The Effects of Small Horizontal Vibrations of the Mother Ship on Tethered Systems with Changing Length

Riko Ogawara, Yoshiaki Terumichi and Stefan Kaczmarczyk

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R. Ogawara*, **Y. Terumichi** #, **S. Kaczmarczyk**†

* Graduate School of Science and Technology, Sophia University, 1028554, Tokyo, Japan, r-ogawara-vlx@eagle.sophia.ac.jp

Faculty of Science and Technology, Sophia University, 1028554, Tokyo, Japan, y-terumi@sophia.ac.jp

† Dept of Engineering, Division of Technology, Faculty of Arts, Science and Technology, University of Northampton, MN1 5PH, Northampton, UK, Stefan.Kaczmarczyk@northampton.ac.uk

Abstract

A tethered system represents a flexible multi-body system in which components and bodies such as a mothership, probes and equipment, are connected by flexible tether cable. In exploration of the marine environment for expanding scientific discoveries and industrial applications, a new design of a tethered system using a drone and cable is considered. When using such a system, it is assumed that the mothership moves harmonically while changing the tether length, for example when the drone is hovering. This can cause the system resonance, which may result in large deformations and displacements of the tether and large movement of the equipment. Furthermore, the natural frequencies of the system vary depending on the change in tether length. Therefore, clarifying and predicting the system vibration phenomena with time-varying tether length is needed.

In this study, the effects of small vibrations of the mother ship on a tethered system during changing the tether length are studied by using the Absolute Nodal Coordinate Formulation (ANCF), which is widely used for flexible body dynamic simulation [2]. In ANCF, the position vector **r** in the inertia frame for an arbitrary point on a flexible tether modeled as a beam is described using the shape function **S**, which is constant over time and depends on only ξ, and the nodal coordinates **e**, differently from the conventional ANCF, as follows:

$$\mathbf{r} = \mathbf{S}\mathbf{e}, \tag{1}$$

$$\mathbf{e} = [e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5 \quad e_6 \quad e_7 \quad e_8]^{\mathrm{T}} \tag{2}$$

where $\xi = x/l_e$; x is the coordinate of the arbitrary point along the beam axis in the deformed configuration; l_e is the length of the element; e_1, e_2, e_5, and e_6 represent the absolute coordinates of the nodes at the left end and right end of the element, respectively; and

$$e_3 = l_e \frac{\partial \mathrm{r}_1(x=0)}{\partial x}, e_4 = l_e \frac{\partial \mathrm{r}_2(x=0)}{\partial x}, e_7 = l_e \frac{\partial \mathrm{r}_1(x=l_e)}{\partial x}, \text{and } e_8 = l_e \frac{\partial \mathrm{r}_2(x=l_e)}{\partial x} \tag{3}$$

in which r_1 and r_2 are the components of the vector **r** that defines the global position vector of the arbitrary point defined by Eq. (1). By using the Lagrange equation, the equation of motion for an element of the beam is obtained eventually as follows:

$$\mathbf{M}_{e}\ddot{\mathbf{e}} + \mathbf{Q}_{i_e} + \mathbf{Q}_{l_e} + \mathbf{Q}_{t_e} = \mathbf{0} \tag{4}$$

where $\mathbf{M}_e$ is the mass matrix, $\mathbf{Q}_{i_e}$ is the inertia force and $\mathbf{Q}_{l_e}$ and $\mathbf{Q}_{t_e}$ are the elastic force vectors due to axial strain and bending respectively.

In addition, the time-varying length of the flexible body is expressed using Variable-domain Finite Element (VFE) method [1]. In this model, the element length l_e is expressed as the length of a flexible body $L(t)$ divided by the number of elements N,

$$l_e = \frac{L(t)}{N}, \qquad L(t) = L_0 \pm \int V(t)\,dt \tag{5}$$

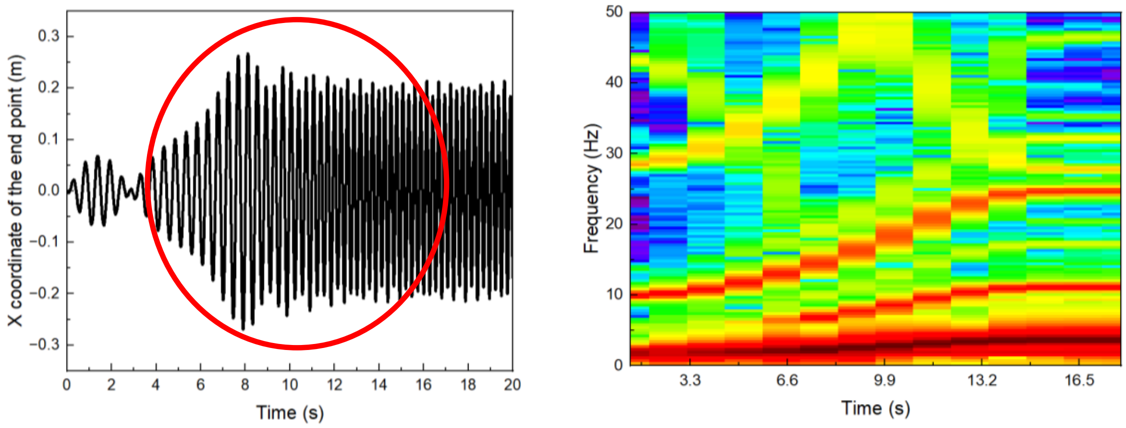
where, L_0 is the initial length of the flexible body and $V(t)$ is the length change velocity.

The analytical model in which a rigid body of mass $M_t = 0.01\ kg$ is connected to the bottom end of a flexible body is used in this study. At the initial state, the flexible body and tether are in the vertical configuration, with the initial length $L_0 = 1.0\ m$, and also the upper end point of flexible body is at the origin of the absolute nodal coordinate system. Then the upper end moves harmonically in the X direction as $x = x_0 \sin \omega t$ where x_0 is the amplitude and ω is the frequency, while changing its length with velocity $V(t)$. The X coordinate of the flexible body end point and the result of Short Time Fourier Transform (SSFT) analysis of the time response record when ω is constant is shown in Fig.1. Here, the natural frequencies of the system are defined by the frequency equation (7). The frequency of the nth bending mode is calculated by equation (8).

$$1+\cos\lambda_n\cosh\lambda_n+\frac{\lambda_n M_t}{\rho AL}(\cos\lambda_n\sinh\lambda_n-\sin\lambda_n\cosh\lambda_n)\\ -\frac{\lambda_n^3 I_t}{\rho AL^3}(\cosh\lambda_n\sin\lambda_n+\sinh\lambda_n\cos\lambda_n)+\frac{\lambda_n^4 M_t I_t}{\rho^2A^2L^4}(1-\cos\lambda_n\cosh\lambda_n)=0 \tag{7}$$

$$f_n=\frac{\lambda_n^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \tag{8}$$

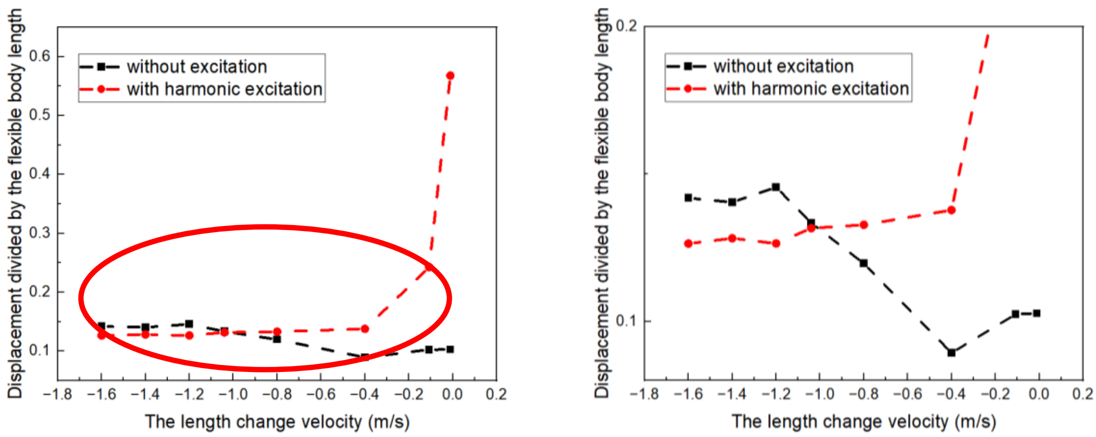
Figure 1 (a) shows the response of the system in terms of the displacements of the tether end point. The frequency of each bending mode varies with the length of tether and the resonance occurs when ω is near the value in the natural frequency. Fig.1 (b) shows the spectrogram obtained by STFT over the time interval shown.



(a) (b)

Figure 1: The displacements when $x_0=0.01m, \omega=12.56rad/s$ while changing the length from 1.0m to 0.6m. The time-varying frequencies of the first, second and third bending modes are shown in the right figure, where each value vary as follows: $f_1=1.54\sim3.89, f_2=10.13\sim26.83, f_3=29.14\sim78.37$.

Figure 2 shows the relationship between the velocity and maximum amplitude of the response. The results for the system under harmonic excitation (red dashed line) demonstrate that the maximum amplitudes are increasing with the decreasing speed of the tether. On the other hand, it is known that in the classical spaghetti problem, the larger the velocity the higher the amplitude [3] as shown by the black dashed line in Figure 2. These relationships lead to the optimal velocity that reduces the effects of resonance and spaghetti problem.



(a) (b)

Figure 2: (a) Relationships between the length change and max value of the amplitude. (b) zoomed area of the relationship.

References

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