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TIME DEPENDENCE OF STOCK PRICE STUDIED BY A STATISTICAL PHYSICS APPROACH

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We study by Monte Carlo (MC) simulations the time evolution of stock markets using a statistical physics approach. We consider an ensemble of agents in which each of them decides to sell or to buy a good according to several factors acting on him/her: the majority of the neighbors, the market atmosphere, the variation of the price and some specific measure applied at a given time. The second part of the work deals with the price variation using a time-dependent mean-field theory. By supposing that the sellers and the buyers belong to two distinct communities with their characteristics different in both intra-group and inter-group interactions, we find the price oscillation with time.

Statistical physics models treat large ensembles of particles interacting with each other via interactions of various kinds. Much has been achieved in the understanding of their properties in different situations[1]. In particular, collective behaviors such as those observed in phase transitions and in correlated dynamics have been demonstrated as consequences of the microscopic underlying interactions between particles, the space dimension and the system symmetry[2].

The interaction of an individual with his neighbors under the social atmosphere (peace or unrest) has been used in many domains such as politics and sociology[3]. Quantitative sociodynamics using stochastic methods and models of social interaction processes has been investigated. The validity of statistical laws in physics and social sciences has been examined. In particular, using interacting social networks, social conflicts have been studied using models of statistical physics[4,5] with the mean-field theory and MC simulations.

Statistical physics used to study economic problems has been called "econophysics". As in sociophysics, the correspondence between physical quantities and parameters can be interpreted in economic terms.

In the present work, we consider the problem of the evolution during the time of the price in a commodity market by examining the effects of several parameters. There is an enormous number of works dealing with the stock markets. T. Lux has given a review[6] on stochastic models borrowed from statistical physics using microscopic interactions between a larger number of traders to deduce macroscopic regularities of the market, independent of microscopic details. On the same line, R. Cont[7] has shown various statistical properties of asset returns with emphasis on properties common to a wide variety of markets and instruments. We need a general model which does not use empirical rules in the course of calculation. This motivates our present work.

We introduce here a new model inspired from models of statistical physics but with modifications where they should be for econophysics. Our model contributes to the family of already abundant stock price variation models, but as seen below, it gives rise to new features not seen before in the price variation. Our model uses an assembly of agents interacting with each other and with the economic temperature as well as with economic measures taken by the government. This model is studied using MC simulations and by the time-dependence mean-field theory.

The aim of this work is to study the variation of the price of a good under the effects of the interaction strength between agents, the economic environment (economic crisis, economic agitation, ...) and boosting measures taken by the government.

We note that the change of price directly yields gross returns. Our objective is to show that the price dynamics which can be used to deduce the stylized facts. The first one is the case where the price of time t is independent of that at time $(t-1)$, real-time fluctuations yields the time average over a lapse of time independent of the time.

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This stylized fact is described by our model. The second stylized fact concerns autocorrelated fluctuations such as volatility clustering.
We will show in particular that a specific measure taken by the government or an economic organization to boost or to lower the market price can have a long-lasting effect. The absence of a model to explain why a temporary shock can generate persistent effects has motivated our present work. Our model presented below explains this stylized fact.
In the simulation, we assume that each agent has several states which can be represented by a spin. In this work we take a spin with an integer amplitude S but, unlike the Ising spin, it has $2S+1$ states: -S, $-S+1$, $-S+2$,..., 0, 1, 2,..., S. For $S=1$, one has thus three states $M=-1,0,1$. In econophysics, a spin state is an economic action of an agent: selling, buying or waiting, for example. A spin σ_i describes the state of an agent i. σ_i has several states, for example $\sigma_i=-1,0,1$. Each state represents an action: let's define $\sigma_i=1$ for buying action, $\sigma_i=-1$ for selling, and $\sigma_i=0$ for waiting. The action of agent i is the result of different mechanisms described in what follows. If $S=2$, each agent has $2S+1=5$ states from -2S to 2S, namely -2, -1, 0, 1, 2, expressing respectively a strong desire to sell (-2), a moderate desire to sell (-1), waiting (0), a moderate desire to buy (+1) and a strong desire to buy (+2). Therefore, the more states the more degrees of will.
In econophysics, the market temperature T expresses the economic atmosphere resulting from many factors such as political situation, economic crisis and international conflicts. In econophysics, low T means stability, high T means unstable situation. Agents interact with each other via an imitation interaction J which leads to some order or collective structure, in contrast with T which favors disorder. Therefore, there is a competition between T and J.
We consider an ensemble of N agents. We suppose that the energy associated with agent i is governed by three terms which govern the market dynamics. The first term is the tendency of an agent to imitate the majority of his neighbors. The second term describes the fact that an agent will react to the price variation: if the price increases he would like to sell to get benefits, if the price decreases he would buy to earn benefits in the future. The third term is an external stimulus (governmental measures) to change the market tendency. No need to say, these three mechanisms describe essentially the price dynamics, but there may be other secondary motivations of an agent to buy or to sell. To simplify the analysis we take only the three terms in Eq. (1) below and deploy the efficient Monte Carlo method (and the mean-field theory) to study the dynamics stemming from it:

$$E_i(t)=\sigma_i(t)\ [-J\sum_j \sigma_j(t-1)] + a\ \sigma_i(t)\ [N(\mathrm{up},t-1)-N(\mathrm{down},t-1)] - H\ \sigma_i(t) \tag{1}$$

where $a>0$. Let us explain each term of the above equation:

i) The first term represents the sum of the influence on $\sigma_i(t)$ at the time t by his neighbors' attitudes at the time $(t-1)$. For simplicity, we assume here all neighbors have the same interaction J with $\sigma_i(t)$. This will not alter general aspects of the model. The agent imitates the majority of his neighbors

ii) In addition to the influence of neighbors given by the first term, $E_i(t)$ also depends on the price tendency given by the second term: let $N(\mathrm{up},t-1)$ ($N(\mathrm{down},t-1)$) be the number of the people who wants to buy (sell) at the previous time $t-1$. The price is proportional to $N(\mathrm{up},t-1)-N(\mathrm{down},t-1)$, namely

* if $N(\mathrm{up},t-1)>N(\mathrm{down},t-1)$, i. e. more people who buy, so the price is high (increasing), σ_i may take the value -1 (sell), against the buying tendency of the first term, to take benefits of selling at a high price

* if $N(\mathrm{up},t-1)<N(\mathrm{down},t-1)$, i. e. more people who sell, so the price is low (decreasing), σ_i may take the value $+1$ (buy), against the imitation tendency of the first term, to take advantage of buying at a low price

iii) The third term is a market-oriented measure to boost buying if H is positive, or to favor selling if H is negative. This measure can be applied for a lapse of time and is removed to leave the market evolve.

Note that the decision of $\sigma_i(t)$ at a given T (to buy, to sell or to wait) depends on the balance of the three terms in Eq. (1). It is the total sum that matters.

We note that the price $P(t)$ can be defined as

$$P(t)=a\ [N(\mathrm{up},t)-N(\mathrm{down},t)]+A \tag{2}$$

where a is a proportional constant and A denotes the stable price determined by the market clearing, namely when $N(\mathrm{up},t)=N(\mathrm{down},t)$.
We perform MC simulations using a network of $N=4\times 12^3=6912$ agents on a face-centered cubic lattice of linear dimension $L=12$ where each agent has 12 nearest neighbors (NN) which is the largest number of NN in all lattices. We suppose that each agent interacts with his/her NN with the same interaction strength J as indicated in Eq. (1). In MC simulations the dynamics is the Glauber's dynamics: we calculate the energy E of a spin at time t, then we try to change its state and calculate its trial energy E'. If $E'<E$ then the spin flips to the trial state, if $E'>E$ the

trial state is accepted only with a probability proportional to $\exp[-(E'-E)/k_BT]$. This causes the motion of that spin. We consider next all spins in the same manner.
It is interesting to examine first the stability of the market as a function of T. By stability we mean that the interaction term in Eq. (1) dominates, namely the collective effect with correlation among agents is present. This occurs when T is lower than a value T_c beyond which agents are independent of each other because the agitation of T which breaks the correlation. This temperature T_c is called "transition temperature" which is seen by anomalies in various quantities such as the system energy E, the order parameter M, fluctuations of E and M called calorific capacity C_V and susceptibility χ in statistical physics. We show in Fig. 1 some physical quantities as functions of T, taking $J=1$, $a=3$ and $H=0$. These quantities show the role of T which allows us to choose appropriate temperature regions when examining the dynamics of the system. As seen in the figure, the energy changes the curvature and the order parameter falls to zero at $T_c = 6.6957$. The fluctuations are very strong, C_V shows a peak at T_c, indicating the transition from the low-T phase to high-T disordered phase. We will see below that it is in the critical region just below T_c that interesting dynamics occurs.

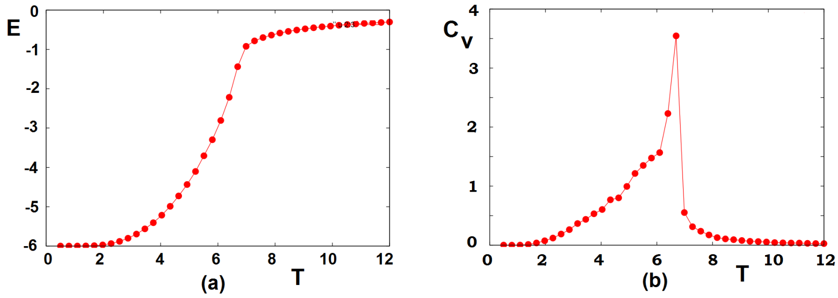

FIG. 1: (a) Energy per individual $E = \langle E\rangle/N$, (b) specific heat per individual C_V versus T. Parameters in Eq. (1): $J=1$, a=3, $H=0$. See text for comments.

The time evolution of the price P is shown in Fig. 2 at several market temperatures with and without a boosting measure.

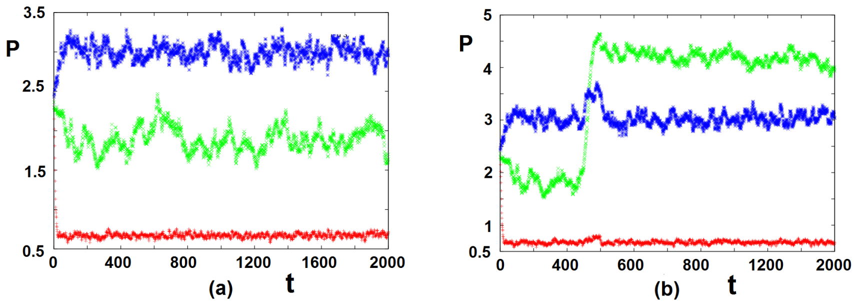

FIG. 2: Time evolution of price P with initial condition 60% sellers, 40% buyers. (a) Price variation when $H=0$: red data are for $T=5.513$, green for $T=6.692$ (just below T_c), blue for $T=7.872$ (after market clearing) (b) Effect of $H=0.2$ applied between $t=400$ and 600. The variation of the price P is shown: red data are for $T=5.513$, green for $T=6.692$, blue for $T=7.872$. Parameters: $J=1$, $a=3$, and the market clearing price A is fixed arbitrarily at 3, see Eq. (2).

Let us summarize some main results of our model treated by MC simulations:

- The results show that the main features do not depend on the number of individual states q: we have similar qualitative results for $q=3$, 5 and ∞.
- We have shown the primordial role of the economic temperature T: there is a critical value T_c corresponding to the market clearing point. At and above T_c the two populations buyers and sellers are equal, the price is stabilized.
- In the region just below T_c the fluctuations are very strong, namely strong exchange of stock market shares. In this region, volatility is much higher. More generally, different temperatures correspond to different volatilities,

and infrequent changes in the temperature generate volatility clustering (a well-known stylized fact). The long autocorrelation occurs in this "critical" region is well-known in statistical physics as the critical-slowing-down phenomenon occurring near the second-order phase transition[1].

- It is also in the region below T_c that under a temporary shock the price can have a persistent effect if the shock is strong enough. In a view point from statistical physics, the persistent state is related to a metastable state: the field H drives the system to a local minimum in the free-energy landscape. The system stays in that minimum for a long time because the barrier is so high to allow the system to climb up to get out.

The above results are interesting because they come from a single energy model [Eq. (1)] without approximation.

We show in Fig. 3 the variation of the price at low and high temperatures obtained by mean-field theory. The model in this work uses the assumption that the sellers and the buyers belong to two communities with different intra-group interactions and different inter-group interactions. This non-symmetric interactions give rise to the price oscillations in a region of market temperature. The price at the market clearing has been arbitrarily fixed to 3. The price oscillates for the market temperature between T^{c1} and T^{c2}. For $T > T^{c2} \simeq 12.10$, the price decays to the market clearing price.

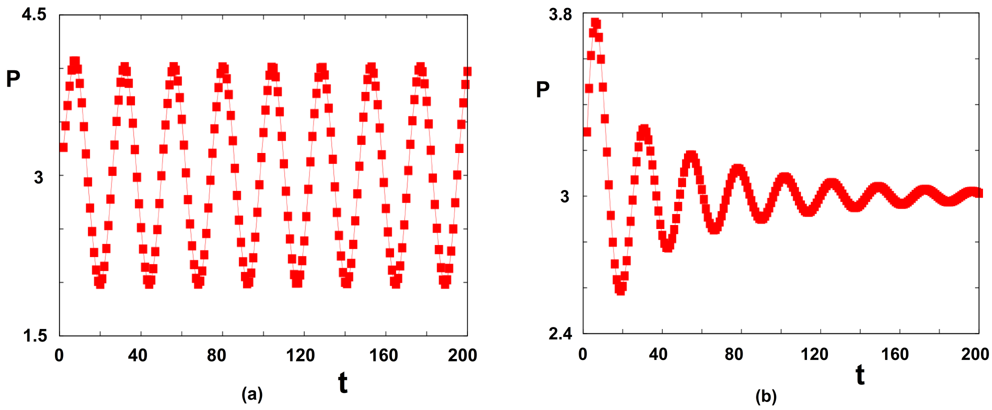

FIG. 3: Price variation versus time for (a) $T = 10.82$, (b) $T = 12.62$. See text for comments.

We emphasize that the mean-field model used here is a simplest model aiming at showing the non trivial price oscillation in a region of market temperature. Such a regular oscillation stems certainly from the simplification of the model. We believe however that such a price oscillation bears an important feature of the market reality.

For details of this work, the reader is referred to our recent publication[8].

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