$$u^t_{s,NG,CG} \leq \overline{U}^t_{s,NG}(1 - z^t_{s,NG}) \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T}, \quad (6)$$

$$D^t_{CG}(u^t_{s,CG} + u^t_{s,NG,CG}) \leq CAP_{CG} m^t_{s,CG} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T},\ \forall\, g \in \mathcal{G}, \quad (7)$$

$$D^t_{NG} u^t_{s,NG,NG} \leq CAP_{NG} m^t_{s,NG} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T},\ \forall\, g \in \mathcal{G}, \quad (8)$$

$$u^t_{s,CG} = u^{t-1}_{s,CG} - \sum_{\sigma \in \mathcal{K}} \sum_{c \in \mathcal{C}} f_{\sigma,c}\, \delta^t_{\sigma,c} u^{t-1}_{s,CG} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T}, \quad (9)$$

$$u^t_{s,NG} = u^{t-1}_{s,NG} + \sum_{\sigma \in \mathcal{K}} \sum_{c \in \mathcal{C}} f_{\sigma,c} \delta^t_{\sigma,c} u^{t-1}_{s,CG} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T}, \quad (10)$$

$$\sum_{s \in \mathcal{S}} u^{\bar{t}}_{s,NG,NG} \geq \underline{QoE}\Big(\sum_{s \in \mathcal{S}} U^0_{s,NG} + U^0_{s,CG}\Big), \quad (11)$$

$$\alpha^{\bar{t}} \geq \underline{\alpha}, \quad (12)$$

$$\sum_{\sigma \in \mathcal{K}} \sum_{c \in \mathcal{C}} \delta^t_{\sigma,c} = 1 \qquad \forall\, t \in \mathcal{T}, \quad (13)$$

$$\sum_{\sigma \in \mathcal{K}} \delta^t_{\sigma,c} \leq 1 + U_c - \alpha^{t-1} \qquad \forall\, t \in \mathcal{T},\ \forall\, c \in \mathcal{C}, \quad (14)$$

$$\sum_{\sigma \in \mathcal{K}} \delta^t_{\sigma,c} \leq 1 + \alpha^{t-1} - L_c \qquad \forall\, t \in \mathcal{T},\ \forall\, c \in \mathcal{C}, \quad (15)$$

$$u^0_{s,o} = U^0_{s,o} \qquad \forall\, s \in \mathcal{S},\ \forall\, o \in \mathcal{G}, \quad (16)$$

$$m^0_{s,g} = M^0_{s,g} \qquad \forall\, s \in \mathcal{S},\ \forall\, g \in \mathcal{G}, \quad (17)$$

$$z^0_{s,NG} = Z^0_{s,NG} \qquad \forall\, s \in \mathcal{S}, \quad (18)$$

$$\alpha^t N_S = \sum_{s \in S} z^t_{s,NG} \qquad \forall\, t \in \mathcal{T} \cup \{0\}, \quad (19)$$

$$m_{s,g}^t \in \mathbb{Z} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T} \,\cup\, \{0\},\ \forall\, g \in \mathcal{G}, \tag{20}$$
$$m_{s,g}^t \in \mathbb{Z} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T} \,\cup\, \{0\},\ \forall\, g \in \mathcal{G}, \tag{21}$$
$$z_{s,NG}^t \in \{0,1\} \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T} \,\cup\, \{0\}, \tag{22}$$
$$u_{s,o}^t \geq 0 \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T} \,\cup\, \{0\},\ \forall\, o \in \mathcal{G}, \tag{23}$$
$$u_{s,o,g}^t \geq 0 \qquad \forall\, s \in \mathcal{S},\ \forall\, t \in \mathcal{T},\ \forall\, o, g \in \mathcal{G}^2, \tag{24}$$
$$\delta_{\sigma,c}^t \in \{0,1\} \qquad \forall\, t \in \mathcal{T},\ \forall\, \sigma \in \mathcal{K},\ \forall\, c \in \mathcal{C}. \tag{25}$$

We denote this formulation by $\mathcal{M}$. The objective function (1) minimizes both subscribers migration costs and network investments. The first term stands for the offered subsidies (user upgrades), the second term for adding new modules for increasing the capacity (densification), and the third term for the deployment of the newest technology NG (coverage extension).

Constraints (2)–(4) refer to the network dynamic. Constraints (2)–(3) define the upper bounds on the numbers of modules for each technology deployed on each site. These constraints also ensure that if a technology is not deployed, no corresponding modules can be added. Constraints (4) impose the number of modules of each technology to be non-decreasing during the time horizon.

Constraints (5)–(8) are the network dimensioning constraints, in charge of making the link between the network dynamic and the subscriber dynamic. Constraints (5) and (6) ensure the load-balancing rule. Constraints (7) and (8) are the capacity constraints: the installed capacities of each technology on each site have to be sufficient for providing services for all users located at this site and having to be served by this technology. They also ensure the technical incompatibility stating that CG subscribers cannot be served by NG technology.

Constraints (9)–(10) are the subscriber dynamic constraints. They define the total number of subscribers to CG and NG technologies at each site and each time period, taking into account former CG subscribers who decide to shift to NG technology, thanks to subsidies and coverage improvements.

Constraints (11)–(12) stand for the model strategic guidelines and refer to the end of the time horizon. Constraint (11) ensures the threshold of subscribers covered by the newest technology is met. The indicator is proportional to the quality of experience which measures the percentage of users having access to the new technology throughput. Constraint (12) imposes that the threshold on the number of sites on which NG is deployed is met.

Constraints (13) ensure that one and only one subsidy from the set $\mathcal{K}$ is offered at each time period, the case when no subsidy is given being represented by $\sigma = 0$. Constraints (14) and (15) ensure that, for each time period, variables $\delta_{\sigma,c}^t$ are set according to the coverage. Constraints (14) (resp. (15)) set all δ related to a range to 0 if the coverage is greater (resp. smaller) than the upper (resp. lower) bound of the range.

Constraints (16)– (18) refer to the initial conditions and constraints (19)–(25) define the domain of all variables in the formulation.

To linearize this formulation, we introduce variables $\pi_{\sigma,c,s}^t = \delta_{\sigma,c}^t u_{s,CG}^{t-1}$, $\forall\, t \in \mathcal{T}$, $\forall\, s \in \mathcal{S}$, $\forall\, \sigma \in \mathcal{K}$, $\forall\, c \in \mathcal{C}$ and use the classical linearization of the product of a binary variable by a continous one.

3 Strengthening of the formulation and upper bound

We designed several families of valid inequalities in order to improve our formulation.

Proposition 1 *The following sets of inequalities are valid for formulation $\mathcal{M}$:*

$$z_{s,NG}^t \leq z_{s,NG}^{t+1} \qquad \forall\, t \in \mathcal{T},\ \forall\, s \in \mathcal{S}, \tag{26}$$
$$\sum_{\sigma\in\mathcal{K}} \sum_{c'<c} \delta_{\sigma,c'}^{t'} \leq 1 - \sum_{\sigma\in\mathcal{K}} \sum_{c'\geq c} \delta_{\sigma,c'}^{t} \qquad \forall\, t,t' \in \mathcal{T}^2,\ \forall\, c \in \mathcal{C},\ \forall\, s \in \mathcal{S}, \tag{27}$$

$$\lceil N_S L_c \rceil \sum_{\sigma \in \mathcal{K}} \delta^t_{\sigma,c} \leq \sum_{s \in \mathcal{S}} z^t_{s,NG} \qquad \forall\, t \in \mathcal{T},\ \forall\, c \in \mathcal{C}, \tag{28}$$

$$\left\lceil \frac{D^t_{NG} U^t_{s,NG}}{CAP_{NG}} \right\rceil z^t_{s,NG} \leq m^t_{s,NG} \qquad \forall\, t \in \mathcal{T},\ \forall\, s \in \mathcal{S}, \tag{29}$$

$$m^t_{s,NG} \leq \max(M^0_{s,NG}, \left\lceil \frac{D^t_{NG} \overline{U}^t_{s,NG}}{CAP_{NG}} \right\rceil) z^t_{s,NG} \qquad \forall\, t \in \mathcal{T},\ \forall\, s \in \mathcal{S}, \tag{30}$$

$$\sum_{\sigma \in \mathcal{K}} \sum_{c \in \mathcal{C}} \pi^t_{\sigma,c,s} = u^{t-1}_{s,CG} \qquad \forall\, t \in \mathcal{T},\ \forall\, s \in \mathcal{S}. \tag{31}$$

Remark 1 *Constraints* (26)–(28) *refer to the fact the coverage is increasing. Constraints* (29) *and* (30) *are based on upper and lower bounds of the number of subscribers. Constraints* (31) *are the direct application of RLT technique (multiplication of constraints* (13) *by variable* $u^{t-1}_{s,CG}$*, for each period* $t \in \mathcal{T}$ *and each site* $s \in \mathcal{S}$*), significantly reinforcing the linearization (see [7]). Details are given in [3].*

We also propose a heuristic approach for the MMP problem, tackling the main difficulties of formulation $\mathcal{M}$ which come from the non-linearity of the user dynamic. Consequently, our heuristic fixes σ^t (the amount of the subsidy offered to CG subscribers) and c^t (the coverage range) to specific values $\tilde{\sigma}^t \in \mathcal{C}$ and $\tilde{c}^t \in \mathcal{K}$, for each period $t \in \mathcal{T}$, and solve the resulting problem optimally. Let us now denote the problem where $\sigma^t = \tilde{\sigma}^t$ and $c^t = \tilde{c}^t$ as the MMP$(\tilde{\sigma}, \tilde{c})$. Our heuristic is described by Algorithm 1.

Algorithm 1: Heuristic algorithm

INPUT : $\mathcal{W} \subset (\mathcal{K} \times \mathcal{C})^{\bar{t}}$;
for $(\tilde{\sigma}, \tilde{c}) \in \mathcal{W}$ **do** $cost(\tilde{\sigma}, \tilde{c}) \leftarrow$ optimal solution cost of $MMP(\tilde{\sigma}, \tilde{c})$;
return $\min_{(\tilde{\sigma}, \tilde{c}) \in \mathcal{W}} cost(\tilde{\sigma}, \tilde{c})$

We denote by C the highest range of coverage. For tractability reasons we assume that

$$\tilde{c} = (c_{init}, C, \ldots, C) \text{ for each } (\tilde{\sigma}, \tilde{c}) \in \mathcal{W}. \tag{32}$$

Assumption (32) implies that all network investments are performed in the first time period, enabling us to remove index t on variables m and z (see[3] for the details). This assumption and the knowledge of the subsidies decisions enable us to considerably simplify the solution of MMP$(\tilde{\sigma}, \tilde{c})$ as summarized next. Number of subscribers $U^t_{s,CG}$ and $U^t_{s,NG}$ are now fixed constants. We reformulate capacity and load-balancing constraints as a set of constraints depending only on variables $m_{s,g}$ and $z_{s,NG}$. We conclude that we can use these constraints to compute a closed form for the optimal value taken by variables $m_{s,g}$ depending on the values taken by variables $z_{s,NG}$, which enable us to remove variables m. We know and introduce indeed for each site $s \in \mathcal{S}$ the number of modules installed at the end of the time horizon:

- for CG technology:
  - if NG is already installed ($Z^0_{s,NG} = 1$): $\tilde{m}^{AI}_{s,CG} = \max\left(\left\lceil \frac{\max_{i \in \mathcal{T}} D^i_{CG} U^i_{s,CG}}{CAP_{CG}} \right\rceil, M^0_{s,CG}\right)$ (only CG subscribers are served by CG technology),
  - if NG is not installed over the time horizon ($z_{s,NG} = 0$): $\tilde{m}^{NI}_{s,CG} = \max\left(\left\lceil \frac{D^{\bar{t}}_{CG} U^{TOT}_s}{CAP_{CG}} \right\rceil, M^0_{s,CG}\right)$ (all subscribers are served by CG technology),
- for NG technology when it is installed ($z_{s,NG} = 1$):

$$\tilde{m}_{s,NG} = \max\left(\left\lceil \frac{D^{\bar{t}}_{NG} U^{\bar{t}}_{s,NG}}{CAP_{NG}} \right\rceil, M^0_{s,NG}\right).$$

Note that if, on a site $s \in \mathcal{S}$, $\tilde{m}^{AI}_{s,CG} > \overline{M}_{CG}$ then $z^{\bar{t}}_{s,NG} = 1$ and if $\tilde{m}_{s,NG} > \overline{M}_{NG}$ then $z^{\bar{t}}_{s,NG} = 0$. If both happen, the instance of $MMP(\tilde{\sigma}^t, \tilde{c}^t)$ is infeasible.

We define by $\mathcal{S}_{CG} \subset \mathcal{S}$ the subset of the sites where NG is not installed at the beginning of the time horizon. We also remove from set $\mathcal{S}_{CG}$ the sites for which we already know if we will install NG technology or not due to infeasibilities. Note that we only have to solve the problem on sites of set $\mathcal{S}_{CG}$. All modules costs for sites of set $\mathcal{S} \setminus \mathcal{S}_{CG}$ are labeled as constant $netcost$. Constant N_{inst} labels the number of sites where we know NG is installed at the end of the time horizon. For each site $s \in \mathcal{S}_{CG}$, $C1_s = CA_{NG} + \tilde{m}_{s,NG}CM_{NG}$ denotes the cost implied by deciding to install NG technology and $C2_s = \left(\tilde{m}^{NI}_{s,CG} - \tilde{m}^{AI}_{s,CG}\right) CM_{CG}$ the cost implied by deciding the opposite. Therefore, the MMP($\tilde{\sigma}, \tilde{c}$) can be reformulated as the following bidimensional knapsack problem:

$$\min \sum_{s \in \mathcal{S}_{CG}} C1_s z^{\bar{t}}_{s,NG} + C2_s(1 - z^{\bar{t}}_{s,NG}) + upgradecost + netcost \tag{33}$$

$$s.t. \sum_{s \in \mathcal{S}_{CG}} U^{\bar{t}}_{s,NG} z^{\bar{t}}_{s,NG} \geq QoE \sum_{s \in \mathcal{S}} U^{TOT}_s - \sum_{s \in \mathcal{S} \setminus \mathcal{S}_{CG}} U^{\bar{t}}_{s,NG}, \tag{34}$$

$$\sum_{s \in \mathcal{S}_{CG}} z^{\bar{t}}_{s,NG} \geq \max(\underline{\alpha}, L_C N_S) - N_{inst}, \tag{35}$$

$$z^{\bar{t}}_{s,NG} \in \{0,1\} \quad \forall s \in \mathcal{S}_{CG}. \tag{36}$$

Proposition 2 *The MMP($\tilde{\sigma}, \tilde{c}$) can be solved in* $\mathcal{O}(NS + |\mathcal{S}_{CG}|^2(CA_{NG} + \overline{M}_{NG}CM_{NG}))$ *when* $\tilde{c}^t = C, \; \forall\, t \in \{2, \ldots, \bar{t}\}$.

Proof: This result comes from the fact that the problem becomes a knapsack problem which can be solved by a dynamic programming algorithm. □

4 Numerical experiments

The purpose of this section is two-fold:

1. Assessing the scalability of formulation $\mathcal{M}$ and the relevance of the valid inequalities on real-life instances.

2. Assessing the performance of the heuristic from Section 3, and in particular the quality of the best solution found when the heuristic solution is used as MIPstart.

TAB. 3: Final gaps for 12 instances (4 sizes, 3 densities) tested with each family of valid inequalities

Instance		Final gap							
N_S	density	$\mathcal{M}$	+ (26)	+ (27)	+ (31)	+ (28)	+ (29)	+ (30)	+ (26)-(30)
50	R	6.00	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	S	5.97	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	U	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	R	10.62	4.52	3.13	3.59	6.33	*1.90*	6.46	**1.14**
	S	14.92	4.81	3.17	*2.55*	3.60	3.67	4.37	**2.50**
	U	7.18	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
150	R	16.71	9.65	5.62	*4.43*	7.91	6.87	6.87	**4.12**
	S	20.72	10.49	4.34	*3.91*	4.13	10.45	7.71	**3.47**
	U	7.22	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
200	R	16.73	10.53	7.99	*3.59*	10.55	8.84	9.80	**2.48**
	S	18.85	10.06	12.22	**2.77**	11.03	13.59	12.28	*3.06*
	U	7.73	1.83	1.01	*0.49*	1.96	1.86	2.27	**0.07**

Both tests are performed on the real-life instances which are detailed in [3].

First, for assessing the valid inequalities, we extract from 50 to 200 sites of the real life instances, on which we test different scenarios. Instances features are displayed in Table 3, column "N_S" standing for the number of sites and column "density" standing for the density scenario (rural R, suburban S or urban U). The final gap is computed with and without the valid inequalities from Section 3. More precisely, we test formulations ($\mathcal{M}$), ($\mathcal{M}$ + each family of valid inequality) and ($\mathcal{M}$ + all families of valid inequalities). The corresponding final gap is displayed in Table 3. The time-limit is set to 1800 seconds. The best value for the final gap is in bold and the second best is in italic. We observe that inequalities (31) are the most efficient ones for reducing the final gap (see Table 3), but combining with the other valid inequalities is the best strategy. These results hence highlight the relevance of our valid inequalities.

Second, we assess the interest of the heuristic of Section 3 for finding feasible solutions on 10 real-sized instances corresponding to different French territorial divisions. Two regions: Bretagne (divided into 4 departments: Finistère, Côtes d'Armor, Morbihan and Ile et Vilaine) and part of Pays de la Loire (divided into 3 departments: Mayenne, Sarthe, Maine et Loire) are hence considered. Instances features are displayed in Table 3, column "Ter. Div" refer to the territorial division considered, column "N_S" to the number of sites and column "α^0" to the initial coverage. As mentioned previously, we look for a solution where the range ("high") is reached over the first period and we enumerate the subsidy amount $\tilde{\sigma} \in \mathcal{K}$ (ten possibilities if we do not restrict) at each time period (five) so as to solve each resulting problem MMP($\tilde{\sigma}, \tilde{c}$) with the pseudo-polynomial model provided in Section 3. This means that we have to solve 10^5 MMP($\tilde{\sigma}, \tilde{c}$) problems, which we cannot afford. Hence, in our heuristic, we enumerate all $\tilde{\sigma} \in \{0, 100, 150, 200, 250\}$ and we solve the problem only if the reaction is sufficient to reach the threshold. This gives at most $5^5 = 3125$ MMP($\tilde{\sigma}, \tilde{c}$) problems to solve.

In a second step, the solution found by the heuristic is used as an initial solution (MIPstart) for the solver. The time limit given to the solver is 7200 seconds minus the time of the heuristic. This limit has been chosen so as to be comparable with the MIP solving in 7200 seconds without an initial solution provided.

Results are presented in Table 4. The column "heuristic" stands for the algorithm described above, the column "MIP" for the MIP without initial solution provided and the column "MIP-start" for the MIP with the heuristic solution provided as MIPstart. The column "gapMIP" reports the gap between the heuristic value and the best solution found by the MIP. Column "f-gap" refer to the final gap and "sol" to the value of the best solution found.

TAB. 4: Solution and final gap for large instances

Instance			heuristic		gapMIP	MIP		MIPstart	
Ter. Div.	N_S	α^0	sol	time		sol	f-gap	sol	f-gap
Finistère	210	36	13406	505	0	13406	4.91	13406	3.14
Côtes d'Armor	149	29	10420	617	0	10420	1.94	10420	1.64
Morbihan	168	38	11178	551	0	11178	3.32	11178	2.06
Ile et Vilaine	214	43	12115	776	0	12115	2.73	12115	2.32
Mayenne	73	31	4879	127	0	4879	0.92	4879	0.00
Sarthe	116	33	7729	186	0	7729	2.38	7229	0.00
Maine et Loire	145	28	9877	221	0	9877	4.06	9877	0.72
Bretagne	741	37	47106	3197	-63.41	128109	100.00	47106	3.51
Pays de la Loire	334	30	22467	4113	-0.01	22470	4.26	22464	3.01
Full instance	1075	35	69497	5997	-59.00	169968	92.80	69497	5.42

We observe that the heuristic finds very good quality solutions for all instances in two hours of total computation time (heuristic + MIPstart). For the two largest instances, these solutions are far better (around 60% savings) than the best solution found without heuristic by the MIP in two hours. These solutions are not improved afterward by CPLEX but using the heuristic as MIPstart enables us to obtain the proof of convergence for the two smallest instances and to have all final gaps below 6%.

5 Conclusions

In this work, we introduced the problem of multi-year investment planning for a telecommunication operator. Encompassing several real aspects faced by operators, our problem consists in optimizing network and subscriber dynamics under capacity and strategic constraints. In particular, we have modeled the fraction of subscribers adopting a new technology as depending on the coverage of that technology. In addition, the operator can provide subsidies to encourage the subscribers to shift faster to that technology. We have provided a non-linear MIP formulation for this problem, which we linearize and reinforce with several sets of valid inequalities. Computational tests have been made for a real 3G/4G case-study. The efficiency of the valid inequalities in improving the performances has been underlined, as well as the relevance of the branch-and-bound procedure performed on the tightened MIP for solving scaled real-life instances. For the largest instances, the solver struggles to find a feasible solution and so we have proposed a heuristic to find a good-quality primal feasible solution. This heuristic is based on fixing the upgrade parameters (subsidies amount and state of coverage) and enables us to find very good quality solutions while running much faster than the branch-and-bound procedure on the exact MIP.

References

[1] Frank M. Bass. A new product growth for model consumer durables. *Management Science*, 15(5):215–227, 1969.

[2] Frank M. Bass. Comments on "a new product growth for model consumer durables the bass model". *Management Science*, 50(12_supplement):1833–1840, 2004.

[3] Adrien Cambier, Matthieu Chardy, Rosa Figueiredo, Adam Ouorou, and Michael Poss. Optimizing the investments in mobile networks and subscriber migrations for a telecommunication operator. https://hal.archives-ouvertes.fr/hal-02019853, October 2019.

[4] CISCO. Cisco visual networking index: Global mobile data traffic forecast update, 2016–2021 white paper, 2017. https://www.cisco.com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/mobile-white-paper-c11-520862.html (Accessed 10/30/2018).

[5] John A. Norton and Frank M. Bass. A diffusion theory model of adoption and substitution for successive generations of high-technology products. *Management Science*, 33(9):1069–1086, 1987.

[6] Orange. Mobile network worlwide activity (orange group webpage), 2018. https://www.orange.com/en/Group/Activities/Networks/Folder/Mobile-network (Accessed 10/30/2018).

[7] H.D. Sherali and W.P. Adams. *A Reformulation-Linearization Technique for Solving Discrete and Continuous Nonconvex Problems*. Nonconvex Optimization and Its Applications. Springer US, 1998.