



Study of the Numerical Simulation of the Electrostatic Separation of Plastic Waste (Case of Ternary Plastic Mixtures)

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Study of the numerical simulation of the electrostatic separation of plastic waste (Case of ternary plastic mixtures)

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Abstract: Numerical simulation has proven to be a powerful tool in the research and development of new electrostatic processes. In a previous article, the authors presented a simple mathematical model to simulate the result of a novel tribo-aero-electrostatic separation process for a ternary mixture of granular materials. The mathematical model assumes that the probability of a granule to be separated can be expressed as a function of the number of impacts with granules belonging to other classes of materials and with the wall. The process is characterized by the fact that the feed of the granules is produced in a fluidized bed device, in the presence of a field. The aim of this article is to simulate the continuity of the operation of such a device as a function of several parameters. such as the composition of the granular mixture, the effect of the walls, the air flow...

Résumé: La simulation numérique s'est révélée être un outil puissant dans la recherche et le développement de nouveaux procédés électrostatiques. Dans un article précédent, les auteurs ont présenté un modèle mathématique simple pour simuler le résultat d'un nouveau procédé de séparation tribo-aéro-électrostatique pour un mélange ternaire de matériaux granulaires. Le modèle mathématique suppose que la probabilité de séparation d'un granulé peut être exprimée en fonction du nombre d'impacts avec des granulés appartenant à d'autres classes de matériaux et avec la paroi. Le procédé est caractérisé par le fait que l'alimentation des granulés est réalisée dans un dispositif à lit fluidisé, en présence d'un champ. Le but de cet article est de simuler la continuité du fonctionnement d'un tel dispositif en fonction de plusieurs paramètres comme la composition du mélange granulaire, l'effet des murs, le flux d'air...

Keywords: Electrostatic separation, Ternary granular mixing, digital models, triboelectricity

Mots-clés : Séparation électrostatique, mélange granulaire ternaire, modèles numériques, triboélectricité.

Codes de classification Jel : XNN; XNN,

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Introduction :

Triboelectric separation represents a solution for the selective sorting of mixtures of plastics, whenever the components have a similar size, shape, density, magnetic permeability and electrical conductivity. The effectiveness of this technique has already been proven.

In a triboelectric separator the plastic granules are charged by triboelectric effect and separated by electric field forces. Researchers and practitioners alike need an easier-to-use simulation tool to perform the feasibility studies that precede the development of a new application. Therefore, the aim of this work is to introduce a simple mathematical model to simulate the result of a tribo-aeroelectrostatic separation process for mixtures of three granular plastics.

The study focuses on the separation of a mixture of three granular materials, denoted A, B and C. The use of this model assumes that we are dealing with perfectly elastic collisions. In the case of mixtures of three different materials, several types of collisions can occur. The first type is the one that is useful for electrostatic separation, which is the inter-granular contact (between A, B and C). The other collisions, considered "parasitic", occur between particles of material A, B or C and the wall, as well as between particles of the same material. It is also assumed that the two closest materials in the triboelectric series are charged with the same sign and they will be collected in the same bin while the third material is charged with the opposite sign and it will be collected in the other bin of the collector .

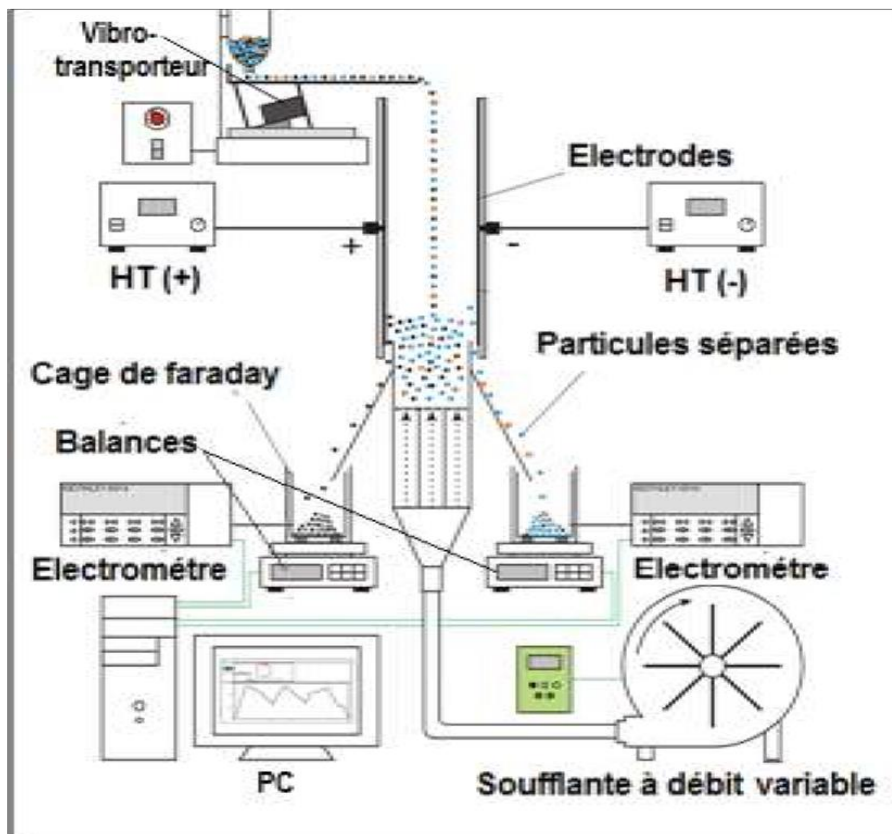


Figure 1 : Tribo-aero-electrostatic separator with electrode parallel plates

1. Mathematical model

The mathematical model is based on the assumption that the probability of separating a granule can be expressed as a function of the number of impacts with granules belonging to another class of

materials and with the walls of the fluidized bed. each granule under goes N (tk) collisions per unit of time: $t = k.\Delta t$, with $k = 1, 2, 3 \dots j$, where i is a positive integer ($j \geq 1$) .
 At $t = 0$, the total mass $M(t)$ of the materials in the fluidized bed is:

$$M(t) = M_A(t) + M_B(t) + M_C(t) + M_P \quad (1)$$

Where M_p is the fictitious mass of the walls, which is constant over time. Under these circumstances, the respective concentrations of the materials A, B, C and the wall p can be expressed as:

$$c_A(t) = M_A(t)/M(t), \quad c_B(t) = M_B(t)/M(t), \quad c_C(t) = M_C(t)/M(t), \quad c_P = M_P/M(t) \quad (2)$$

In a fluidized bed of known geometry and a controllable air flow, it is assumed that the speed of the fluidized bed is not significantly modified by changes in mass $M(t)$, which means that $N(t)$ is constant over time, i.e. $N(t) = N_I$. For the case of a mixture where the three types of particles have the concentrations $c_A(t)$, $c_B(t)$ and $c_C(t)$, each particle of type c undergoes $c_A(t) N$ contacts with particles of type A and $c_B(t) N$ contacts with type B particles.

- a) C is between A and P then $0 \leq \lambda_{PA} \leq 1$, $0 \leq \lambda_{PB} \leq 1$ et $0 \leq \lambda_{PC} \leq 1$ (granule-wall collisions have less effect than those between two granules)
- b) B and C are between A and P, then $\lambda_{PA} > 1$ (a granule A charges better in a collision with the wall P than with a granule B or C) and $0 \leq \lambda_{PB} \leq 1$ (the wall has less effect on a granule B which charges better in a collision with A) and $\lambda_{PC} < 0$ (the charge exchanges C with the wall have an opposite sign to those exchanged with A).
- c) A and C are between B and P, then $\lambda_{PB} > 1$ (a granule B charges better in a collision with the wall P than with a granule A) and $0 \leq \lambda_{PA} \leq 1$ (the wall has less of effect on a granule A which charges better in a collision with B or C) and $\lambda_{PC} < 0$ (the charge exchanges of a granule C with the wall have an opposite sign to those exchanged with A).

Thus, during a period of time t , the number of collisions X_A exchanged from a granule A with all the particles B and C per unit of time is:

$$X_A(t) = \int_0^t [c_B(t) + \lambda_{AC} c_C(t) + \lambda_{PA} c_P] N(t) dt \quad (3)$$

Likewise, the number of X_B collisions between each granule B and the granules A and C can be expressed as:

$$X_B(t) = \int_0^t [c_A(t) + \lambda_{BC} c_C(t) + \lambda_{PB} c_P] N(t) dt \quad (4)$$

The number of X_C collisions between each C granule and the A and B granules is:

$$X_C(t) = \int_0^t [(\lambda_{CA} c_A(t) - \lambda_{CB} c_B(t)) + \lambda_{PC} c_P] N(t) dt \quad (5)$$

Let $P(X_A) = p(x_A)$, $x_A = X_A / N_I$, be the probability that the aggregate A collides with the granules B and C, then collected at the electrode, under well-defined conditions (nature and size of the three classes of granules, geometry of the fluidized bed, air flow of this), the high voltage applied to the electrodes, etc ...). The probability $P(X_A)$, is given by Gauss's law:

$$P(X_A) = P(x_A) = \Pi((x_A - x_{Aav})/s_{xA}) = p(x_A) \quad (6)$$

2. Simulation algorithms

An iterative algorithm can be used to obtain the estimate $X_{Ae}(i, j)$, $X_{Be}(i, j)$ and $X_{Ce}(i, j)$ of the respective numbers of collisions $X_A(t)$, $X_B(t)$ and $X_C(t)$ in step j by the granules introduced into the fluidized bed in step i and the estimates $M_{Ase}(j)$, $M_{Bse}(j)$ and $M_{Cse}(j)$ of the respective masses $M_{As}(t)$, $M_{Bs}(t)$ and $M_{Cs}(t)$. For $t = j\Delta t$, where i and j are positive integers and Δt is sufficiently small compared to the masses $M_A(t)$, $M_B(t)$ and $M_C(t)$, at the concentrations $c_A(t)$, $c_B(t)$, $c_C(t)$ and the frequency of collisions $N(t)$ to be considered constant:

$$M_{Ae}(k) = M_A((k-1)\Delta t), M_{Be}(k) = M_B((k-1)\Delta t), M_{Ce}(k) = M_C((k-1)\Delta t). \quad (7)$$

$$c_{Ae}(k) = c_A((k-1)\Delta t), c_{Be}(k) = c_B((k-1)\Delta t), c_{Ce}(k) = c_C((k-1)\Delta t). \quad (8)$$

$N_e(k) = N((k-1)\Delta t)$ pour chaque intervalle $((k-1)\Delta t \leq t < (k+1)\Delta t)$ for $k = 1, 2, \dots, i$.

Step 1: The estimated number of charges $X_A(I)$ exchanged by each granule A with the granules B and C, $X_B(I)$ exchanged by each granule B with the granules A and C and $X_C(I)$ exchanged by each granule C with the granules A and B can be calculated from equations (3), (4) and (5) as follows

$$X_{Ae}(1,1) = [(c_{Be}(1) + \lambda_{AC} c_{Ce}(1)) + \lambda_{PAC} P] N_{e1} \quad (9)$$

$$X_{Be}(1,1) = [(c_{Ae}(1) + \lambda_{BC} c_{Ce}(1)) + \lambda_{PBC} P] N_{e1} \quad (10)$$

$$X_{Ce}(1,1) = [(\lambda_{AC} c_{Ae}(1) - \lambda_{BC} c_{Be}(1)) + \lambda_{PC} P] N_{e1} \quad (11)$$

Avec

$x_{Ae}(1,1) = X_{Ae}(1,1)/N_{e1}$, $x_{Be}(1,1) = X_{Be}(1,1)/N_{e1}$ et $x_{Ce}(1,1) = X_{Ce}(1,1)/N_{e1}$, estimated masses $M_{Ase}(1)$, $M_{Bse}(1)$ and $M_{Cse}(1)$ separate granules respectively A, B et C at Δt , is obtained from the equation (6)

$$M_{Ase}(1) = p(X_{Ae}(1,1)) M_{Ae}(1) = p(x_{Ae}(1,1)) m_{Ae}(1,1) \quad (12a)$$

$$M_{Bse}(1) = p(X_{Be}(1,1)) M_{Be}(1) = p(x_{Be}(1,1)) m_{Be}(1,1) \quad (12b)$$

$$M_{Cse}(1) = p(X_{Ce}(1,1)) M_{Ce}(1) = p(x_{Ce}(1,1)) m_{Ce}(1,1) \quad (12c)$$

With $m_{Ae}(1,1) = M_A(0)$, $m_{Be}(1,1) = M_B(0)$ et $m_{Ce}(1,1) = M_C(0)$ are the initial masses at $t=0$. The masses remaining after the first step are calculated from the initial mass:

$$m_{Ae}(1,2) = m_{Ae}(1,1) - p(x_{Ae}(1,1))m_{Ae}(1,1) \quad (13a)$$

$$m_{Be}(1,2) = m_{Be}(1,1) - p(x_{Be}(1,1))m_{Be}(1,1) \quad (13b)$$

$$m_{Ce}(1,2) = m_{Ce}(1,1) - p(x_{Ce}(1,1))m_{Ce}(1,1) \quad (13c)$$

In the above formulas, $m_{Ae}(1,2)$, $m_{Be}(1,2)$ and $m_{Ce}(1,2)$ are the masses of the particles which have collisions suffered in the first stage and they will undergo further collisions in the during the second iteration. Therefore, the probability of separation is greater compared to the new particles which will be introduced into the fluidized bed in the next step.

Step j ($j \geq 2$): soit $m(j,j)$ la masse m ajoutée à chaque étape j :

$$m_e(j, j) = m; m_{Ae}(j, j) = m_A; m_{Be}(j, j) = m_B; m_{Ce}(j, j) = m_C$$

Where $m_{Ae}(j,j)$, $m_{Be}(j,j)$ and $m_{Ce}(j,j)$ are the masses of the new granules introduced into the fluidized bed. The total mass in the fluidized bed in step j can be expressed as follows:

$$M_{Ae}(j) = \sum_{i=1}^j m_{Ae}(i, j) \quad (14)$$

Similar formulas can be written $M_{Be}(j)$ and $M_{Ce}(j)$. The total mass estimated at step j is:

$$M_e(j) = M_{Ae}(j) + M_{Be}(j) + M_{Ce}(j) + M_P \quad (15)$$

Masses $m_{Ae}(i,j)$, $m_{Be}(i,j)$ et $m_{Ce}(i,j)$ introduced in step i and those still present in the fluidized bed at the start of step j are calculated separately:

$$m_{Ae}(i, j) = m_{Ae}(i, j-1) - p(x_{Ae}(i, j-1)) * m_{Ae}(i, j-1) \quad (16a)$$

$$m_{Be}(i, j) = m_{Be}(i, j-1) - p(x_{Be}(i, j-1)) * m_{Be}(i, j-1) \quad (16b)$$

$$m_{Ce}(i, j) = m_{Ce}(i, j-1) - p(x_{Ce}(i, j-1)) * m_{Ce}(i, j-1) \quad (16c)$$

The $c_{Ae}(j)$, $c_{Be}(j)$ and $c_{Ce}(j)$ concentrations of granules A , B and C can be determined as follows:

$$c_{Ae}(j) = M_{Ae}(j)/M(j), c_{Be}(j) = M_{Be}(j)/M(j), c_{Ce}(j) = M_{Ce}(j)/M(j) \quad (17)$$

$$x_{Ae}(i, j) = X_{Ae}(i, j)/N_{e1}; x_{Be}(i, j) = X_{Be}(i, j)/N_{e1}; x_{Ce}(i, j) = X_{Ce}(i, j)/N \quad (18)$$

With these notations, the standardized unit charges exchanged by the granules A , B and C of the initial masses $m_{Ae}(1,1)$, $m_{Be}(1,1)$ and $m_{Ce}(1,1)$ in the fluidized bed up to the instant $t = j\Delta t$ can be expressed as follows:

$$X_{Ae}(1, j) = \{[(c_{Be}(1) + \lambda_{AC}c_{Ce}(1)) + \lambda_{PAC}p_e] + [(c_{Be}(2) + \lambda_{AC}c_{Ce}(2)) + \lambda_{PAC}p_e]M_e(2)/M_e(1) + \dots + [(c_{Be}(j) + \lambda_{AC}c_{Ce}(j)) + \lambda_{PAC}p_e]M_e(j)/M_e(1)\} \Delta t \quad (19a)$$

$$X_{Be}(1, j) = \{[(C_{Ae}(1) + \lambda_{BCCe}(1)) + \lambda_{PBCPe}] + [(C_{Ae}(2) + \lambda_{BCCe}(2)) + \lambda_{PBCPe}]M_e(2)/M_e(1) + \dots + [(C_{Ae}(j) + \lambda_{BCCe}(j)) + \lambda_{PBCPe}]M_e(j)/M_e(1)\} \Delta t \quad (19b)$$

$$X_{Ce}(1, j) = \{[(\lambda_{ACCAe}(1) - \lambda_{BCCBe}(1))\lambda_{PCCPe}] + [(\lambda_{ACCAe}(2) - \lambda_{BCCBe}(2)) + \lambda_{PCCPe}]M_e(2)/M_e(1) + \dots + [(\lambda_{ACCAe}(j) - \lambda_{BCCBe}(j)) + \lambda_{PCCPe}]M_e(j)/M_e(1)\} \Delta t \quad (19c)$$

The new particles introduced at each time step Δt have the possibility of being charged by starting with step j :

$$\begin{aligned} X_{Ae}(j, j) &= \{[(C_{Be}(j) + \lambda_{ACCCe}(j)) + \lambda_{PACPe}]M_e(j)/M_e(1)\} \Delta t \\ X_{Be}(j, j) &= \{[(C_{Ae}(j) + \lambda_{BCCe}(j)) + \lambda_{PBCPe}]M_e(j)/M_e(1)\} \Delta t \\ X_{Ce}(j, j) &= \{[(\lambda_{ACCAe}(j) - \lambda_{BCCBe}(j)) + \lambda_{PCCPe}]M_e(j)/M_e(1)\} \Delta t \end{aligned} \quad (20)$$

In général:

$$x_{Ae}(i, j) = \{[(C_{Be}(i) + \lambda_{ACCCe}(i)) + \lambda_{PACPe}] + [(C_{Be}(i+1) + \lambda_{ACCCe}(i+1)) + \lambda_{PACPe}]M_e(i+1)/M_e(1) + \dots + [(C_{Be}(j) + \lambda_{ACCCe}(j)) + \lambda_{PACPe}]M_e(j)/M_e(1)\} \Delta t$$

$$x_{Be}(i, j) = \{[(C_{Ae}(i) + \lambda_{BCCe}(i)) + \lambda_{PBCPe}] + [(C_{Ae}(i+1) + \lambda_{BCCe}(i+1)) + \lambda_{PBCPe}]M_e(i+1)/M_e(1) + \dots + [(C_{Ae}(j) + \lambda_{BCCe}(j)) + \lambda_{PBCPe}]M_e(j)/M_e(1)\} \Delta t$$

$$x_{Ce}(i, j) = \{[(\lambda_{ACCAe}(i) - \lambda_{BCCBe}(i)) + \lambda_{PCCPe}] + [(\lambda_{ACCAe}(i+1) - \lambda_{BCCBe}(i+1)) + \lambda_{PCCPe}]M_e(i+1)/M_e(1) + \dots + [(\lambda_{ACCAe}(j) - \lambda_{BCCBe}(j)) + \lambda_{PCCPe}]M_e(j)/M_e(1)\} \Delta t \quad (21)$$

The estimated masses recovered in the collector A under the effect of the air current can be calculated with the following formulas:

$$\begin{aligned} M_{Aseair}(A)(j) &= [c_{airAC Ae}(1)]M(1, j) + [c_{airAC Ae}(2)]M(2, j) + \dots [c_{airAC Ae}(i)]M(i, j) \\ M_{Bseair}(A)(j) &= [c_{airBC Be}(1)]M(1, j) + [c_{airBC Be}(2)]M(2, j) + \dots [c_{airBC Be}(i)]M(i, j) \\ M_{Cseair}(A)(j) &= [c_{airCC Ce}(1)]M(1, j) + [c_{airCC Ce}(2)]M(2, j) + \dots [c_{airCC Ce}(i)]M(i, j) \end{aligned} \quad (22)$$

The estimated masses accidentally recovered in collector B can be calculated in the same way:

$$\begin{aligned} M_{Aseair}(B)(j) &= [c_{airAc Ae}(1)]M(1, j) + [c_{airAc Ae}(2)]M(2, j) + \dots [c_{airAc Ae}(i)]M(i, j) \\ M_{Bseair}(B)(j) &= [c_{airBc Be}(1)]M(1, j) + [c_{airBc Be}(2)]M(2, j) + \dots [c_{airBc Be}(i)]M(i, j) \\ M_{Cseair}(B)(j) &= [c_{airCc Ce}(1)]M(1, j) + [c_{airCc Ce}(2)]M(2, j) + \dots [c_{airCc Ce}(i)]M(i, j) \end{aligned} \quad (23)$$

Consequently and by using the equations (12) - (22) and (23) we obtain:

$x_{Ce}(i,j) > 0$:

$$M_{bac(-)}(j) = M_{Ase}(j) + M_{Aseair(A)}(j) + M_{Bseair(A)}(j) + M_{Cseair(A)}(j) \quad (24 a)$$

$$M_{bac(+)}(j) = M_{Bse}(j) + M_{Cse}(j) + M_{Aseair(B)}(j) + M_{Bseair(B)}(j) + M_{Cseair(B)}(j) \quad (24 b)$$

Si $x_{Ce}(i) < 0$:

$$M_{bac(-)}(i) = M_{Ase}(i) + M_{Cse}(i) + M_{Aseair(A)}(i) + M_{Bseair(A)}(i) + M_{Cseair(A)}(i) \quad (25 a)$$

$$M_{bac(+)}(i) = M_{Bse}(i) + M_{Aseair(B)}(i) + M_{Bseair(B)}(i) + M_{Cseair(B)}(i) \quad (25 b)$$

3. Simulation results

The calculations were performed with a program written in MATLAB 7.0. The iteration was $\Delta t = 1$ s, the mass introduced into the device at the start of the operation is equal to $M_1 = 200$ g (with an $M_{lim} = 30$ g) and the mass we have assumed that $m(j, j)$ introduced at each step was 5 g / s.

The charges exchanged between the particles A, B, C and the wall are respectively $\lambda_{PA} = 0,2$, $\lambda_{PB} = 0,6$ and $\lambda_{PC} = 0,2$

3.1 Effect of the composition of the mixture

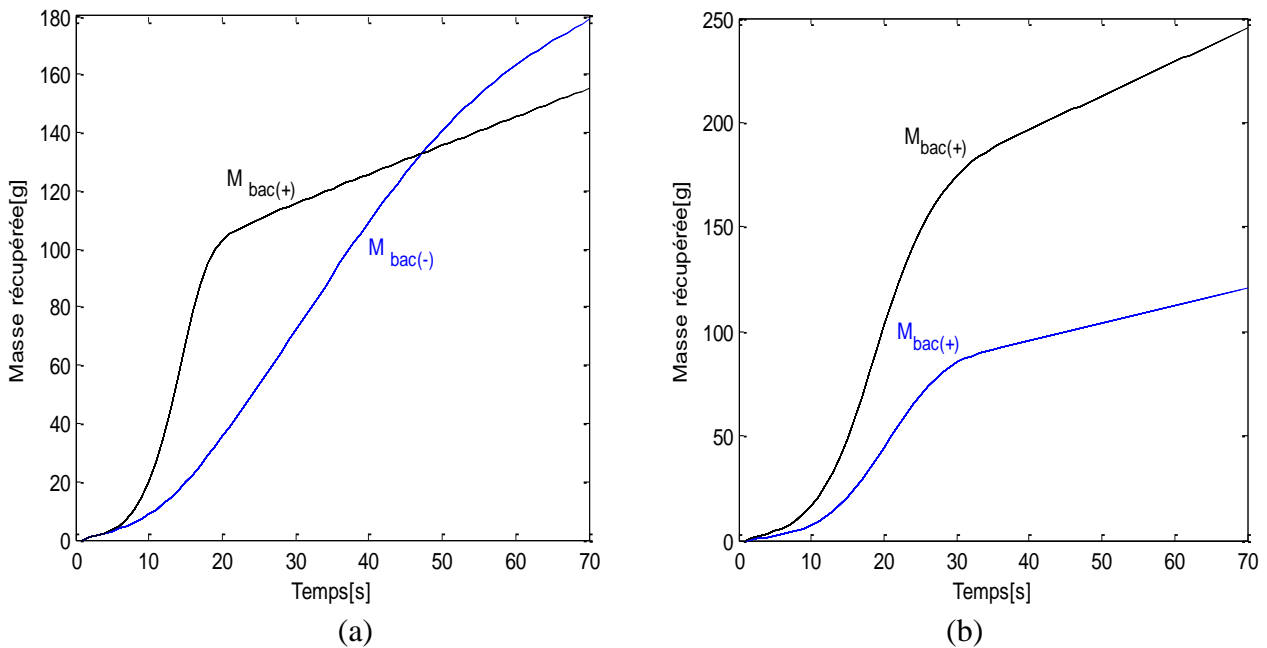


Figure 2 Evolution of the estimated masses recovered depending of the composition of the mixture

a) 60% A + 30% B + 10% C

b) 33% A + 33% B + 33% C

Figure 2.a shows that a granule of A has a lower probability than a granule of B of colliding with a granule of opposite polarity.

The triboelectric charge process is more efficient for minority particles which acquire sufficient charge to separate them. Beyond $t = 18s$, the granules of A are collected in a greater quantity than the granules of B and C, this is explained by the fact that the materials B and C exist in small quantities in the fluidized bed. balanced mixture (figure 2.b), the mass recovered in the tank (+) represents double the quantity recovered in the tank (-), because all the particles C will be recovered in the tank (+) with the granules B

3.2 Effect of the nature of the walls

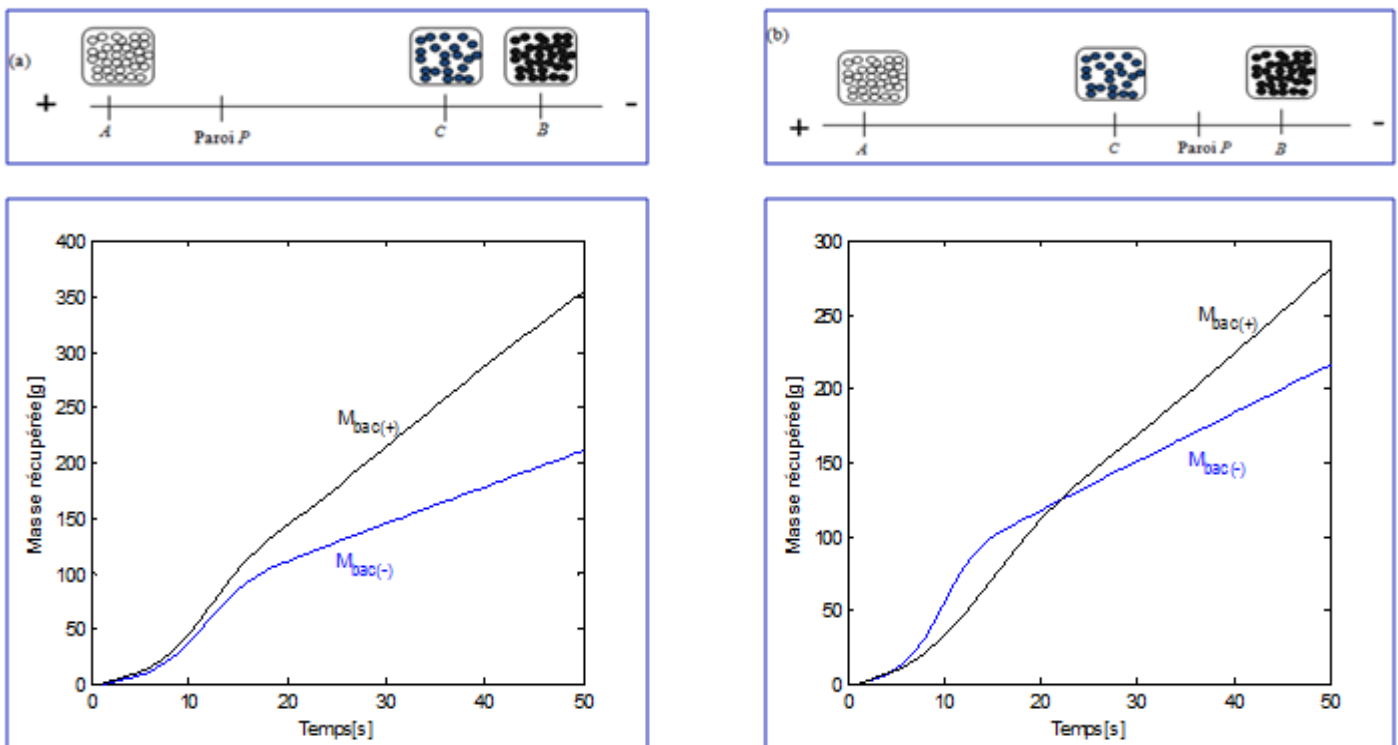


Figure 3 Evolution of the estimated masses recovered depending of the nature of the walls

The aim of this study was initially to observe whether the walls of the fluidized bed influence the electrostatic separation process for a mixture of three granular materials noted A, B and C in continuous mode. Figure 3 shows the evolution of the mass recovered for two examples of the position of the walls relative to the position of the granules in the tribo-electric series for a mixture of three granular materials 33% A + 33% B + 33% C . We notice that the materials closer to the walls are loaded and recovered better than materials located far from the walls because by increasing the particle load the recovery rate increases.

3.3 Effect of the nature of material flow

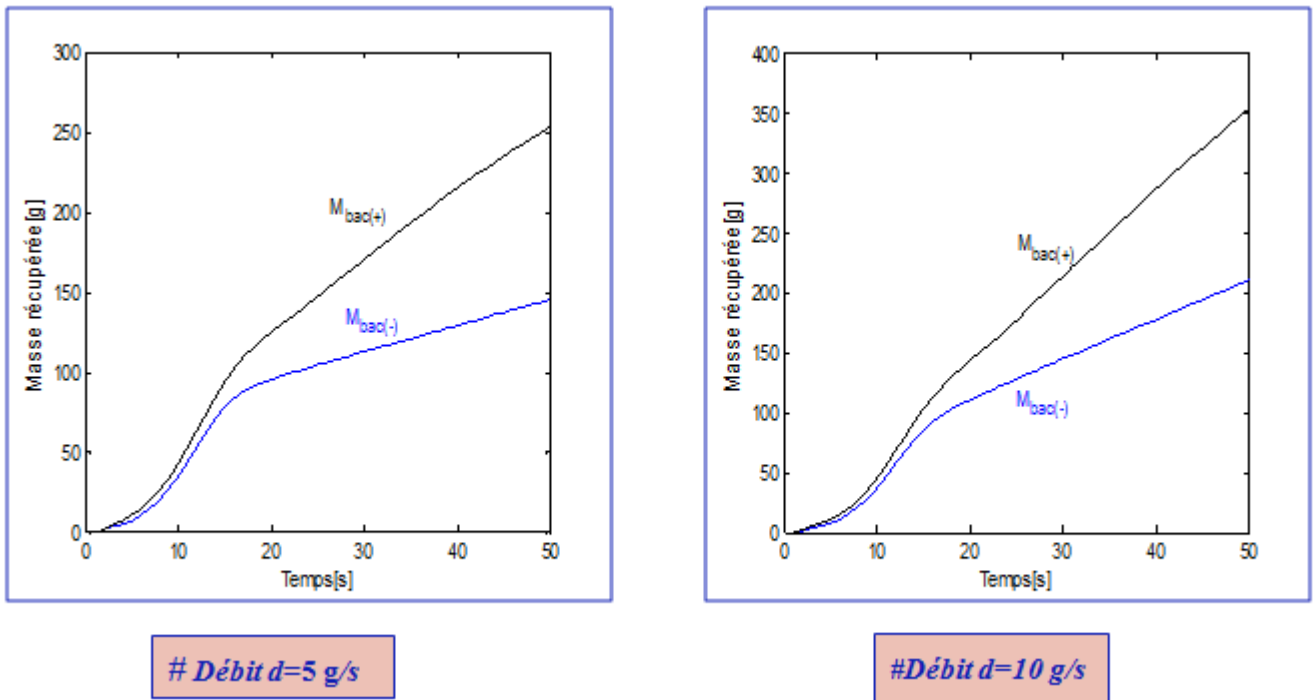


Figure 4 Evolution of the estimated masses recovered depending of the material flow

Figure. 4 illustrates the masses recovered and its variations with respect to time for two values of the material flow rate Flow rate = 5 g / s; Flow = 10g / s. A gradual increase in this mass is observed with the increase in the flow of material.

Conclusion:

The objective of this work was to simulate the operation of a tribo-aeroelectrostatic separator for a mixture consisting of three granular plastic materials.

The results of numerical calculations have shown that the operation of this type of device can be accurately predicted by numerical simulation, based on a simple mathematical model.

The continuous operation of the separator depends on several factors which influence the efficiency of the device such as :

- The composition of the mixture to be separated: the minor particles charge more quickly and are easily separated from the mixture
- The mass flow rate introduced instantly (variation of 5..10 [g / s]).
- The nature of the material used for the walls.
- Study of the effect of the walls on the materials to be separated according to their respective positions in the triboelectric series.

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