Axiomatical Computation of Model Composites of Enactment Logic.

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AXIOMATICAL COMPUTATION OF MODEL COMPOSITES OF ENACTMENT LOGIC.

(Expressive Enactment Logic)

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Extended Abstract[+]. This research is on axiomatical expressions using Kleene Axiom schema. Nine propositional formulas from enactment logic are expressed in terms of axioms based on schema means. This will result in about 40 axiomatical expressions. Expressive enactment logic is proposed as axiomatical expression of model composites of enactment propositions.

Keywords. model, composites, enactment, logic, expression, syntactic, axiom.

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1 INTRODUCTION

This research is based on axiomatical schemas of Kleene[3] that gives axiomatic expressions based on formulas of enactment models[1]. Kleene give the following axiom schemas:

(1a) $\alpha \to (\beta \to \alpha)$.

(1b) $(\alpha \to \beta) \to (\alpha \to (\beta \to \gamma) \to (\alpha \to \gamma))$.

(2a) $\alpha \to (\beta \to (\alpha \wedge \beta))$,

(2b) $(\alpha \to \beta) \to \alpha$.

(2c) $(\alpha \wedge \beta) \to \beta$.

(3a) $\alpha \to (\alpha \vee \beta)$,

(3b) $\beta \to (\alpha \vee \beta)$,

(3c) $(\alpha \to \gamma) \to ((\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma))$.

(4a) $(\alpha \to \beta) \to ((\alpha \to \neg \beta) \to \neg \alpha)$,

(4b) $(\neg\neg\alpha) \to \alpha$.

Results of Work[1]:

The models of enactment logic are:

(1) $enact_E \to enact_L$

(2) $a_i \to a$

(3) $a_j \to l$

(4) $rank_i \to t$

(5) $enact_E \to (enact_E \to a_i)$

(6) $eanct_L \rightarrow (a_j \rightarrow l)$

(7) $a_i \rightarrow (a_j \rightarrow l)$

(8) $a_j \rightarrow (l \rightarrow rank_i)$

(9) $l \rightarrow (rank_i \rightarrow t)$

Axiom is produced by replacing the Greek variable of an axiom schema by formula. Models (1) to (9) will be expressed using Kleene expression axioms (1a) to (4b).

2 AXIOMATIC EXPRESSIONS

This section will produce 40 axiomatic expressions of enactment logic with Kleene axiom schema.

Let $Enact_E$ to be represented by E_E .

Let $Enact_L$ to be represented by E_L .

Axiom (1a):

$(1) E_E \rightarrow (E_L \rightarrow E_E),$
$(2) a_i \rightarrow (a \rightarrow a_i),$
$(3) a_j \rightarrow (l \rightarrow a_j),$
$(4) rank_i \rightarrow (t \rightarrow rank_i).$

Axiom (2a)

$(1) E_E \rightarrow (E_L \rightarrow E_E \wedge E_L),$
$(2) a_i \rightarrow (a \rightarrow a_i \wedge a),$
$(3) a_j \rightarrow (l \rightarrow a_j \wedge l),$
$(4) rank_i \rightarrow (t \rightarrow rank_i \wedge t).$

Axiom (1b):

$$(1)(E_E \to E_L) \to ((E_E \to (E_L \to a_i)) \to (E_E \to a_i)),$$
$$(2)(a_i \to a) \to ((a_i \to (a \to a_i)) \to (a \to a_i)),$$
$$(3)(a_j \to l) \to ((a_j \to (l \to a_j)) \to (l \to a_j)),$$
$$(4)(rank_i \to t) \to ((rank_i \to (t \to rank_i)) \to (t \to rank_i)).$$

Axiom (2b):

$$(1)(E_E \wedge E_L) \to E_E,$$
$$(2)(a_i \wedge a) \to a_i,$$
$$(3)(a_j \wedge l) \to a_j,$$
$$(4)(rank_i \wedge t) \to rank_{i.}$$

Axiom (2c):

$$(1)(E_E \wedge E_L) \to E_L,$$
$$(2)(a_i \wedge a_i) \to a_i,$$
$$(3)(a_j \wedge l) \to l,$$
$$(4)(rank_i \wedge t) \to t.$$

Axiom (3a):

$$(1) E_E \to (E_E \vee E_L),$$
$$(2) a_i \to (a_i \vee a),$$
$$(3) a_j \to (a_j \vee l),$$
$$(4) rank_i \to (rank_i \vee t).$$

Axiom (3b):

$$(1) E_L \to (E_E \vee E_L),$$
$$(2) a \to (a_i \vee a),$$
$$(3) l \to (a_j \vee l),$$
$$(4) t \to (rank_i \vee t).$$

Axiom (3c):

$$(1)(E_E \to a_i) \to ((E_L \to a_i) \to ((E_E \vee E_L) \to a_i)),$$
$$(2)(a_i \to a) \to ((a_i \to a) \to ((a_i \vee a) \to a_i)),$$
$$(3)(a_j \to l) \to ((a_j \to l) \to ((a_j \vee l) \to a_j)),$$
$$(4)(rank_i \to t) \to ((rank_i \to t) \to ((rank_i \vee t) \to rank_i)).$$

Axiom (4a):

$$(1)(E_E \to E_L) \to ((E_E \to \neg E_L) \to \neg E_E),$$
$$(2)(a_i \to a) \to ((a_i \to \neg a) \to \neg a_i),$$
$$(3)(a_j \to l) \to ((a_j \to \neg l) \to \neg a_j),$$
$$(4)(rank_i \to t) \to ((rank_i \to \neg t) \to \neg rank_i).$$

Axiom (4b):

$$(1) \neg\neg E_E \to E_E,$$
$$(2) \neg\neg E_L \to E_L,$$
$$(3) \neg\neg a_i \to a_i,$$
$$(4) \neg\neg a_j \to a_j,$$
$$(5) \neg\neg l \to l,$$
$$(6) \neg\neg t \to t,$$
$$(7) \neg\neg rank_i \to rank_{i.}$$

3 CONCLUSION

This research work concludes on axiomatical expressions made from Kleene axiom schemas. In here, 10 axiom schemas were used to express enactment axioms modeled in [1]. There is now 40(forty) axiom expressions in an "E*xpressive Enactment Logic (EEL)"*. Expressive Enactment Logic is an axiomatical expression of a propositional model composite of enactment. Appiah-Kleene Axioms are labelled as *Axiom (1a)* to *Axiom (4b).*

Compliance with Ethical Standards

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Conflict of Interest:

Author, Dr. Frank Appiah declares that he has no conflict of interest .

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