Computationally Efficient Indirect Kalman Filter for Hydraulic Machinery

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Abstract

Hydraulic machinery such as cranes and excavators are multidisciplinary systems that combine mechanics, hydraulics, and electronics. Their computer simulation often includes data fusion from real sensors using various information fusing techniques such as state and parameter estimations. Out of various methods, the error-state extended Kalman filter, also known as the indirect Kalman filter, is often used in the literature because of their high computational efficiency for multibody systems [1]. However, their application to hydraulically actuated multibody systems utilizes a numerical approach as in [2] and consequently, it affects to the computational efficiency.

The objective of this study is to introduce an efficient indirect Kalman filter that utilizes a semi-analytical approach in computing the state-transition matrix in the framework of hydraulic machinery. The introduced methodology is compared with the numerical approach available in the literature [2]. As a case example, a hydraulic crane is considered, where the mechanics is modeled using the index-3 augmented Lagrangian-based semi-recursive formulation and the hydraulics is modeled using the classical lumped fluid theory. The two filters are compared based on the accuracy of the state estimations and the associated computational efficiencies.

In the modeling of hydraulic machinery, the mechanics and hydraulics can be coupled using a monolithic method, and accordingly, the combined equations of motion can be written as [2]

$$\left.\begin{aligned}\bar{\mathbf{M}}^{\Sigma}\ddot{\mathbf{z}}+\mathbf{\Phi}_{\mathbf{z}}^{\mathrm{T}}\alpha\mathbf{\Phi}+\mathbf{\Phi}_{\mathbf{z}}^{\mathrm{T}}\lambda=\bar{\mathbf{Q}}^{\Sigma}\left(\mathbf{z},\dot{\mathbf{z}},\mathbf{p}\right)\\ \lambda^{(h+1)}=\lambda^{(h)}+\alpha\mathbf{\Phi}^{(h+1)}\\ \dot{\mathbf{p}}=\mathbf{u}_0\left(\mathbf{z},\dot{\mathbf{z}},\mathbf{p}\right)\end{aligned}\right\}, \tag{1}$$

where $\mathbf{z}$, $\dot{\mathbf{z}}$, $\ddot{\mathbf{z}}$ are the respective vectors of the relative joint positions, velocities, and accelerations, $\bar{\mathbf{M}}^{\Sigma}$ and $\bar{\mathbf{Q}}^{\Sigma}$ are the accumulated mass matrix and external force vector, respectively, $\mathbf{\Phi}$ is the vector of loop-closure constraints and $\mathbf{\Phi}_{\mathbf{z}}$ is its Jacobian matrix, λ is the vector of iterated Lagrange multipliers, α is the penalty factor, h is the iteration step, $\mathbf{p}$ and $\dot{\mathbf{p}}$ are the respective vectors of pressures and pressure build-ups, and $\mathbf{u}_0$ is the function of pressure variation equations.

In this study, the state vector of the filters is $\mathbf{x}=\left[\left(\Delta\mathbf{z}^{\mathrm{i}}\right)^{\mathrm{T}},\ \left(\Delta\dot{\mathbf{z}}^{\mathrm{i}}\right)^{\mathrm{T}},\ \left(\Delta\mathbf{p}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, where Δ represents the error in the respective states and the superscript $^{\mathrm{i}}$ represents the independent coordinates. The prediction stage of the filter can be written as [1]

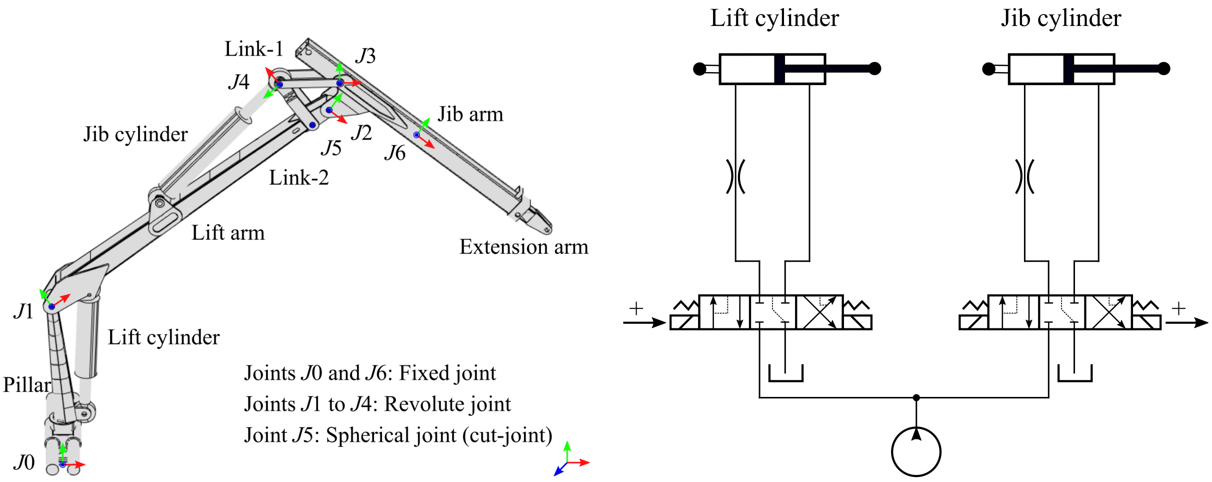
$$\hat{\mathbf{x}}_k^- = \mathbf{0} \qquad \text{and} \qquad \mathbf{P}_k^- = \left(\bar{\mathbf{f}}_{\mathbf{x}}\right)_{k-1}\mathbf{P}_{k-1}^{+}\left(\bar{\mathbf{f}}_{\mathbf{x}}\right)_{k-1}^{\mathrm{T}}+\left(\textstyle\sum^{P}\right)_{k-1}, \tag{2}$$

where $\hat{\mathbf{x}}^-$ is the predicted state mean, $\mathbf{P}^-$ is the associated covariance matrix, k is a time-step, $\sum^P$ is the plant covariance matrix, and $\bar{\mathbf{f}}_{\mathbf{x}}$ is the state-transition matrix that can be written as

$$\bar{\mathbf{f}}_{\mathbf{x}}=\frac{\partial\bar{\mathbf{f}}}{\partial\hat{\mathbf{x}}}=\frac{\partial}{\partial\left[\hat{\mathbf{z}}^{\mathrm{i}},\ \hat{\dot{\mathbf{z}}}^{\mathrm{i}},\ \hat{\mathbf{p}}\right]}\begin{bmatrix}\hat{\mathbf{z}}^{\mathrm{i}}+\hat{\dot{\mathbf{z}}}^{\mathrm{i}}\Delta t+\frac{1}{2}\ddot{\mathbf{z}}^{\mathrm{i}}\Delta t^2\\ \hat{\dot{\mathbf{z}}}^{\mathrm{i}}+\ddot{\mathbf{z}}^{\mathrm{i}}\Delta t\\ \hat{\mathbf{p}}+\dot{\mathbf{p}}\Delta t\end{bmatrix}\simeq\left[\begin{array}{c:c:c}\mathbf{I}_{N_f}+\frac{1}{2}\frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\mathbf{z}^{\mathrm{i}}}\Delta t^2 & \mathbf{I}_{N_f}\Delta t+\frac{1}{2}\frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\dot{\mathbf{z}}^{\mathrm{i}}}\Delta t^2 & \frac{1}{2}\frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\mathbf{p}}\Delta t^2\\ \hdashline \frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\mathbf{z}^{\mathrm{i}}}\Delta t & \mathbf{I}_{N_f}+\frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\dot{\mathbf{z}}^{\mathrm{i}}}\Delta t & \frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\mathbf{p}}\Delta t\\ \hdashline \mathbf{0}_{N_p\times N_f} & \mathbf{0}_{N_p\times N_f} & \mathbf{I}_{N_p}\end{array}\right], \tag{3}$$

where $\bar{\mathbf{f}}$ is the state transition model, N_f is the mechanism degrees of freedom, N_p is the number of pressure elements, and Δt is the time-step size. In the semi-analytical approach, the partial derivatives $\frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\mathbf{z}^{\mathrm{i}}}$ and $\frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\dot{\mathbf{z}}^{\mathrm{i}}}$ can be computed analytically using the multibody equations of motion and the constraint manifold as in [3], whereas $\frac{\partial\Delta\ddot{\mathbf{z}}^{\mathrm{i}}}{\partial\mathbf{p}}$ is computed numerically. In the numerical approach, all the partial derivatives are computed numerically using the forward differentiation rule [2]. Furthermore, the correction stage of the filter has a similar set of equations as in the extended Kalman filter [1].

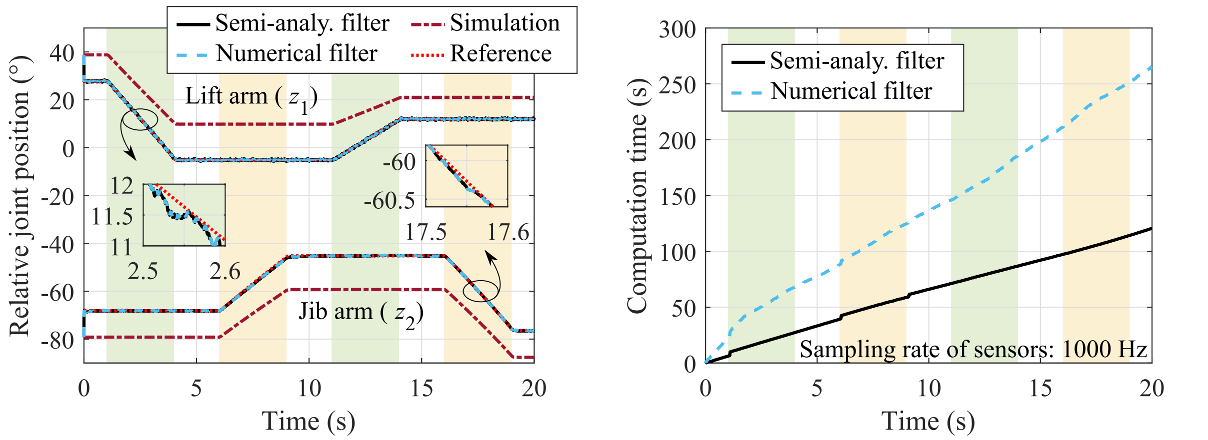
As a case example, a hydraulic crane is considered as shown in Fig. 1. This study utilizes a *"reference model"* that provides the ground truth, and the semi-analytical and numerical filters are applied on the *"simulation model"*, which is an imperfect version of the reference model.



(a) Mechanics of the hydraulic crane. (b) Hydraulics of the hydraulic crane.

Figure 1: The hydraulic crane model considered in this study.

In the state estimation of the hydraulic crane, the accuracy and computational efficiency of the filters are provided in Fig. 2. It can be seen that at the same level of accuracy at the position level, the semi-analytical filter is more efficient than the numerical filter.



(a) Estimations with encoders on the lift arm and link-1. (b) Efficiency with encoders and gyros on both the arms.

Figure 2: Comparing the semi-analytical and numerical filters at 1000 Hz sampling rate of the sensors.

This study introduced a novel and efficient indirect Kalman filter based on a semi-analytical approach in the framework of hydraulic machinery. The result demonstrated that the semi-analytical filter was 54% more efficient than the numerical filter at the same level of estimation accuracy.

References

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