# Optimization-Based Urban Network Traffic Management with Mixed Autonomy Incorporating Dynamic Saturation Rates

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**SHORT SUMMARY**

This work introduces a novel optimization-based control framework for managing traffic flow in a network with mixed autonomy, where both Connected and Automated Vehicles (CAVs) and Human-Driven Vehicles coexist. The proposed model extends the store-and-forward model by incorporating a dynamic saturation flow rate, which considers the autonomy level of queues. The problem is formulated as a non-convex Quadratic Program (QP), which accounts for the dynamic aspects of the traffic network in terms of queue lengths, spillback, green time allocation, routing of CAVs, and dynamic saturation flow rate. To solve the non-convex QP problem, we employ a computationally efficient heuristic algorithm, which treats the dynamic saturation flow rate as a parameter outside the optimization framework, converting the non-convex problem into a series of convex subproblems. Numerical results on a grid network demonstrate the performance of the proposed methodology.

**Keywords**: Mixed traffic, store-and-forward modelling, multi-commodity traffic, connected and automated vehicles.

## 1 INTRODUCTION

Road traffic congestion continues to negatively affect cities worldwide, demanding innovative solutions. One promising approach lies in leveraging emerging technologies, such as Connected and Automated Vehicles (CAVs), via appropriate management strategies to improve traffic conditions (Papamichail et al., 2019). In fact, (semi-)automated driving has the potential to significantly decrease urban traffic congestion (Foxx et al., 2017). Since vehicles are equipped with technologies that enable, e.g., platooning, via adaptive cruise control or cooperative adaptive cruise control, the inclusion of CAVs in the urban traffic setting may allow an increase of the network capacity by maintaining shorter headways (Lioris, Pedarsani, Tascikaraoglu, & Varaiya, 2017; Roncoli, 2019; Lazar, Coogan, & Pedarsani, 2020). However, mixed autonomy, i.e., the coexistence of CAVs with Human Driven Vehicles (HDVs) poses management challenges, not only in terms of traffic characteristics but also in the definition of integrated traffic management strategies. Therefore, developing integrated management strategies for mixed traffic is crucial because it can enable a positive impact on efficiency, safety, mobility, and ultimately sustainability of the overall transportation systems (Taiebat, Brown, Safford, Qu, & Xu, 2018; Mavromatis, Tassi, Piechocki, & Sooriyabandara, 2020).

This work deals with mixed traffic in urban traffic networks by formulating an integrated problem for optimizing traffic signals and CAV routing. We propose an extension to the store-and-forward model, which incorporates a multi-commodity component, for CAVs, coupled with a single-commodity component, for HDVs. Furthermore, to account for the different driving characteristics and platooning effects, we account for time-and space-varying saturation rates. The proposed model is integrated into an optimization problem, which is then solved via a heuristic approach. The final aim is to pave the way for a more efficient and responsive urban traffic management system in a mixed autonomy environment.

Table 1: Mathematical notation

| Symbol | Description | Symbol | Description |
|---|---|---|---|
| $Z$ | Set of links | $J$ | Set of nodes (intersections) |
| $D \subset Z$ | Set of destinations (CAVs and HDVs) | $\bar{D}$ | Set of destinations (except HDVs): $\bar{D} = D - \{0\}$ |
| $C$ | Cycle time | $K$ | Total number of time steps |
| $L_{(j)}$ | Lost time (red phase) | $T$ | Discrete time step period |
| $k$ | Discrete time index ($k = 0, 1, \dots, K$) | $w_1,\ w_2,\ w_3,\ w_4$ | Weights of the penalty terms in the optimization objective |
| $F_{(z,d)}$ | Network parameter for shortest path between link $z$ and destination $d$ | $x_{(z,d,k)}$ | Queue length of link $z$ directed to destination $d$ |
| $x^{\max}_{(z)}$ | Maximum queue length of link $z$ | $q_{(z,d,k)}$ | Outflow of link $z$ directed to destination $d$ |
| $p_{(z,d,k)}$ | Inflow of link $z$ directed to destination $d$ | $b_{(z,d,k)}$ | Demand flow entering link $z$ directed to destination $d$ |
| $r_{(z,d,k)}$ | Exit flow leaving link $z$ directed to destination $d$ | $g_{(j,i,k)}$ | Green time at intersection $j$ for phase $i$ |
| $g^{\min}_{(j,i)}$ | Minimum green time at intersection $j$ for phase $i$ | $A_{(j)}$ | Set of phases pertaining to intersection $j$ |
| $B_{(z)}$ | Set of phases admitting right of way (r.o.w) to link $z$ | $O_{(j)}$ | Set of outgoing links at intersection $j$ |
| $I_{(j)}$ | Set of incoming links at intersection $j$ | $JD_{(z)}$ | Downstream intersection pertaining to link $z$ |
| $JU_{(z)}$ | Upstream intersection pertaining to link $z$ | $f_{(z,m,d,k)}$ | Transport-flow vector of link $z$ towards downstream link $m$ directed to destination $d$ |
| $G_{(z,m,d,k)}$ | Operational green time of link $z$ towards downstream link $m$ directed to destination $d$ | $s_{(z,k)}$ | Dynamic saturation flow rate of link $z$ |
| $h_{\text{HDV}}$ | Headway of HDVs | $h_{\text{CAV}}$ | Headway of CAVs |
| $t_{(z,m)}$ | Turning rate of HDVs traversing link $z$ towards link $m$ | $e_{(z,m)}$ | Exit turning rate of HDVs at link $z$ |
| $\Theta$ | Measure of autonomy level of link | $\varepsilon$ | Small value to avoid division-by-zero |

The parameters and optimization variables that are indexed by $k$ refer to value at time instant $k$.

# 2 METHODOLOGY

## 2.1 Mathematical modelling

We consider a digraph representing the urban network, which is composed of a directed set of arcs (links) $z \in Z$ and a set of nodes (intersections) $j \in J$. The links $z$ are connecting intersections and certain links are free at one end as they are entry or exiting links. Each intersection $j$ is signalized (controlled traffic signal) and is associated with incoming links $i \in I_{(j)}$ and outgoing links $m \in O_{(j)}$. Furthermore, we consider that all intersections observe the same cycle time $C_{(j)} = C, \forall\, j \in J$, which is assumed constant and equal to the time step of our model $C = T$ (Aboudolas, Papageorgiou, & Kosmatopoulos, 2009). The model is formulated in discrete time, indexed by $k = 0, 1, 2, \dots, K$, where each time instant represents a signal cycle of duration $T$; thus, the overall horizon is $[0, KT]$.

The signal control plan for intersection $j$ (including the fixed lost time $L_{(j)}$) is assigned as a fixed number of phases from the set $A_{(j)}$, while $B_{(z)}$ indicates the set of phases where link $z$ has right of way (r.o.w.).

Moreover, we consider that each link is characterized by the queue length (number of vehicles), defined separately for CAVs and HDVs, and, for the former also divided by destination, which is updated according to the store-and-forward model (Aboudolas et al., 2009). In this modeling approach, we determine for each intersection the traffic signal plan, i.e., green time duration, which in turn is distributed to the operational green times of the links according to their destination and vehicular type. In particular, we assume that the destination of HDVs is not known (and they are distributed to outgoing links according to measured turning rates), while CAVs can be redirected to their destination following specific routing commands.

Therefore, to devise the resulting mathematical framework of multi-destination mixed traffic store and forward modeling, extending the works by (Aboudolas et al., 2009; Han, Hegyi, Yuan, Roncoli, & Hoogendoorn, 2018; De Souza, Carlson, Müller, & Ampountolas, 2020), we present the following framework, as:

$$\begin{aligned} \min \quad & w_1 \sum_{k=0}^{K} \sum_{z \in Z} \sum_{d \in \bar{D}} \frac{x^2_{(z,d,k)}}{x_{max(z)}} + w_2 \sum_{k=0}^{K} \sum_{z \in Z} \frac{x^2_{(z,0,k)}}{x_{max(z)}} \\ & + w_3 \sum_{z \in Z} \sum_{d \in \bar{D}} F_{(z,d)} x_{(z,d,K)} + w_4 \sum_{k=1}^{K} \sum_{j \in J} \sum_{i \in A_{(j)}} \left(g_{(j,i,k)} - g_{(j,i,k-1)}\right)^2 \end{aligned} \tag{1.1}$$

s.t.

$$x_{(z,d,k+1)} = x_{(z,d,k)} + C\left(p_{(z,d,k)} - q_{(z,d,k)} + b_{(z,d,k)} - r_{(z,d,k)}\right), \qquad z \in Z, d \in D \tag{1.2}$$

$$\sum_{i \in A(j)} g_{(j,i,k)} = C - L_{(j)}, \qquad j \in J \tag{1.3}$$

$$g_{(j,i,k)} \geq g_{min(j,i)}, \qquad i \in A_{(j)}, j \in J \tag{1.4}$$

$$0 \leq \sum_{m \in O_{(j)}, d \in D} G_{(z,m,d,k)} \leq \sum_{i \in B_{(z)}} g_{(j,i,k)}, \qquad j = JD_{(z)} \tag{1.5}$$

$$f_{(z,m,d,k)} = \begin{cases} \frac{G_{(z,m,d,k)} s_{(z,k)}}{C}, & z \neq d, m \in O_{(j)}, j = JD_{(z)} \\ t_{(z,m)} q_{(z,d,k)}, & d = 0 \end{cases} \tag{1.6}$$

$$q_{(z,d,k)} = \begin{cases} \frac{x_{(z,d,k)}}{C}, & z = d \\ \sum_{m \in O_{(j)}} f_{(z,m,d,k)}, & z \neq d, d \in D, j = JD_{(z)} \end{cases} \tag{1.7}$$

$$p_{(z,d,k)} = \sum_{i \in I_{(j)}} f_{(i,z,d,k)}, \qquad z \in Z, d \in D, j = JU_{(z)} \tag{1.8}$$

$$r_{(z,d,k)} = e_{(z)} \frac{x_{(z,d,k)}}{C}, \qquad z \in Z \tag{1.9}$$

$$0 \leq \sum_{d \in D} x_{(z,d,k)} \leq x_{max(z)}, \qquad z \in Z \tag{1.10}$$

$$x_{(z,d,k=0)} = x_{0(z,d,k)}, \qquad z \in Z, d \in D \tag{1.11}$$

$$s_{(z,k)} = \frac{\sum_{d \in D} x_{(z,d,k)}}{h_{CAV} \sum_{d \in \bar{D}} x_{(z,d,k)} + h_{HDV} x_{(z,0,k)}}, \qquad z \in Z \tag{1.12}$$

$$\frac{1}{h_{HDV}} \leq s_{(z,k)} \leq \frac{1}{h_{CAV}}, \qquad z \in Z \tag{1.13}$$

The optimization problem is defined over the discrete-time indices of $k = 0, ..., K-1$.

The presented optimization problem (1), is a non-convex Quadratic Program (QP) that is NP-hard in terms of computational complexity (Van Leeuwen, 1991).

## 2.2 Objective Function

The objective minimizes a cost function (1.1) consisting of four terms: the first and second terms are the main components of the optimal control problem, which attempts to reduce and harmonize the relative total queue length for each link: the first term relates to the CAVs while the second term relates to the HDVs. The purpose of the third term is as a terminal cost for CAVs (which are also routed), where $F_{(z,d)}$ is the result of a modified version of the Floyd–Warshall algorithm to compute the distance between each link-destination pair; this value is multiplied by the queue length of CAVs and it intends to bring vehicles to their respective destination while following the shortest possible path(s). The fourth term is a penalty term to suppress fluctuations in the signal

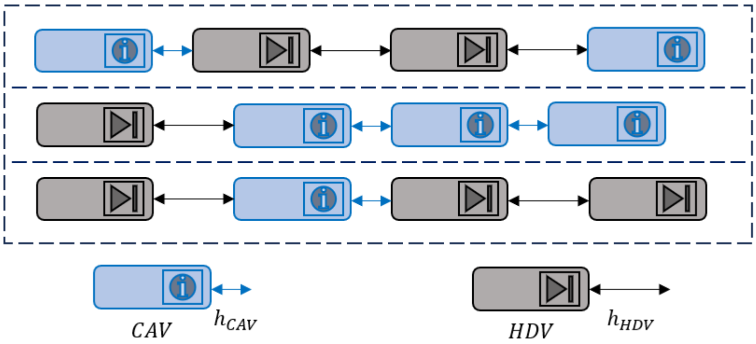



Figure 1: It shows that CAVs can platoon behind any vehicle, and they take a nominal time-headway of $h_{CAV}$, while HDVs are taking a longer time-headway of $h_{HDV}$.

control timing at each intersection $j$ and for each phase $i$ over consecutive time steps: this term ensures that the controller mitigates abrupt distributions of signal timings. Each term has attached weights $w_1$, $w_2$, $w_3$, and $w_4$, which can be adjusted, e.g., via trial-and-error.

### 2.3 Constraints

Constraint (1.2) defines the dynamics of the queue lengths of each link as a conservation equation, where each link has associated inflows and outflows from either other links, i.e., $p$ and $q$, respectively, or from the outside, i.e., $b$ and $r$, respectively, where the latter sink is only defined for HDVs leaving the network. The constraints (1.3) - (1.4) are associated with the controller signal timing for each intersection $j$ and the set of phases $i \in A_{(j)}$. Constraint (1.3) ensures that the green times are distributed during the entire cycle, while (1.4) acts as the lower bound of each phase and intersection. Constraint (1.5) allows the controller to distribute the allocated time of intersection for a particular link (which has r.o.w.) over the outgoing links $m \in O_{(j)}$ and destinations $d$. In constraint (1.6), a transport-flow vector $f_{(z,m,d,k)}$ is defined to compute the outflow from link $z$ towards link $m$ for CAVs and HDVs. The first segment utilizes operational green time with varying saturation to determine the respective outflow for CAVs and HDVs alike, while the second segment dictates that only HDVs ($d = 0$) must follow predefined (measured) turning rates $t_{(z,m)}$. Constraints (1.7)–(1.8) are defined to compute outflow $q_{(z,d,k)}$ and inflow $p_{(z,d,k)}$ by employing the transport-flow vector. In constraint (1.7), the first segment ensures that vehicles leave the network once they reach their respective destination (only for CAVs). Constraint (1.9) defines the sinks for HDVs according to the (predefined) exiting turning rates $e_{(z)}$ at each link. Constraint (1.10) describes that queue length is shared among the vehicles, i.e., CAVs heading to their desired destination and HDVs have a certain limit $x_{max(z)}$ (storage capacity) for a particular link. Constraint (1.11) imposes the initial condition to the optimization problem. Finally, constraint (1.12), defines the rule for varying saturation rates as a function of the queue length and headways of CAVs and HDVs.

Since we assume that saturation flow rates, $s_{(z,k)}$, are dynamic, varying in space, i.e., by link, and in time, we define that the saturation flow rate can be represented by the inverse summation of average discharging headways (Urbanik et al., 2015), as:

$$s = \frac{1}{\sum_{i=1}^{N} h_i}, \tag{2}$$

where, $h_i$ denotes the time-headway of the $i_{th}$ vehicle and $N$ is the total number of vehicles in a link. We further assume that CAVs can maintain shorter time headways and can platoon behind any vehicle, as sketched in Figure 1. Assume that $h_{CAV}$ and $h_{HDV}$ denote the time headways of CAVs and HDVs respectively, thus by employing the mixed traffic autonomy function from (Roncoli, 2019; Lazar et al., 2020), we obtain

$$s = \frac{1}{\Theta\, h_{CAV} + (1-\Theta)h_{HDV}}, \tag{3}$$

where $\Theta$ defines the autonomy level of the link or road segment defined as

$$\Theta = \frac{\sum_{CAV,HDV} X}{\sum_{CAV} X + \sum_{HDV} X}, \tag{4}$$

where $X$ is the queue length vector of a link that contains CAVs and HDVs vehicles combined. Thus, by substituting (4) in (3), we obtain constraint (1.13).

### 2.4 Implementation

In order to compute the optimal control defined by the traffic timing and system optimal routes, we solve optimization problem (1), assuming that the availability of the initial states (queue lengths), demand vector for CAVs and HDVs, and (constant) turning rates for routing and exiting the network of HDVs; the latter can be calculated via an estimation method (Tettamanti, Varga, Kulcsár, & Bokor, 2008) or using online loop detectors (Qi, Dai, Tang, & Hu, 2020). Furthermore, we assume the routing information of CAVs is instructed to each vehicle connected to the system via V2I urban-infrastructure controller.

To solve the non-convex QP problem, we devise a heuristc algorithm based on an iterative procedure to derive the optimal solution for the problem of (1) while varying the saturation rate (as a parameter) outside the framework of optimization as described in Algorithm 1. Note that this fixed point iteration method recasts the non-convex nature of (1) to convex.

**Algorithm 1** Fixed-Point Iteration for Saturation-Flow Rate Update

**Input**: $\theta_{(z,k)} = \sum_{d \in \bar{D}} b_{(z,d,k)} / \left(\sum_{d \in D} b_{(z,d,k)} + \varepsilon\right)$
**Initialize**: $s_{0(z,k)} = 1/\left(h_{CAV}\theta_{(z,k)} + h_{HDV}(1 - \theta_{(z,k)})\right)$
$i \leftarrow 0$
**while** $i \leq$ Max Iterations **do**
    Feed the $s_{opt.(z,k)} = s_{i(z,k)}$
    Solve the Optimization Problem (1)* with $s_{opt.(z,k)}$
    Extract the Optimal Values for $x_{(z,d,k)}$
    Update the Saturation Rate Values:
    $s_{i+1(z,k)} = \sum_{d \in D} x_{(z,d,k)} / \left(h_{CAV} \sum_{d \in \bar{D}} x_{(z,d,k)} + h_{HDV} x_{(z,0,k)}\right)$
    $i \leftarrow i + 1$
    **if** $\left|s_{i(z,k)} - s_{i+1(z,k)}\right| \leq \sigma$ **then**
        Stop
    **end if**
**end while**
**Output**: Optimal Solution

*The saturation flow rate in (1) is considered as a parameter in the implementation of the fixed-point iterative algorithm rather than optimizer variable

## 3 NUMERICAL EXPERIMENTS

We consider a network with $|Z| = 40$ links and $|J| = 16$ intersections, with eight entry and eight exit links along with 24 interlinks as shown in Figure 2. All links have each phase of green time which is distributed categorically as per their orientation, i.e., vertically and horizontally. The optimization horizon is $K = 40$ steps, and $C = 90$ s, implying a total simulation time of 1 hour. Furthermore, we define $h_{CAV} = 1/4000$ veh/h $= 0.9$ s, $h_{HDV} = 1/2000$ veh/h $= 1.8$ s, $L_{(j)} = 5$ s for all $j \in J$. Also, we assume that $g_{min(j,i)} = 15$ s for $j \in J, i \in F_{(j)}$ and $x_{max(z)} = 100$ veh for all $z \in Z$. The traffic demand is illustrated in Figure 3, along with their Orgin-Destination (OD) pairs for CAVs, while HDVs with Origin (O) links.

The proposed strategy is implemented in Python, while AMPL (Fourer, Gay, & Kernighan, 1987) instances are employed for building the optimization problem; the selected solver is Gurobi (Gurobi Optimization, LLC, 2023).

Firstly, we present in Figure (4) the cumulative sum of CAVs and HDVs at the network level. It is shown that all CAVs reach their respective destinations, while the HDVs are all leaving the network according to the assigned exit-turning rates.

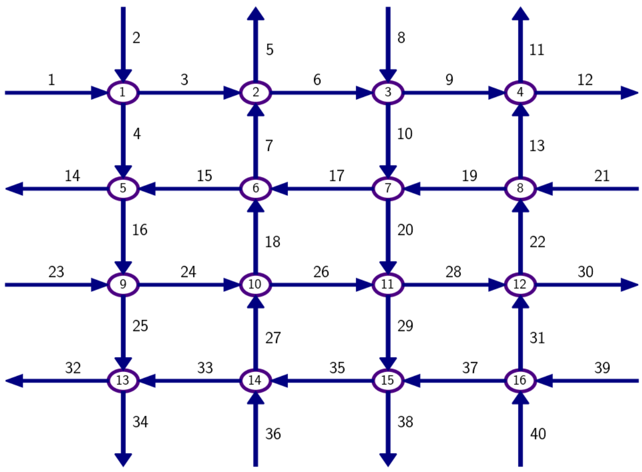


Figure 2: The designed grid network for testing the proposed approach.

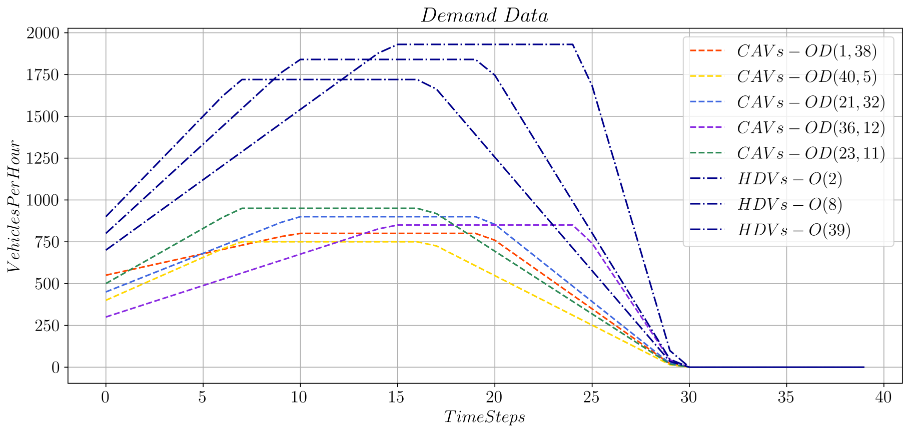


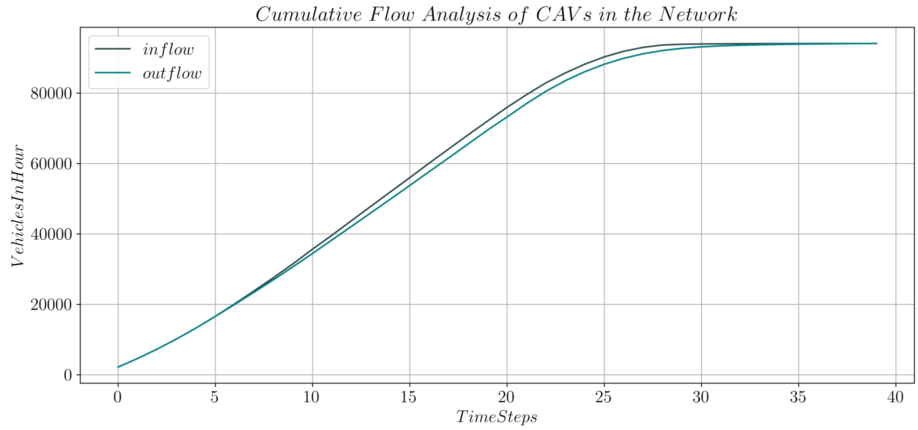

Figure 3: The demand matrix over the selected timespan.

Secondly, queue length along with saturation flow rate is illustrated in Figure 5, where one can see that the queue length of CAVs and HDVs dictates the dynamic nature of the saturation rate, which is changes based on the headways of CAVs and HDVs.

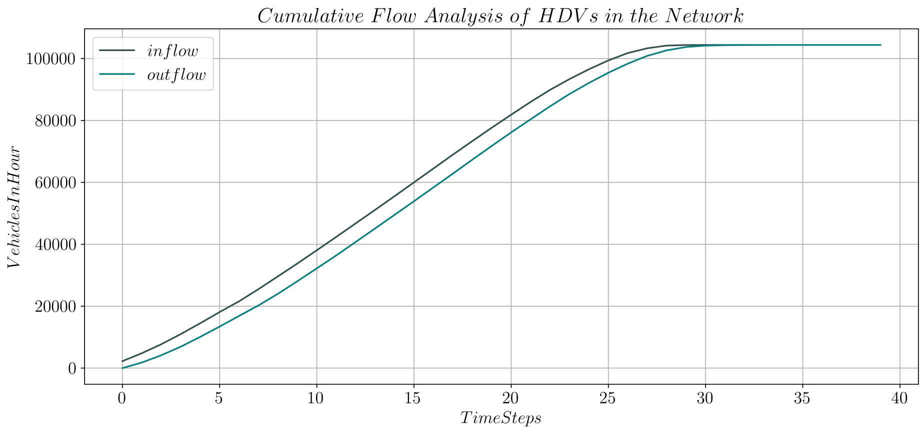
Finally, the trajectory of the controller green times for intersections $j = 2, 10, 11$, and 13 are outlined in Figure 6, where one can observe that the signal time in the network changes as per the horizontal and vertical admitting flow to each link, i.e. the outflow from each subsequent links connected to a particular node will effect the inflow of the next link which in turn affect the optimizer decision for green time allocation. Note that, the changes in the controlled green times can be further smoothed by adjusting (tuning) the weights of the objective in (1).

# 4 CONCLUSIONS

We proposed a novel approach to optimize traffic flow in mixed-autonomy environments by exploiting the signal controller timing for CAVs and HDVs at the intersections and supplementing the CAVs system routing, while we also account for the effect of mixed autonomy on the saturation flow rate. We incorporated the multi-destination-based store-and-forward modelling of CAVs with a single-commodity component, thus accounting for behaviors of both vehicle types. The proposed model was integrated in an optimization problem, solved via a fixed-point iterative algorithm that solves at each time a simpler quadratic convex problem. The results show preliminary results obtained via the presented framework. For future work, we will improve the optimization method, develop alternate algorithms, compare the solution with that obtained by a nonlinear solver, and define a more realistic case study.

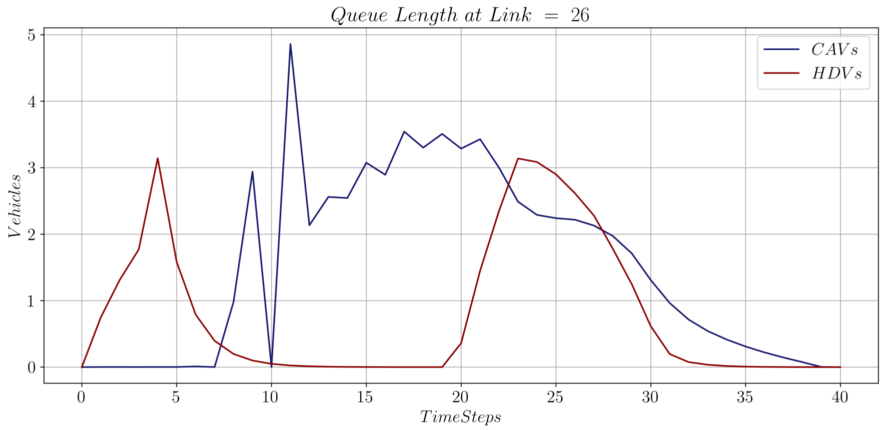



(a)

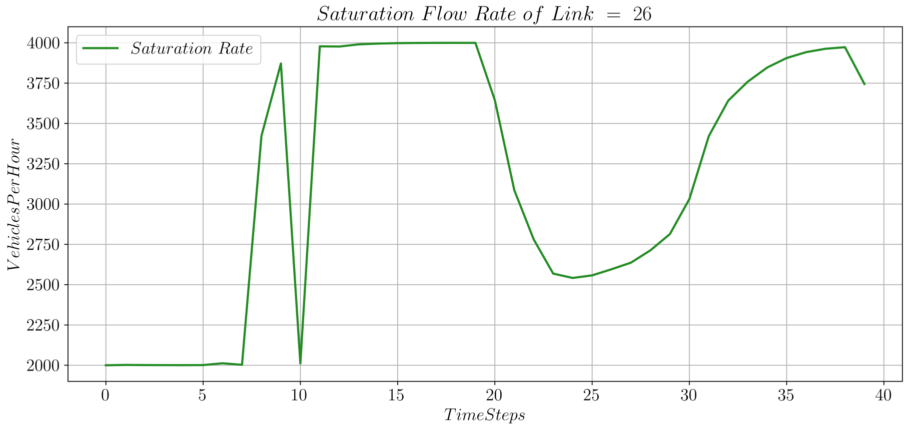



(B)

Figure 4: The network analysis by an illustration of the network-level cumulative inflow and outflow of CAVs (a) and HDVs (b) .




(a)




(b)

Figure 5: The queue lengths (i.e., number of vehicles) of CAVs and HDVs over the time horizon (a), and the dynamic saturation flow rate (in green) for the same link (b).

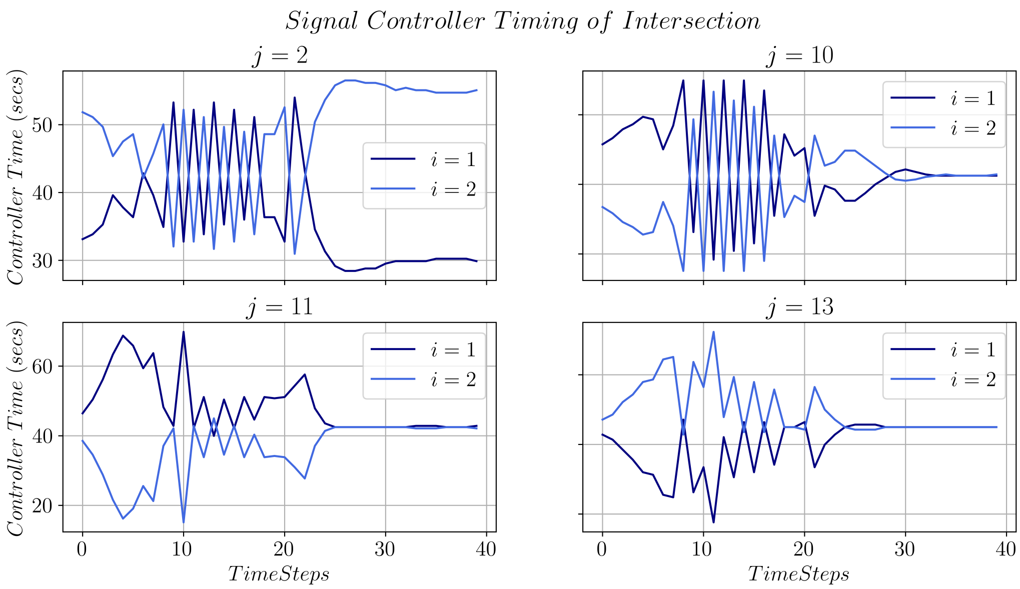



Figure 6: The signal controller timing for selected intersections. Both phases of the traffic signal allocation, $i = 1$ and $i = 2$ are shown in the figures.

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