Hyperboctys

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Abstract: This study introduces the Hyperboctys - Sieve of Primes, Quadratic Sequences of Primes, and Divisors.
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1 Introduction

From the Paraboctys study, we arrived at "The Hyperbolic Sieve of Primes and Products xy" [5] study. From the Hyperbolic Sieve of Primes, we arrived at the present study. The name "Hyperboctys" comes from the hyperbolic structure shown in the Hyperbolic Sieve of Primes and Products xy and Paraboctys name.

Hyperboctys is the complement of the Paraboctys and vice-versa. The two together show us that all polynomial sequences are connected. They form a unique mosaic or tessellation in 3 dimensions.

The combination Paraboctys and Hyperboctys explain how and why prime number sequences appear in polynomials. When we change a parameter of a polynomial sequence, at the same time infinite others are changed. All this happens to keep the integers in the lattice-grid. You change the mosaic or tessellation, but you never change the 3D structure that supports the properties (finite difference method) of the polynomials.

Paraboctys and hyperboctys show us why Goldbach's conjecture and Landau's problem are the same problems and how to prove them.

In this introductory study, we will expand the TMT - Triangular Multiplication Table that we found in the study The Hyperbolic Sieve of Primes and Products xy. We will introduce 5 ways to analyze the Composite number density in the multiplication table. We will start with the usual known TMT - Triangular Multiplication Table and end in the FMT - Full Multiplication Table. TMT has only the non-negative products, but FMT has all Integer numbers products.

Later we will show that the same properties existing in the TMT - Triangular Multiplication Table apply in quadratic sequences of prime numbers.

Then we will formally define what Hyperboctys is, as well as its notation. We will show some algebraic operations and possible rotations along with some examples.

Because of the operations with hyperboctys, we can add constants to “kill” the zeroes of FMT and find the Prime sequences. This idea comes from the covering system in Paraboctys: all polynomial and irreducible sequences without Zero as an element will have an infinite number of primes. It is impossible to cover all elements of a non-composite generator sequence with the possible composite generators. So, always will appear an infinite number of Prime elements.

Then we will formally define what Composite Generator is as well as where they appear in the Multiplication Table.

We will show the behavior and characteristics of Hyperboctys rotations using the Multiplication Table as an example.

Then, we will present an introduction of the polynomial sequences that form the repeated composites in the Multiplication Table.

Also, FMT and its rotations are the “cutters” or the “limiters” of quadratic Prime sequences. They have the composite generators to limit the Prime sequences.

In Paraboctys we see that it is not possible to cover any non-composite generator completely. This explains why any Prime generator sequence is not a prime-free sequence, is not a composite generator, is a non-composite generator, and has an infinite number of primes.

A comparative study of the density of prime numbers in sequences and the density of divisors in integers is still lacking.

We will study the hyperboctys variations when $Y[-1]$ and $Y[1]$ vary both in the same direction or no variation between alternatives.

Finally, we will show where the sequences of prime numbers in the Hyperboctys are found.

Enjoy yourself!

1.1 Previous conventions:

Please, as reference consult the *Conventions, notations, and abbreviations* study [2]. The latest version at https://1drv.ms/b/s!Arslv070x3WjjYUpsGLsNeWwfH6OdA?e=K1C4q5

2 The Multiplication Table (MT)

One of the results of *The Hyperbolic Sieve of Primes and Products xy study* [5] was the Multiplication Table in the hyperbolic lattice-grid.

See the multiplication table below:

10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	63	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
0	1	2	3	4	5	6	7	8	9	10

Figure 1. The 10 X 10 one quadrant multiplication table. The axis of symmetry is the sequence A000290 The Square numbers.

It is not possible to think we can divide it in exactly two triangles the n X n square multiplication table. This thought deceives us. The reason is simple:

a) any Integer x has two forms in the square multiplication table as $x * 1$ and $1 * x$.

b) any product in the multiplication table appears as $x * y$ and $y * x$.

c) but the Square numbers cannot invert the multiplicand with the multiplier.

Therefore, we need to study two triangular multiplication tables:

1) TMTS is a triangular multiplication table with the A000290 Square numbers on one edge. We can construct two TMTS. One TMTS has sides $(y, yx, y(y-0))$ and the other has sides $(x, xy, x(x-0))$.

10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	63	72	81	
8	8	16	24	32	40	48	56	64		
7	7	14	21	28	35	42	49			
6	6	12	18	24	30	36				
5	5	10	15	20	25					
4	4	8	12	16						
3	3	6	9							
2	2	4								
1	1									
0	1	2	3	4	5	6	7	8	9	10

10										100
9									81	90
8								64	72	80
7							49	56	63	70
6						36	42	48	54	60
5					25	30	35	40	45	50
4				16	20	24	28	32	36	40
3			9	12	15	18	21	24	27	30
2		4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
0	1	2	3	4	5	6	7	8	9	10

Figure 1. The 10 X 10 triangular multiplication table with the Square numbers on one edge. The sides of the triangle are (y, yx, y^2) and (x, xy, x^2).

2) TMTO is a triangular multiplication table with the Oblong numbers on one edge. We can construct two TMTS. One TMTS has sides $(y, yx, y(y-1))$ and the other has sides $(x, xy, x(x-1))$.

10	10	20	30	40	50	60	70	80	90	
9	9	18	27	36	45	54	63	72		
8	8	16	24	32	40	48	56			
7	7	14	21	28	35	42				
6	6	12	18	24	30					
5	5	10	15	20						
4	4	8	12							
3	3	6								
2	2									
1										
0	1	2	3	4	5	6	7	8	9	10

10										
9										90
8									72	80
7								56	63	70
6							42	48	54	60
5						30	35	40	45	50
4					20	24	28	32	36	40
3				12	15	18	21	24	27	30
2			6	8	10	12	14	16	18	20
1		2	3	4	5	6	7	8	9	10
0	1	2	3	4	5	6	7	8	9	10

Figure 1. The 10 X 10 triangular multiplication table with the Oblong numbers on one edge. The sides of the triangle are $(y, yx, y(y-1))$ and $(x, xy, x(x-1))$.

Now, let's study all types of Multiplication Table (MT) dividing it into 5 parts:

1. The TMTS - Triangular Multiplication Table with Squares.
2. The TMTO - Triangular Multiplication Table with Oblongs.
3. The QMT - One Quadrant Square Multiplication Table
4. The QMTLZ - One Quadrant Square Multiplication Table Less the Zeroes
5. The FMT - Full Multiplication Table

This will be important to see a new prime density approach in Prime quadratic sequences.

2.1 The TMTS - Triangular Multiplication Table with Square numbers

The TMTS triangular multiplication table with square numbers is the multiplication table with columns equal in size to the column value plus one row to include the row Zero.

In this TMTS, we are considering $C \geq 0$ disregarding the negative columns, and $R \geq 0$ disregarding the negative rows.

See below the triangular multiplication table picture showing the multiplier and the multiplicand for each product, with Y-axis inverted:

TMTS TRIANGULAR MULTIPLICATION TABLE WITH SQUARE NUMBERS: (Column C is the multiplier) * (Row R is the multiplicand) = Product								
C = Multiplier	0*R <= 0^2	1*R <= 1^2	2*R <= 2^2	3*R <= 3^2	4*R <= 4^2	5*R <= 5^2	6*R <= 6^2	7*R <= 7^2
R = Multiplicand	0	1	2	3	4	5	6	7
A000004	0 * 0 = 0	1 * 0 = 0	2 * 0 = 0	3 * 0 = 0	4 * 0 = 0	5 * 0 = 0	6 * 0 = 0	7 * 0 = 0
A000027		1 * 1 = 1	2 * 1 = 2	3 * 1 = 3	4 * 1 = 4	5 * 1 = 5	6 * 1 = 6	7 * 1 = 7
2^2+2(C-2)			2 * 2 = 4	3 * 2 = 6	4 * 2 = 8	5 * 2 = 10	6 * 2 = 12	7 * 2 = 14
3^2+3(C-3)				3 * 3 = 9	4 * 3 = 12	5 * 3 = 15	6 * 3 = 18	7 * 3 = 21
4^2+4(C-4)					4 * 4 = 16	5 * 4 = 20	6 * 4 = 24	7 * 4 = 28
5^2+5(C-5)						5 * 5 = 25	6 * 5 = 30	7 * 5 = 35
6^2+6(C-6)							6 * 6 = 36	7 * 6 = 42
7^2+7(C-7)								7 * 7 = 49

8 * 0 = 0	9 * 0 = 0	10 * 0 = 0	11 * 0 = 0	12 * 0 = 0	13 * 0 = 0	14 * 0 = 0	15 * 0 = 0	16 * 0 = 0
8 * 1 = 8	9 * 1 = 9	10 * 1 = 10	11 * 1 = 11	12 * 1 = 12	13 * 1 = 13	14 * 1 = 14	15 * 1 = 15	16 * 1 = 16
8 * 2 = 16	9 * 2 = 18	10 * 2 = 20	11 * 2 = 22	12 * 2 = 24	13 * 2 = 26	14 * 2 = 28	15 * 2 = 30	16 * 2 = 32
8 * 3 = 24	9 * 3 = 27	10 * 3 = 30	11 * 3 = 33	12 * 3 = 36	13 * 3 = 39	14 * 3 = 42	15 * 3 = 45	16 * 3 = 48
8 * 4 = 32	9 * 4 = 36	10 * 4 = 40	11 * 4 = 44	12 * 4 = 48	13 * 4 = 52	14 * 4 = 56	15 * 4 = 60	16 * 4 = 64
8 * 5 = 40	9 * 5 = 45	10 * 5 = 50	11 * 5 = 55	12 * 5 = 60	13 * 5 = 65	14 * 5 = 70	15 * 5 = 75	16 * 5 = 80
8 * 6 = 48	9 * 6 = 54	10 * 6 = 60	11 * 6 = 66	12 * 6 = 72	13 * 6 = 78	14 * 6 = 84	15 * 6 = 90	16 * 6 = 96
8 * 7 = 56	9 * 7 = 63	10 * 7 = 70	11 * 7 = 77	12 * 7 = 84	13 * 7 = 91	14 * 7 = 98	15 * 7 = 105	16 * 7 = 112
8 * 8 = 64	9 * 8 = 72	10 * 8 = 80	11 * 8 = 88	12 * 8 = 96	13 * 8 = 104	14 * 8 = 112	15 * 8 = 120	16 * 8 = 128
	9 * 9 = 81	10 * 9 = 90	11 * 9 = 99	12 * 9 = 108	13 * 9 = 117	14 * 9 = 126	15 * 9 = 135	16 * 9 = 144
		10 * 10 = 100	11 * 10 = 110	12 * 10 = 120	13 * 10 = 130	14 * 10 = 140	15 * 10 = 150	16 * 10 = 160
			11 * 11 = 121	12 * 11 = 132	13 * 11 = 143	14 * 11 = 154	15 * 11 = 165	16 * 11 = 176
				12 * 12 = 144	13 * 12 = 156	14 * 12 = 168	15 * 12 = 180	16 * 12 = 192
					13 * 13 = 169	14 * 13 = 182	15 * 13 = 195	16 * 13 = 208
						14 * 14 = 196	15 * 14 = 210	16 * 14 = 224
							15 * 15 = 225	16 * 15 = 240
								16 * 16 = 256

Map of colors:
A000004 The Zero number, in red web color #FF0000. They appear only in row 0.
A000012 The One number in blue light web color #3399CC. It appears only one in row 1 with column 1.
A000040 The Prime numbers, in blue web color #336699. Each Prime appears once in row 1.
A000290 The Square numbers (except Zero and One), in yellow web color #FFFF00.
A002378 The Oblong numbers (except Zero and Two), in red-dark web color #993333.
A005563 The Square minus One numbers (except Zero and minus One), in Orange-dark web color #FF6600.
DISTINCT COMPOSITES (entries computed in A333995) and REPEATED COMPOSITES (entries computed in A108407):
This color represents the DISTINCT COMPOSITES that will be repeated in just 1 column ahead in row 1. They are A323644 Composites with 3 or 4 divisors. Semiprime or Prime^3. {4, 6, 8, 9, 10, 14, 15, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 49, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91,
This color represents the REPEATED COMPOSITES from only 1 previous Prime column or column with product Prime^3. They appear only in row 1. They are A323644 Composites semiprime or Prime^3. {4, 6, 8, 9, 10, 14, 15, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 49, 51, 55, 57, 58, 62, 65, 69, 74, 77,
This color represents the DISTINCT COMPOSITES that will be repeated in more than 1 column ahead until it is repeated last in row 1. They are A058080 Composites with more than 4 divisors. {12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 45, 48, 50, 52, 54, 56, 60, 63, 64, 66, 68, 70, 72, 75, 76, 78,
This color represents the REPEATED COMPOSITES of the previous column(s) and will be repeated last in row 1. They are A058080 Composites with more than 4 divisors. {12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 45, 48, 50, 52, 54, 56, 60, 63, 64, 66, 68, 70, 72, 75, 76, 78, 80, 81, 84, 88, 90, 92, 96,

Figure 1. The Triangular Multiplication Table with Square numbers - TMTS. Multiplications xy from 0 to 16.

Let's call *repeated composites* that have already appeared at least once in a previous column.

Let's call *distinct composites* that did not appear in any of the previous columns.

Then, the TMTS has the Composite numbers classified into four types:

- Distinct Semiprimes or $Prime^3$ numbers.
- Repeated Semiprimes or $Prime^3$ numbers.
- Distinct Composites with more than 4 divisors numbers.
- Repeated Composites with more than 4 divisors numbers.

Semiprimes or $Prime^3$ numbers have only two pairs of multiplications between its divisors. These are the composites that appear only twice in the multiplication table: once as distinct and once as repeated composite.

We are including the line of Zeros in TMTS because we show that the element Zero is the only element in a polynomial sequence that allows us to classify the polynomial as a *Composite Generator*.

Another fact that makes us consider the line of zeroes is that we will explore many results doing rotations with the FMT - Full Multiplication Table.

In the rotations of FMT, the line of the row of Zeros does not change. The line of Zeros behaves as a reference for the rotations.

The Hyperbolic Sieve of Primes and Products xy study shows us that TMTS is a hyperbolic structure where all Repeated Composites are connected by a hyperbolic line. Consequently, when we extend the TMTS to a full table, we also extend the hyperbolic lines.

2.1.1 Triangular Multiplication Table with Squares conclusions

We get the following results:

OEIS	Column C in the TMTS -->	0	1	2	3	4	5	6	7	8	9	10
A000012	Number of Zero numbers in column C.	1	1	1	1	1	1	1	1	1	1	1
A063524	Number of Unit number in column C.	0	1	0	0	0	0	0	0	0	0	0
A010051	Number of Prime numbers in column C.	0	0	1	1	0	1	0	1	0	0	0
A113638	Number of Composite numbers in column C.	0	0	1	2	4	4	6	6	8	9	10
A000027	Number of terms in column C (not counting the Zeros).	0	1	2	3	4	5	6	7	8	9	10
A000027	Number of terms in column C.	1	2	3	4	5	6	7	8	9	10	11
.	Number of distinct Semiprimes or Prime^3 in column C.	0	0	1	2	1	3	0	4	0	1	0
.	Number of repeated Semiprimes or Prime^3 in column C.	0	0	0	0	1	0	1	0	1	1	1
.	Number of distinct Composites with more than 4 divisors in column C.	0	0	0	0	2	1	4	2	5	5	6
.	Number of repeated Composites with more than 4 divisors in column C.	0	0	0	0	0	0	1	0	2	2	3
.	Number of Semiprimes or Prime^3 in column C.	0	0	1	2	2	3	1	4	1	2	1
.	Number of Composites with more than 4 divisors in column C.	0	0	0	0	2	1	5	2	7	7	9
A333995	Number of distinct Composites in column C.	0	0	1	2	3	4	4	6	5	6	6
A108407	Number of repeated Composites in column C.	0	0	0	0	1	0	2	0	3	3	4
.	Number of Even numbers in column C (not counting the Zeros).		0	2	1	4	2	6	3	8	4	10
A142150	Number of Odd numbers in column C.		1	0	2	0	3	0	4	0	5	0

OEIS	Column C in the TMTS -->	0	1	2	3	4	5	6	7	8	9	10
A000027	Number of Zero numbers until column C.	1	2	3	4	5	6	7	8	9	10	11
A057427	Number of Unit number until column C.	0	1	1	1	1	1	1	1	1	1	1
A000720	Number of Prime numbers until column C.	0	0	1	2	2	3	3	4	4	4	4
A333996	Number of Composite numbers until column C.	0	0	1	3	7	11	17	23	31	40	50
A000217	Number of terms until column C (not counting the Zeros).	0	1	3	6	10	15	21	28	36	45	55
A000217	Number of terms until column C.	1	3	6	10	15	21	28	36	45	55	66
.	Number of distinct Semiprimes or Prime^3 until column C.	0	0	1	3	4	7	7	11	11	12	12
.	Number of repeated Semiprimes or Prime^3 until column C.	0	0	0	0	1	1	2	2	3	4	5
.	Number of distinct Composites with more than 4 divisors until column C.	0	0	0	0	2	3	7	9	14	19	25
.	Number of repeated Composites with more than 4 divisors until column C.	0	0	0	0	0	0	1	1	3	5	8
.	Number of Semiprimes or Prime^3 until column C.	0	0	1	3	5	8	9	13	14	16	17
.	Number of Composites with more than 4 divisors until column C.	0	0	0	0	2	3	8	10	17	24	33
A334454	Number of distinct Composites until column C.	0	0	1	3	6	10	14	20	25	31	37
A334455	Number of repeated Composites until column C.	0	0	0	0	1	1	3	3	6	9	13
A335624	Number of Even numbers until column C (not counting the Zeros).		0	2	3	7	9	15	18	26	30	40
.	Number of Odd numbers until column C.		1	1	3	3	6	6	10	10	15	15

Percentage	Column C in the TMTS -->	0	1	2	3	4	5	6	7	8	9	10
A%	Percentage of Zero numbers until column C.	100%	67%	50%	40%	33%	29%	25%	22%	20%	18%	17%
C%	Percentage of Unit number until column C.		33%	17%	10%	7%	5%	4%	3%	2%	2%	2%
D%	Percentage of Prime numbers until column C.		0%	17%	20%	13%	14%	11%	11%	9%	7%	6%
E%	Percentage of Composite numbers until column C.		0%	17%	30%	47%	52%	61%	64%	69%	73%	76%
B%	Percentage of terms until column C.		100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
F%	Percentage of distinct Semiprimes or Prime^3 until column C.			100%	100%	57%	64%	41%	48%	35%	30%	24%
G%	Percentage of repeated Semiprimes or Prime^3 until column C.			0%	0%	14%	9%	12%	9%	10%	10%	10%
H%	Percentage of distinct Composites with more than 4 divisors until column C.			0%	0%	29%	27%	41%	39%	45%	48%	50%
I%	Percentage of repeated Composites with more than 4 divisors until column C.			0%	0%	0%	0%	6%	4%	10%	13%	16%
J%	Percentage of Semiprimes or Prime^3 until column C.			100%	100%	71%	73%	53%	57%	45%	40%	34%
K%	Percentage of Composites with more than 4 divisors until column C.			0%	0%	29%	27%	47%	43%	55%	60%	66%
L%	Percentage of distinct Composites until column C.			100%	100%	86%	91%	82%	87%	81%	78%	74%
M%	Percentage of repeated Composites until column C.			0%	0%	14%	9%	18%	13%	19%	23%	26%
.	Number of Even numbers until column C (not counting the Zeros).		0%	67%	50%	70%	60%	71%	64%	72%	67%	73%
.	Number of Odd numbers until column C.		100%	33%	50%	30%	40%	29%	36%	28%	33%	27%

Figure 1. Results from the TMTS.

Disregarding the Zero numbers in row 0, the TMTS is a triangular pattern where:

$$A000217[C] = 1 + A000720[C] + A333996[C]$$

$$A333996[C] = A108407[C] + A333995[C]$$

$$B = C + D + E$$

$$E = F + G + H + I = J + K = L + M$$

$$J = E + G$$

$$K = H + I$$

$$L = F + H$$

$$M = G + I$$

At TMTS, the x and y divisors that generate the repeated composite products appear only once. No pair of divisors is repeated.

2.2 TMTO - Triangular Multiplication Table with Oblong numbers

The *TMTO - Triangular Multiplication Table with Oblong numbers* is also a triangular multiplication table.

We make TMTO from TMTS by removing the sequence of Square numbers that form the diagonal side.

TMTO has columns equal in size to the column value minus One plus one row to include the Zero column.

So, TMTO has columns equal in size to the column value.

In TMTO, we are also disregarding the negative rows and columns.

TMTO TRIANGULAR MULTIPLICATION TABLE WITH OBLONG NUMBERS								
C = Multiplier	0*R < 0^2	1*R < 1^2	2*R < 2^2	3*R < 3^2	4*R < 4^2	5*R < 5^2	6*R < 6^2	7*R < 7^2
R = Multiplicand	0	1	2	3	4	5	6	7
A000004		1 * 0 = 0	2 * 0 = 0	3 * 0 = 0	4 * 0 = 0	5 * 0 = 0	6 * 0 = 0	7 * 0 = 0
A000027			2 * 1 = 2	3 * 1 = 3	4 * 1 = 4	5 * 1 = 5	6 * 1 = 6	7 * 1 = 7
2^2+2(C-2)				3 * 2 = 6	4 * 2 = 8	5 * 2 = 10	6 * 2 = 12	7 * 2 = 14
3^2+3(C-3)					4 * 3 = 12	5 * 3 = 15	6 * 3 = 18	7 * 3 = 21
4^2+4(C-4)						5 * 4 = 20	6 * 4 = 24	7 * 4 = 28
5^2+5(C-5)							6 * 5 = 30	7 * 5 = 35
6^2+6(C-6)								7 * 6 = 42

8*R < 8^2	9*R < 9^2	10*R < 10^2	11*R < 11^2	12*R < 12^2	13*R < 13^2	14*R < 14^2	15*R < 15^2	16*R < 16^2
8	9	10	11	12	13	14	15	16
8 * 0 = 0	9 * 0 = 0	10 * 0 = 0	11 * 0 = 0	12 * 0 = 0	13 * 0 = 0	14 * 0 = 0	15 * 0 = 0	16 * 0 = 0
8 * 1 = 8	9 * 1 = 9	10 * 1 = 10	11 * 1 = 11	12 * 1 = 12	13 * 1 = 13	14 * 1 = 14	15 * 1 = 15	16 * 1 = 16
8 * 2 = 16	9 * 2 = 18	10 * 2 = 20	11 * 2 = 22	12 * 2 = 24	13 * 2 = 26	14 * 2 = 28	15 * 2 = 30	16 * 2 = 32
8 * 3 = 24	9 * 3 = 27	10 * 3 = 30	11 * 3 = 33	12 * 3 = 36	13 * 3 = 39	14 * 3 = 42	15 * 3 = 45	16 * 3 = 48
8 * 4 = 32	9 * 4 = 36	10 * 4 = 40	11 * 4 = 44	12 * 4 = 48	13 * 4 = 52	14 * 4 = 56	15 * 4 = 60	16 * 4 = 64
8 * 5 = 40	9 * 5 = 45	10 * 5 = 50	11 * 5 = 55	12 * 5 = 60	13 * 5 = 65	14 * 5 = 70	15 * 5 = 75	16 * 5 = 80
8 * 6 = 48	9 * 6 = 54	10 * 6 = 60	11 * 6 = 66	12 * 6 = 72	13 * 6 = 78	14 * 6 = 84	15 * 6 = 90	16 * 6 = 96
8 * 7 = 56	9 * 7 = 63	10 * 7 = 70	11 * 7 = 77	12 * 7 = 84	13 * 7 = 91	14 * 7 = 98	15 * 7 = 105	16 * 7 = 112
	9 * 8 = 72	10 * 8 = 80	11 * 8 = 88	12 * 8 = 96	13 * 8 = 104	14 * 8 = 112	15 * 8 = 120	16 * 8 = 128
		10 * 9 = 90	11 * 9 = 99	12 * 9 = 108	13 * 9 = 117	14 * 9 = 126	15 * 9 = 135	16 * 9 = 144
			11 * 10 = 110	12 * 10 = 120	13 * 10 = 130	14 * 10 = 140	15 * 10 = 150	16 * 10 = 160
				12 * 11 = 132	13 * 11 = 143	14 * 11 = 154	15 * 11 = 165	16 * 11 = 176
					13 * 12 = 156	14 * 12 = 168	15 * 12 = 180	16 * 12 = 192
						14 * 13 = 182	15 * 13 = 195	16 * 13 = 208
							15 * 14 = 210	16 * 14 = 224
								16 * 15 = 240

Map of colors:
A000004 The Zero number, in red web color #FF0000. They appear only in row 0.
A000012 The One number in blue light web color #3399CC. It appears only one in row 1 with column 1.
A000040 The Prime numbers, in blue web color #336699. Each Prime appears once in row 1.
A000290 The Square numbers (except Zero and One), in yellow web color #FFFF00.
A002378 The Oblong numbers (except Zero and Two), in red-dark web color #993333.
A005563 The Square minus One numbers (except Zero and minus One), in Orange-dark web color #FF6600.
DISTINCT COMPOSITES (entries computed in A333995) and REPEATED COMPOSITES (entries computed in A108407):
This color represents the DISTINCT COMPOSITES that will be repeated in just 1 column ahead in row 1. They are A323644 Composites with 3 or 4 divisors. Semiprime or Prime^3. {4, 6, 8, 9, 10, 14, 15, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 49, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91,
This color represents the REPEATED COMPOSITES from only 1 previous Prime column or column with product Prime^3. They appear only in row 1. They are A323644 Composites semiprime or Prime^3. {4, 6, 8, 9, 10, 14, 15, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 49, 51, 55, 57, 58, 62, 65, 69, 74, 77,
This color represents the DISTINCT COMPOSITES that will be repeated in more than 1 column ahead until it is repeated last in row 1. They are A058080 Composites with more than 4 divisors. {12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 45, 48, 50, 52, 54, 56, 60, 63, 64, 66, 68, 70, 72, 75, 76, 78,
This color represents the REPEATED COMPOSITES of the previous column(s) and will be repeated last in row 1. They are A058080 Composites with more than 4 divisors. {12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 45, 48, 50, 52, 54, 56, 60, 63, 64, 66, 68, 70, 72, 75, 76, 78, 80, 81, 84, 88, 90, 92, 96,

Figure 1. The TMTO - Triangular Multiplication Table with Oblong numbers.

2.2.1 Conclusions from the TMTO - Triangular Multiplication Table with Oblong numbers

We get the following results:

OEIS	Column C in the TMTO-->	0	1	2	3	4	5	6	7	8	9	10
A057427	Number of Zero numbers in column C.	0	1	1	1	1	1	1	1	1	1	1
	Number of Unit number in column C.	0	0	0	0	0	0	0	0	0	0	0
A010051	Number of Prime numbers in column C.	0	0	1	1	0	1	0	1	0	0	0
	Number of Composite numbers in column C.	0	0	0	1	3	3	5	5	7	8	9
A001477	Number of terms in column C.	0	1	2	3	4	5	6	7	8	9	10
	Number of distinct Semiprimes or Prime^3 in column C.	0	0	0	1	0	2	0	3	0	1	0
	Number of repeated Semiprimes or Prime^3 in column C.	0	0	0	0	1	0	1	0	1	1	1
	Number of distinct Composites with more than 4 divisors in column C.	0	0	0	0	2	1	3	2	4	4	5
	Number of repeated Composites with more than 4 divisors in column C.	0	0	0	0	0	0	1	0	2	2	3
	Number of Semiprimes or Prime^3 in column C.	0	0	0	1	1	2	1	3	1	2	1
	Number of Composites with more than 4 divisors in column C.	0	0	0	0	2	1	4	2	6	6	8
A333995	Number of distinct Composites in column C.	0	0	0	1	2	3	3	5	4	5	5
A108407	Number of repeated Composites in column C.	0	0	0	0	1	0	2	0	3	3	4

OEIS	Column C in the TMTO-->	0	1	2	3	4	5	6	7	8	9	10
A001477	Number of Zero numbers until column C.	0	1	2	3	4	5	6	7	8	9	10
	Number of Unit number until column C.	0	0	0	0	0	0	0	0	0	0	0
A000720	Number of Prime numbers until column C.	0	0	1	2	2	3	3	4	4	4	4
	Number of Composite numbers until column C.	0	0	0	1	4	7	12	17	24	32	41
A000217	Number of terms until column C.	0	1	3	6	10	15	21	28	36	45	55
	Number of distinct Semiprimes or Prime^3 until column C.	0	0	0	1	1	3	3	6	6	7	7
	Number of repeated Semiprimes or Prime^3 until column C.	0	0	0	0	1	1	2	2	3	4	5
	Number of distinct Composites with more than 4 divisors until column C.	0	0	0	0	2	3	6	8	12	16	21
	Number of repeated Composites with more than 4 divisors until column C.	0	0	0	0	0	0	1	1	3	5	8
	Number of Semiprimes or Prime^3 until column C.	0	0	0	1	2	4	5	8	9	11	12
	Number of Composites with more than 4 divisors until column C.	0	0	0	0	2	3	7	9	15	21	29
	Number of distinct Composites until column C.	0	0	0	1	3	6	9	14	18	23	28
	Number of repeated Composites until column C.	0	0	0	0	1	1	3	3	6	9	13

Percentage	Column C in the TMTO-->	0	1	2	3	4	5	6	7	8	9	10
A	Percentage of Zero numbers until column C.		100%	67%	50%	40%	33%	29%	25%	22%	20%	18%
B	Percentage of Unit number until column C.		0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
C	Percentage of Prime numbers until column C.		0%	33%	33%	20%	20%	14%	14%	11%	9%	7%
D	Percentage of Composite numbers until column C.		0%	0%	17%	40%	47%	57%	61%	67%	71%	75%
E	Percentage of terms until column C.		100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
F	Percentage of distinct Semiprimes or Prime^3 until column C.				100%	25%	43%	25%	35%	25%	22%	17%
G	Percentage of repeated Semiprimes or Prime^3 until column C.				0%	25%	14%	17%	12%	13%	13%	12%
H	Percentage of distinct Composites with more than 4 divisors until column C.				0%	50%	43%	50%	47%	50%	50%	51%
I	Percentage of repeated Composites with more than 4 divisors until column C.				0%	0%	0%	8%	6%	13%	16%	20%
J	Percentage of Semiprimes or Prime^3 until column C.				100%	50%	57%	42%	47%	38%	34%	29%
K	Percentage of Composites with more than 4 divisors until column C.				0%	50%	43%	58%	53%	63%	66%	71%
L	Percentage of distinct Composites until column C.				100%	75%	86%	75%	82%	75%	72%	68%
M	Percentage of repeated Composites until column C.				0%	25%	14%	25%	18%	25%	28%	32%

Figure 1. Results from the TMTO - Triangular Multiplication Table Less the Square hypotenuse.

At TMTO, the x and y divisors that generate the repeated composite products also appear only once, but it is not complete. It is missing the divisors of the Squares. No pair of divisors is repeated.

2.3 The QMT - One Quadrant Square Multiplication Table

The "*QMT - One Quadrant Square Multiplication Table*" is square.

It is made by the combination of TMT plus TMTO.

In QMT, we are also disregarding the negative rows and columns.

20	0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18	0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17	0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16	0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 1. The QMT - One Quadrant Square Multiplication Table.

2.3.1 Conclusions from the QMT - One Quadrant Square Multiplication Table

We get the following results:

OEIS	Column C in the QMT Quadrant Square Multiplication Table -->	0	1	2	3	4	5	6	7	8	9	10
A040000	Number of Zero numbers in column C.	1	2	2	2	2	2	2	2	2	2	2
A063524	Number of Unit number in column C.	0	1	0	0	0	0	0	0	0	0	0
.	Number of Prime numbers in column C.	0	0	2	2	0	2	0	2	0	0	0
.	Number of Composite numbers in column C.	0	0	1	3	7	7	11	11	15	17	19
A005408	Number of terms in column C.	1	3	5	7	9	11	13	15	17	19	21
.	Number of distinct Semiprimes or Prime^3 in column C.	0	0	1	3	1	5	0	7	0	2	0
.	Number of repeated Semiprimes or Prime^3 in column C.	0	0	0	0	2	0	2	0	2	2	2
.	Number of distinct Composites with more than 4 divisors in column C.	0	0	0	0	4	2	7	4	9	9	11
.	Number of repeated Composites with more than 4 divisors in column C.	0	0	0	0	0	0	2	0	4	4	6
.	Number of Semiprimes or Prime^3 in column C.	0	0	1	3	3	5	2	7	2	4	2
.	Number of Composites with more than 4 divisors in column C.	0	0	0	0	4	2	9	4	13	13	17
.	Number of distinct Composites in column C.	0	0	1	3	5	7	7	11	9	11	11
.	Number of repeated Composites in column C.	0	0	0	0	2	0	4	0	6	6	8

OEIS	Column C in the QMT Quadrant Square Multiplication Table -->	0	1	2	3	4	5	6	7	8	9	10
A005843	Number of Zero numbers until column C.	0	2	4	6	8	10	12	14	16	18	20
A057427	Number of Unit number until column C.	0	1	1	1	1	1	1	1	1	1	1
.	Number of Prime numbers until column C.	0	0	2	4	4	6	6	8	8	8	8
.	Number of Composite numbers until column C.	0	0	1	4	11	18	29	40	55	72	91
A005563	Number of terms until column C.	0	3	8	15	24	35	48	63	80	99	120
.	Number of distinct Semiprimes or Prime^3 until column C.	0	0	1	4	5	10	10	17	17	19	19
.	Number of repeated Semiprimes or Prime^3 until column C.	0	0	0	0	2	2	4	4	6	8	10
.	Number of distinct Composites with more than 4 divisors until column C.	0	0	0	0	4	6	13	17	26	35	46
.	Number of repeated Composites with more than 4 divisors until column C.	0	0	0	0	0	0	2	2	6	10	16
.	Number of Semiprimes or Prime^3 until column C.	0	0	1	4	7	12	14	21	23	27	29
.	Number of Composites with more than 4 divisors until column C.	0	0	0	0	4	6	15	19	32	45	62
.	Number of distinct Composites until column C.	0	0	1	4	9	16	23	34	43	54	65
.	Number of repeated Composites until column C.	0	0	0	0	2	2	6	6	12	18	26

Percentage	Column C in the QMT Quadrant Square Multiplication Table -->	0	1	2	3	4	5	6	7	8	9	10
A	Percentage of Zero numbers until column C.		67%	50%	40%	33%	29%	25%	22%	20%	18%	17%
B	Percentage of Unit number until column C.		33%	13%	7%	4%	3%	2%	2%	1%	1%	1%
C	Percentage of Prime numbers until column C.		0%	25%	27%	17%	17%	13%	13%	10%	8%	7%
D	Percentage of Composite numbers until column C.		0%	13%	27%	46%	51%	60%	63%	69%	73%	76%
E	Percentage of terms until column C.		100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
F	Percentage of distinct Semiprimes or Prime^3 until column C.				100%	45%	56%	34%	43%	31%	26%	21%
G	Percentage of repeated Semiprimes or Prime^3 until column C.				0%	18%	11%	14%	10%	11%	11%	11%
H	Percentage of distinct Composites with more than 4 divisors until column C.				0%	36%	33%	45%	43%	47%	49%	51%
I	Percentage of repeated Composites with more than 4 divisors until column C.				0%	0%	0%	7%	5%	11%	14%	18%
J	Percentage of Semiprimes or Prime^3 until column C.				100%	64%	67%	48%	53%	42%	38%	32%
K	Percentage of Composites with more than 4 divisors until column C.				0%	36%	33%	52%	48%	58%	63%	68%
L	Percentage of distinct Composites until column C.				100%	82%	89%	79%	85%	78%	75%	71%
M	Percentage of repeated Composites until column C.				0%	18%	11%	21%	15%	22%	25%	29%

Figure 1. Results from the QMT - One Quadrant Square Multiplication Table.

In QMT the repeated composites with different divisors pair appear twice, but the divisors pair of the Squares appear once.

2.4 QMTLZ - One Quadrant Square Multiplication Table Less the Zeroes

The "*QMTLZ - One Quadrant Square Multiplication Table Less the Zeroes*" is also a square multiplication table.

We make it from QMT disregarding the line and column of the Zeroes.

In QMTLZ, we are also disregarding the negative rows and columns.

20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 1. The QMTLZ - One Quadrant Square Multiplication Table Less the Zeroes.

2.4.1 Conclusions from the QMTLZ - One Quadrant Square Multiplication Table Less the Zeroes

We get the following results:

OEIS	Column C in the QMTLZ Quadrant Square Multiplication Less Zero Table -->	0	1	2	3	4	5	6	7	8	9	10
A000004	Number of Zero numbers in column C.	0	0	0	0	0	0	0	0	0	0	0
A063524	Number of Unit number in column C.	0	1	0	0	0	0	0	0	0	0	0
.	Number of Prime numbers in column C.	0	0	2	2	0	2	0	2	0	0	0
.	Number of Composite numbers in column C.	0	0	1	3	7	7	11	11	15	17	19
A004273	Number of terms in column C.	0	1	3	5	7	9	11	13	15	17	19
.	Number of distinct Semiprimes or Prime^3 in column C.	0	0	1	3	1	5	0	7	0	2	0
.	Number of repeated Semiprimes or Prime^3 in column C.	0	0	0	0	2	0	2	0	2	2	2
.	Number of distinct Composites with more than 4 divisors in column C.	0	0	0	0	4	2	7	4	9	9	11
.	Number of repeated Composites with more than 4 divisors in column C.	0	0	0	0	0	0	2	0	4	4	6
.	Number of Semiprimes or Prime^3 in column C.	0	0	1	3	3	5	2	7	2	4	2
.	Number of Composites with more than 4 divisors in column C.	0	0	0	0	4	2	9	4	13	13	17
.	Number of distinct Composites in column C.	0	0	1	3	5	7	7	11	9	11	11
.	Number of repeated Composites in column C.	0	0	0	0	2	0	4	0	6	6	8

OEIS	Column C in the QMTLZ Quadrant Square Multiplication Less Zero Table -->	0	1	2	3	4	5	6	7	8	9	10
A000004	Number of Zero numbers until column C.	0	0	0	0	0	0	0	0	0	0	0
A057427	Number of Unit number until column C.	0	1	1	1	1	1	1	1	1	1	1
.	Number of Prime numbers until column C.	0	0	2	4	4	6	6	8	8	8	8
.	Number of Composite numbers until column C.	0	0	1	4	11	18	29	40	55	72	91
A000290	Number of terms until column C.	0	1	4	9	16	25	36	49	64	81	100
.	Number of distinct Semiprimes or Prime^3 until column C.	0	0	1	4	5	10	10	17	17	19	19
.	Number of repeated Semiprimes or Prime^3 until column C.	0	0	0	0	2	2	4	4	6	8	10
.	Number of distinct Composites with more than 4 divisors until column C.	0	0	0	0	4	6	13	17	26	35	46
.	Number of repeated Composites with more than 4 divisors until column C.	0	0	0	0	0	0	2	2	6	10	16
.	Number of Semiprimes or Prime^3 until column C.	0	0	1	4	7	12	14	21	23	27	29
.	Number of Composites with more than 4 divisors until column C.	0	0	0	0	4	6	15	19	32	45	62
.	Number of distinct Composites until column C.	0	0	1	4	9	16	23	34	43	54	65
.	Number of repeated Composites until column C.	0	0	0	0	2	2	6	6	12	18	26

Percentage	Column C in the QMTLZ Quadrant Square Multiplication Less Zero Table -->	0	1	2	3	4	5	6	7	8	9	10
A	Percentage of Zero numbers until column C.		0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
B	Percentage of Unit number until column C.		100%	25%	11%	6%	4%	3%	2%	2%	1%	1%
C	Percentage of Prime numbers until column C.		0%	50%	44%	25%	24%	17%	16%	13%	10%	8%
D	Percentage of Composite numbers until column C.		0%	25%	44%	69%	72%	81%	82%	86%	89%	91%
E	Percentage of terms until column C.		100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
F	Percentage of distinct Semiprimes or Prime^3 until column C.				100%	45%	56%	34%	43%	31%	26%	21%
G	Percentage of repeated Semiprimes or Prime^3 until column C.				0%	18%	11%	14%	10%	11%	11%	11%
H	Percentage of distinct Composites with more than 4 divisors until column C.				0%	36%	33%	45%	43%	47%	49%	51%
I	Percentage of repeated Composites with more than 4 divisors until column C.				0%	0%	0%	7%	5%	11%	14%	18%
J	Percentage of Semiprimes or Prime^3 until column C.				100%	64%	67%	48%	53%	42%	38%	32%
K	Percentage of Composites with more than 4 divisors until column C.				0%	36%	33%	52%	48%	58%	63%	68%
L	Percentage of distinct Composites until column C.				100%	82%	89%	79%	85%	78%	75%	71%
M	Percentage of repeated Composites until column C.				0%	18%	11%	21%	15%	22%	25%	29%

Figure 1. Results from the QMTLZ - One Quadrant Square Multiplication Table Less the Zeroes.

The behavior of the composite numbers in table QMT and QMTLZ are identical.

2.5 The FMT - Full Multiplication Table

The "*FMT - Full Multiplication Table*" is the real complete square multiplication table. It has all the Integers: the positive, the negative, and the Zero.

The FMT is the result of 4 QMTLZ plus the vertical and horizontal lines of Zeroes.

So, $FMT = (4 * QMTLZ[C] + 4C) + 1$.

-100	-90	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100
-90	-81	-72	-63	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81	90
-80	-72	-64	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72	80
-70	-63	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70
-60	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60
-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50
-40	-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40
-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30
-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20
30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	-30
40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36	-40
50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54	-60
70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56	-63	-70
80	72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56	-64	-72	-80
90	81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63	-72	-81	-90
100	90	80	70	60	50	40	30	20	10	0	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100

Figure 1. The FMT - Full Multiplication Table.

The FMT is covered by all hyperbolic lines $Y[y] = xy$ in all quadrants.

2.5.1 Conclusions from the FMT - Full Multiplication Table

We get the following results:

OEIS	Column C in the FMT Full Multiplication Table -->	0	1	2	3	4	5	6	7	8	9	10
A123932	Number of Zero numbers in column C.	1	4	4	4	4	4	4	4	4	4	4
.	Number of Unit number in column C.	0	4	0	0	0	0	0	0	0	0	0
.	Number of Prime numbers in column C.	0	0	8	8	0	8	0	8	0	0	0
.	Number of Composite numbers in column C.	0	0	4	12	28	28	44	44	60	68	76
A008590	Number of terms in column C.	1	8	16	24	32	40	48	56	64	72	80
.	Number of distinct Semiprimes or Prime^3 in column C.	0	0	4	12	4	20	0	28	0	8	0
.	Number of repeated Semiprimes or Prime^3 in column C.	0	0	0	0	8	0	8	0	8	8	8
.	Number of distinct Composites with more than 4 divisors in column C.	0	0	0	0	16	8	28	16	36	36	44
.	Number of repeated Composites with more than 4 divisors in column C.	0	0	0	0	0	0	8	0	16	16	24
.	Number of Semiprimes or Prime^3 in column C.	0	0	4	12	12	20	8	28	8	16	8
.	Number of Composites with more than 4 divisors in column C.	0	0	0	0	16	8	36	16	52	52	68
.	Number of distinct Composites in column C.	0	0	4	12	20	28	28	44	36	44	44
.	Number of repeated Composites in column C.	0	0	0	0	8	0	16	0	24	24	32

OEIS	Column C in the FMT Full Multiplication Table -->	0	1	2	3	4	5	6	7	8	9	10
A016813	Number of Zero numbers until column C.	1	5	9	13	17	21	25	29	33	37	41
A297217	Number of Unit number until column C.	0	4	4	4	4	4	4	4	4	4	4
.	Number of Prime numbers until column C.	0	0	8	16	16	24	24	32	32	32	32
.	Number of Composite numbers until column C.	0	0	4	16	44	72	116	160	220	288	364
A016754	Number of terms until column C.	1	9	25	49	81	121	169	225	289	361	441
.	Number of distinct Semiprimes or Prime^3 until column C.	0	0	4	16	20	40	40	68	68	76	76
.	Number of repeated Semiprimes or Prime^3 until column C.	0	0	0	0	8	8	16	16	24	32	40
.	Number of distinct Composites with more than 4 divisors until column C.	0	0	0	0	16	24	52	68	104	140	184
.	Number of repeated Composites with more than 4 divisors until column C.	0	0	0	0	0	0	8	8	24	40	64
.	Number of Semiprimes or Prime^3 until column C.	0	0	4	16	28	48	56	84	92	108	116
.	Number of Composites with more than 4 divisors until column C.	0	0	0	0	16	24	60	76	128	180	248
.	Number of distinct Composites until column C.	0	0	4	16	36	64	92	136	172	216	260
.	Number of repeated Composites until column C.	0	0	0	0	8	8	24	24	48	72	104

Percentage	Column C in the FMT Full Multiplication Table -->	0	1	2	3	4	5	6	7	8	9	10
A	Percentage of Zero numbers until column C.		56%	36%	27%	21%	17%	15%	13%	11%	10%	9%
B	Percentage of Unit number until column C.		44%	16%	8%	5%	3%	2%	2%	1%	1%	1%
C	Percentage of Prime numbers until column C.		0%	32%	33%	20%	20%	14%	14%	11%	9%	7%
D	Percentage of Composite numbers until column C.		0%	16%	33%	54%	60%	69%	71%	76%	80%	83%
E	Percentage of terms until column C.		100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
F	Percentage of distinct Semiprimes or Prime^3 until column C.				100%	45%	56%	34%	43%	31%	26%	21%
G	Percentage of repeated Semiprimes or Prime^3 until column C.				0%	18%	11%	14%	10%	11%	11%	11%
H	Percentage of distinct Composites with more than 4 divisors until column C.				0%	36%	33%	45%	43%	47%	49%	51%
I	Percentage of repeated Composites with more than 4 divisors until column C.				0%	0%	0%	7%	5%	11%	14%	18%
J	Percentage of Semiprimes or Prime^3 until column C.				100%	64%	67%	48%	53%	42%	38%	32%
K	Percentage of Composites with more than 4 divisors until column C.				0%	36%	33%	52%	48%	58%	63%	68%
L	Percentage of distinct Composites until column C.				100%	82%	89%	79%	85%	78%	75%	71%
M	Percentage of repeated Composites until column C.				0%	18%	11%	21%	15%	22%	25%	29%

Figure 1. Results from the FMT - Full Multiplication Table.

The composite numbers behavior of the FMT is identical to the sum of two positive QMT and two negative QMT.

2.5.2 The hyperbolic grid of the FMT

From figure 3. in the study [The Hyperbolic Sieve of Primes and Products xy], we showed only the 1st quadrant. We can now extend it to all 4 quadrants. See the result in the XY plane.

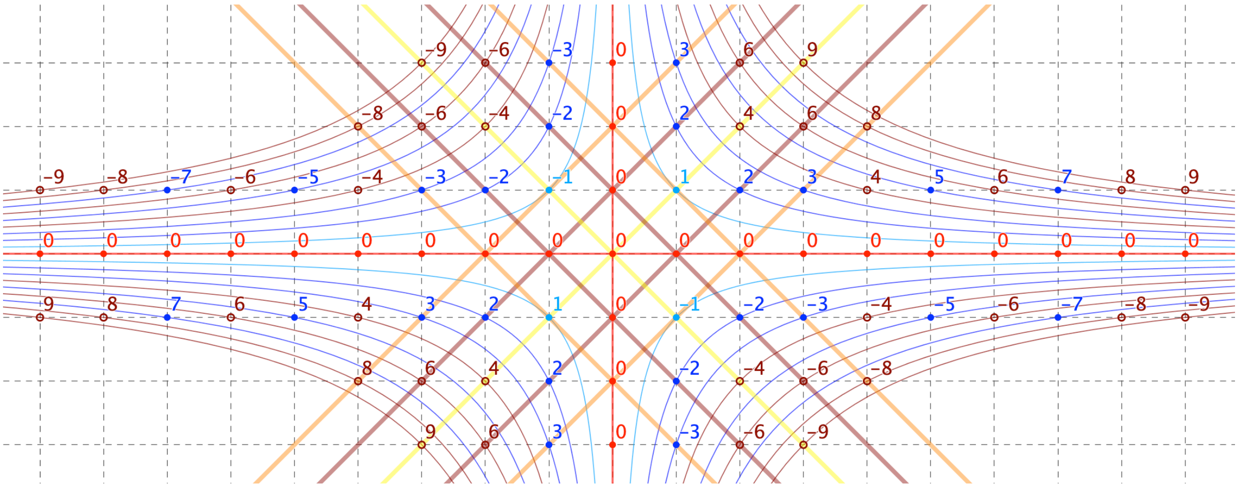

Figure 1. The FMT in XY plane

See this result in table format:

C.G. @ a=0		-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
Δ		100	81	64	49	36	25	16	9	4	1	0	1	4	9	16	25	36	49	64	81	100
|√Δ|		10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10
C. G.		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b		-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
c		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	-200	-180	-160	-140	-120	-100	-80	-60	-40	-20	0	20	40	60	80	100	120	140	160	180	200
	19	-190	-171	-152	-133	-114	-95	-76	-57	-38	-19	0	19	38	57	76	95	114	133	152	171	190
	18	-180	-162	-144	-126	-108	-90	-72	-54	-36	-18	0	18	36	54	72	90	108	126	144	162	180
	17	-170	-153	-136	-119	-102	-85	-68	-51	-34	-17	0	17	34	51	68	85	102	119	136	153	170
	16	-160	-144	-128	-112	-96	-80	-64	-48	-32	-16	0	16	32	48	64	80	96	112	128	144	160
	15	-150	-135	-120	-105	-90	-75	-60	-45	-30	-15	0	15	30	45	60	75	90	105	120	135	150
	14	-140	-126	-112	-98	-84	-70	-56	-42	-28	-14	0	14	28	42	56	70	84	98	112	126	140
	13	-130	-117	-104	-91	-78	-65	-52	-39	-26	-13	0	13	26	39	52	65	78	91	104	117	130
	12	-120	-108	-96	-84	-72	-60	-48	-36	-24	-12	0	12	24	36	48	60	72	84	96	108	120
	11	-110	-99	-88	-77	-66	-55	-44	-33	-22	-11	0	11	22	33	44	55	66	77	88	99	110
	10	-100	-90	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100
	9	-90	-81	-72	-63	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81	90
	8	-80	-72	-64	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72	80
	7	-70	-63	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70
	6	-60	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60
	5	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50
	4	-40	-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40
	3	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30
	2	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20
Y[1]	1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
	-2	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20
	-3	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	-30
	-4	40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36	-40
	-5	50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
	-6	60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54	-60
	-7	70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56	-63	-70
	-8	80	72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56	-64	-72	-80
	-9	90	81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63	-72	-81	-90
	-10	100	90	80	70	60	50	40	30	20	10	0	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100
	-11	110	99	88	77	66	55	44	33	22	11	0	-11	-22	-33	-44	-55	-66	-77	-88	-99	-110
	-12	120	108	96	84	72	60	48	36	24	12	0	-12	-24	-36	-48	-60	-72	-84	-96	-108	-120
	-13	130	117	104	91	78	65	52	39	26	13	0	-13	-26	-39	-52	-65	-78	-91	-104	-117	-130
	-14	140	126	112	98	84	70	56	42	28	14	0	-14	-28	-42	-56	-70	-84	-98	-112	-126	-140
	-15	150	135	120	105	90	75	60	45	30	15	0	-15	-30	-45	-60	-75	-90	-105	-120	-135	-150
	-16	160	144	128	112	96	80	64	48	32	16	0	-16	-32	-48	-64	-80	-96	-112	-128	-144	-160
	-17	170	153	136	119	102	85	68	51	34	17	0	-17	-34	-51	-68	-85	-102	-119	-136	-153	-170
	-18	180	162	144	126	108	90	72	54	36	18	0	-18	-36	-54	-72	-90	-108	-126	-144	-162	-180
	-19	190	171	152	133	114	95	76	57	38	19	0	-19	-38	-57	-76	-95	-114	-133	-152	-171	-190
	-20	200	180	160	140	120	100	80	60	40	20	0	-20	-40	-60	-80	-100	-120	-140	-160	-180	-200

Figure 1. The Hyperbolic Lattice-Grid with its hyperbolas $xy = Integer$ in all quadrants

The equations of the hyperbola's lines are of the form $xy = Integer$.
The equations of the Composite Generators lines are of the form $x = \pm y \pm k$.

Note that we base this entire structure on the three central elements [0,0,0].

In the verticals and horizontals, we have a linear function (or a quadratic function with a coefficient $a = 0$).

In the verticals, we have the sequences $Y[y] = by$. Because $b = x$, then $Y[y] = xy$.

In the horizontals, we have the sequences $Y[y] = yb$. Because $b = x$, then $Y[y] = yx$.

- In column zero and row zero, we have only the Zero numbers.
- In columns and rows ± 1, we have all the Integer numbers OEIS A256958. In these two columns and two rows are the only sequences that will appear all the Prime numbers.
- In columns and rows ± 2, we have all the Integer numbers in the form of $2n$. Positive and negative Two are the only Primes in these sequences. The Primes are next to Zero.
- In columns and rows ± 3, we have all the Integer numbers in the form of $3n$. Positive and negative Three are the only Primes in these sequences. The Primes are next to Zero.
- In columns and rows ± 4, we have all the Integer numbers in the form of $4n$. There are no Primes in these sequences.
- In columns and rows ± 5, we have all the Integer numbers in the form of $5n$. Positive and negative Five are the only Primes in these sequences. The Primes are next to Zero.
- In columns and rows ± 6, we have all the Integer numbers in the form of $6n$. There are no Primes in these sequences.
- In columns and rows ± 7, we have all the Integer numbers in the form of $7n$. Positive and negative Seven are the only Primes in these sequences. The Primes are next to Zero.
- In columns and rows ± 8, we have all the Integer numbers in the form of $8n$. There are no Primes in these sequences.
- And so on…

2.5.3Conclusion

The Full Multiplication Table is a hyperbolic lattice-grid.

3 Definition of *HYPERBOCTYS*:

The FMT gives us a clue of a general hyperbolic structure with the following algorithm:

Columns C --->		-5	-4	-3	-2	-1	0	1	2	3	4	5
a		(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2	(g-2h+i)/2
b		(i-g)/2-5	(i-g)/2-4	(i-g)/2-3	(i-g)/2-2	(i-g)/2-1	(i-g)/2	(i-g)/2+1	(i-g)/2+2	(i-g)/2+3	(i-g)/2+4	(i-g)/2+5
c		h	h	h	h	h	h	h	h	h	h	h
	10	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	9	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	8	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	7	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	6	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	5	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	4	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	3	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	2	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
Y[1]=x_3	1	i-5	i-4	i-3	i-2	i-1	i	i+1	i+2	i+3	i+4	i+5
Y[0]=x_2	0	h	h	h	h	h	h	h	h	h	h	h
Y[-1]=x_1	-1	g+5	g+4	g+3	g+2	g+1	g	g-1	g-2	g-3	g-4	g-5
	-2	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-3	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-4	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-5	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-6	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-7	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-8	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-9	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE
	-10	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE	VALUE

Figure 1. The hyperboctys structure algorithm. The verticals are quadratics based on the 3 consecutive elements $\left[Y[-1],Y[0],Y[1]\right]$.

Each column is a 2nd-degree polynomial equation of the form

$$Y[y]=ay^2+by+c$$

The coefficients are determined by 3 consecutive elements of the vertical sequence $\left[Y[-1],Y[0],Y[1]\right]=[x_1,x_2,x_3]$. The quadratic equation is

$$[x_1,x_2,x_3]\equiv Y[y]=\left(\frac{x_1-2x_2+x_3}{2}\right)y^2+\left(\frac{x_3-x_1}{2}\right)y+x_2$$

As we increase the value of the column from left to right, to maintain the same hyperbolic structure of FMT then, the 3 consecutive elements of the vertical sequence $[Y[-1], Y[0], Y[1]]$ need to vary:

- $Y[1] = x_3$ increase its value by one unit
- $Y[0] = x_2$ keeps its value constant
- $Y[-1] = x_1$ decrease its value by one unit

Then, for each column C we have:

$$\dots$$

$$[\mathrm{g}+3, \mathrm{h}, \mathrm{i}-3] \equiv Y_{@C=-3}[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}-3\right)y + \mathrm{h}$$

$$[\mathrm{g}+2, \mathrm{h}, \mathrm{i}-2] \equiv Y_{@C=-2}[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}-2\right)y + \mathrm{h}$$

$$[\mathrm{g}+1, \mathrm{h}, \mathrm{i}-1] \equiv Y_{@C=-1}[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}-1\right)y + \mathrm{h}$$

$$[\mathrm{g}, \mathrm{h}, \mathrm{i}] \equiv Y_{@C=0}[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}\right)y + \mathrm{h}$$

$$[\mathrm{g}-1, \mathrm{h}, \mathrm{i}+1] \equiv Y_{@C=1}[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+1\right)y + \mathrm{h}$$

$$[\mathrm{g}-2, \mathrm{h}, \mathrm{i}+2] \equiv Y_{@C=2}[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+2\right)y + \mathrm{h}$$

$$[\mathrm{g}-3, \mathrm{h}, \mathrm{i}+3] \equiv Y_{@C=3}[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+3\right)y + \mathrm{h}$$

$$\dots$$

Generically, we will denote any vertical as being

$$[\mathrm{g}-\mathrm{C}, \mathrm{h}, \mathrm{i}+\mathrm{C}] \equiv Y[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y + \mathrm{h}$$

3.1 Hyperboctys notation

Because just 3 central elements from column Zero $[Y[-1], Y[0], Y[1]] = [x_1, x_2, x_3] = [\mathrm{g}, \mathrm{h}, \mathrm{i}]$ determine all the hyperbolic grids, then just the 3 central elements $[Y[-1], Y[0], Y[1]] = [x_1, x_2, x_3] = [\mathrm{g}, \mathrm{h}, \mathrm{i}]$ determine any hyperboctys.

So, we will determine and note any hyperboctys as being

$$HS[Y[-1], Y[0], Y[1]] \text{ or } HS[x_1, x_2, x_3] \text{ or } HS[\mathrm{g}, \mathrm{h}, \mathrm{i}]$$

3.2 The parabolic origin of hyperboctys

Any hyperboctys $HS[g, h, i]$ will have verticals with quadratic equations given by

$$[\mathrm{g}-\mathrm{C}, \mathrm{h}, \mathrm{i}+\mathrm{C}] \equiv Y[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y + \mathrm{h}$$

being C the column of the hyperboctys.

For $i = g$, then

$$Y[y] = (\mathrm{g}-\mathrm{h})y^2 + \mathrm{C}y + \mathrm{h}$$

We will define the quadratic origin of hyperboctys where $i = g$, and $C = 0$.

3.3 Some operations with hyperboctys

We can do several operations with hyperboctys. The rule is to keep the hyperbolic grid. The operations will change the mosaic of the tessellation keeping the hyperbolic grid.

We will see its behavior when we submit for some basic operations.

3.3.1 Addition or subtraction with hyperboctys

Being

$$HS_2[g+n,h+n,i+n] = HS_1[g,h,i]+n$$

Then,

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}$$

$$[\mathrm{g}-\mathrm{C}+\mathrm{n},\mathrm{h}+\mathrm{n},\mathrm{i}+\mathrm{C}+\mathrm{n}] \equiv Y_2[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}+\mathrm{n}$$

So,

$$Y_2[y] = Y_1[y] + \mathrm{n}$$

Example 1: $FMT + 1 = HS[0,0,0] + 1 = HS[1,1,1]$

a		0	0	0	0	0	0	0	0	0	0	0
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		0	0	0	0	0	0	0	0	0	0	0
	10	-50	-40	-30	-20	-10	0	10	20	30	40	50
	9	-45	-36	-27	-18	-9	0	9	18	27	36	45
	8	-40	-32	-24	-16	-8	0	8	16	24	32	40
	7	-35	-28	-21	-14	-7	0	7	14	21	28	35
	6	-30	-24	-18	-12	-6	0	6	12	18	24	30
	5	-25	-20	-15	-10	-5	0	5	10	15	20	25
	4	-20	-16	-12	-8	-4	0	4	8	12	16	20
	3	-15	-12	-9	-6	-3	0	3	6	9	12	15
	2	-10	-8	-6	-4	-2	0	2	4	6	8	10
Y[1]	1	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	5	4	3	2	1	0	-1	-2	-3	-4	-5
	-2	10	8	6	4	2	0	-2	-4	-6	-8	-10
	-3	15	12	9	6	3	0	-3	-6	-9	-12	-15
	-4	20	16	12	8	4	0	-4	-8	-12	-16	-20
	-5	25	20	15	10	5	0	-5	-10	-15	-20	-25
	-6	30	24	18	12	6	0	-6	-12	-18	-24	-30
	-7	35	28	21	14	7	0	-7	-14	-21	-28	-35
	-8	40	32	24	16	8	0	-8	-16	-24	-32	-40
	-9	45	36	27	18	9	0	-9	-18	-27	-36	-45
	-10	50	40	30	20	10	0	-10	-20	-30	-40	-50

a		0	0	0	0	0	0	0	0	0	0	0
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		1	1	1	1	1	1	1	1	1	1	1
	10	-49	-39	-29	-19	-9	1	11	21	31	41	51
	9	-44	-35	-26	-17	-8	1	10	19	28	37	46
	8	-39	-31	-23	-15	-7	1	9	17	25	33	41
	7	-34	-27	-20	-13	-6	1	8	15	22	29	36
	6	-29	-23	-17	-11	-5	1	7	13	19	25	31
	5	-24	-19	-14	-9	-4	1	6	11	16	21	26
	4	-19	-15	-11	-7	-3	1	5	9	13	17	21
	3	-14	-11	-8	-5	-2	1	4	7	10	13	16
	2	-9	-7	-5	-3	-1	1	3	5	7	9	11
Y[1]	1	-4	-3	-2	-1	0	1	2	3	4	5	6
Y[0]	0	1	1	1	1	1	1	1	1	1	1	1
Y[-1]	-1	6	5	4	3	2	1	0	-1	-2	-3	-4
	-2	11	9	7	5	3	1	-1	-3	-5	-7	-9
	-3	16	13	10	7	4	1	-2	-5	-8	-11	-14
	-4	21	17	13	9	5	1	-3	-7	-11	-15	-19
	-5	26	21	16	11	6	1	-4	-9	-14	-19	-24
	-6	31	25	19	13	7	1	-5	-11	-17	-23	-29
	-7	36	29	22	15	8	1	-6	-13	-20	-27	-34
	-8	41	33	25	17	9	1	-7	-15	-23	-31	-39
	-9	46	37	28	19	10	1	-8	-17	-26	-35	-44
	-10	51	41	31	21	11	1	-9	-19	-29	-39	-49

Example 2: $HS[4,0,4] + 41 = HS[45,41,45]$

a		4	4	4	4	4	4	4	4	4	4	4
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		0	0	0	0	0	0	0	0	0	0	0
	10	350	360	370	380	390	**400**	410	420	430	440	450
	9	279	288	297	306	315	**324**	333	342	351	360	369
	8	216	224	232	240	248	**256**	264	272	280	288	296
	7	161	168	175	182	189	**196**	203	210	217	224	231
	6	114	120	126	132	138	**144**	150	156	162	168	174
	5	75	80	85	90	95	**100**	105	110	115	120	125
	4	44	48	52	56	60	**64**	68	72	76	80	84
	3	21	24	27	30	33	**36**	39	42	45	48	51
	2	6	8	10	12	14	**16**	18	20	22	24	26
Y[1]	1	-1	0	1	2	3	**4**	5	6	7	8	**9**
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	**9**	8	7	6	5	**4**	3	2	1	0	-1
	-2	26	24	22	20	18	**16**	14	12	10	8	6
	-3	51	48	45	42	39	**36**	33	30	27	24	21
	-4	84	80	76	72	68	**64**	60	56	52	48	44
	-5	125	120	115	110	105	**100**	95	90	85	80	75
	-6	174	168	162	156	150	**144**	138	132	126	120	114
	-7	231	224	217	210	203	**196**	189	182	175	168	161
	-8	296	288	280	272	264	**256**	248	240	232	224	216
	-9	369	360	351	342	333	**324**	315	306	297	288	279
	-10	450	440	430	420	410	**400**	390	380	370	360	350

a		4	4	4	4	4	4	4	4	4	4	4
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		41	41	41	41	41	41	41	41	41	41	41
	10	391	401	411	421	431	**441**	451	461	471	481	491
	9	320	329	338	347	356	365	374	383	392	401	410
	8	257	265	273	281	**289**	297	305	313	321	329	337
	7	202	209	216	223	230	237	244	251	258	265	272
	6	155	161	167	173	179	185	191	197	203	209	215
	5	116	**121**	126	131	136	141	146	151	156	161	166
	4	85	89	93	97	101	105	109	113	117	**121**	125
	3	62	65	68	71	74	77	80	83	86	89	92
	2	47	**49**	51	53	55	57	59	61	63	65	67
Y[1]	1	40	41	42	43	44	45	46	47	48	**49**	50
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	50	**49**	48	47	46	45	44	43	42	41	40
	-2	67	65	63	61	59	57	55	53	51	**49**	47
	-3	92	89	86	83	80	77	74	71	68	65	62
	-4	125	**121**	117	113	109	105	101	97	93	89	85
	-5	166	161	156	151	146	141	136	131	126	**121**	116
	-6	215	209	203	197	191	185	179	173	167	161	155
	-7	272	265	258	251	244	237	230	223	216	209	202
	-8	337	329	321	313	305	297	**289**	281	273	265	257
	-9	410	401	392	383	374	365	356	347	338	329	320
	-10	491	481	471	461	451	**441**	431	421	411	401	391

3.3.2Multiplication or division of the hyperboctys by a constant

To maintain the hyperbolic grid, multiplication by Integer proceeds like this

$$HS_2[ng, nh, ni] = n * HS_1[g, h, i]$$

Then,

$$[\mathrm{g} - \mathrm{C}, \mathrm{h}, \mathrm{i} + \mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g} - 2\mathrm{h} + \mathrm{i}}{2}\right) y^2 + \left(\frac{\mathrm{i} - \mathrm{g}}{2} + C\right) y + \mathrm{h}$$

$$[\mathrm{ng} - \mathrm{C}, \mathrm{nh}, \mathrm{ni} + \mathrm{C}] \equiv Y_2[y] = \left(\frac{\mathrm{ng} - 2\mathrm{nh} + \mathrm{ni}}{2}\right) y^2 + \left(\frac{\mathrm{ni} - \mathrm{ng}}{2} + C\right) y + \mathrm{nh}$$

So,

$$Y_2[y] = nY_1[y] + \mathrm{Cy}$$

There is a change in the offset (coefficient b).

Example: $HS[13,11,11] * 2 + 1 = HS[26,22,22] + 1 = HS[27,23,23]$

a		1	1	1	1	1	1	1	1	1	1	1
b		-6	-5	-4	-3	-2	-1	0	1	2	3	4
c		11	11	11	11	11	11	11	11	11	11	11
	10	51	61	71	**81**	91	101	111	**121**	131	141	151
	9	38	47	56	65	74	83	92	101	110	119	128
	8	27	35	43	51	59	67	75	83	91	99	107
	7	18	**25**	32	39	46	53	60	67	74	**81**	88
	6	11	17	23	29	35	41	47	53	59	65	71
	5	6	11	**16**	21	26	31	**36**	41	46	51	56
	4	3	7	11	15	19	23	27	31	35	39	43
	3	2	5	8	11	14	17	20	23	26	29	32
	2	3	5	7	**9**	11	13	15	17	19	21	23
Y[1]	1	6	7	8	**9**	10	11	12	13	14	15	**16**
Y[0]	0	11	11	11	11	11	11	11	11	11	11	11
Y[-1]	-1	18	17	**16**	15	14	13	12	11	10	**9**	8
	-2	27	**25**	23	21	19	17	15	13	11	**9**	7
	-3	38	35	32	29	26	23	20	17	14	11	8
	-4	51	47	43	39	35	31	27	23	19	15	11
	-5	66	61	56	51	46	41	**36**	31	26	21	**16**
	-6	83	77	71	65	59	53	47	41	35	29	23
	-7	102	95	88	**81**	74	67	60	53	46	39	32
	-8	123	115	107	99	91	83	75	67	59	51	43
	-9	146	137	128	119	110	101	92	83	74	65	56
	-10	171	161	151	141	131	**121**	111	101	91	**81**	71

a		2	2	2	2	2	2	2	2	2	2	2
b		-7	-6	-5	-4	-3	-2	-1	0	1	2	3
c		22	22	22	22	22	22	22	22	22	22	22
	10	152	162	172	182	192	202	212	222	232	242	252
	9	**121**	130	139	148	157	166	175	184	193	202	211
	8	94	102	110	118	126	134	142	150	158	166	174
	7	71	78	85	92	99	106	113	120	127	134	141
	6	52	58	**64**	70	76	82	88	94	**100**	106	112
	5	37	42	47	52	57	62	67	72	77	82	87
	4	26	30	34	38	42	46	50	54	58	62	66
	3	19	22	**25**	28	31	34	37	40	43	46	**49**
	2	**16**	18	20	22	24	26	28	30	32	34	**36**
Y[1]	1	17	18	19	20	21	22	23	24	**25**	26	27
Y[0]	0	22	22	22	22	22	22	22	22	22	22	22
Y[-1]	-1	31	30	29	28	27	26	**25**	24	23	22	21
	-2	44	42	40	38	**36**	34	32	30	28	26	24
	-3	61	58	55	52	**49**	46	43	40	37	34	31
	-4	82	78	74	70	66	62	58	54	50	46	42
	-5	107	102	97	92	87	82	77	72	67	62	57
	-6	136	130	124	118	112	106	**100**	94	88	82	76
	-7	**169**	162	155	148	141	134	127	120	113	106	99
	-8	206	198	190	182	174	166	158	150	142	134	126
	-9	247	238	229	220	211	202	193	184	175	166	157
	-10	292	282	272	262	252	242	232	222	212	202	192

a		2	2	2	2	2	2	2	2	2	2	2
b		-7	-6	-5	-4	-3	-2	-1	0	1	2	3
c		23	23	23	23	23	23	23	23	23	23	23
	10	153	163	173	183	193	203	213	223	233	243	253
	9	122	131	140	149	158	167	176	185	194	203	212
	8	95	103	111	119	127	135	143	151	159	167	175
	7	72	79	86	93	**100**	107	114	**121**	128	135	142
	6	53	59	65	71	77	83	89	95	101	107	113
	5	38	43	48	53	58	63	68	73	78	83	88
	4	27	31	35	39	43	47	51	55	59	63	67
	3	20	23	26	29	32	35	38	41	44	47	50
	2	17	19	21	23	**25**	27	29	31	33	35	37
Y[1]	1	18	19	20	21	22	23	24	**25**	26	27	28
Y[0]	0	23	23	23	23	23	23	23	23	23	23	23
Y[-1]	-1	32	31	30	29	28	27	26	**25**	24	23	22
	-2	45	43	41	39	37	35	33	31	29	27	**25**
	-3	62	59	56	53	50	47	44	41	38	35	32
	-4	83	79	75	71	67	63	59	55	51	47	43
	-5	108	103	98	93	88	83	78	73	68	63	58
	-6	137	131	125	119	113	107	101	95	89	83	77
	-7	170	163	156	149	142	135	128	**121**	114	107	**100**
	-8	207	199	191	183	175	167	159	151	143	135	127
	-9	248	239	230	221	212	203	194	185	176	167	158
	-10	293	283	273	263	253	243	233	223	213	203	193

3.3.3Full CCW rotation of s steps in hyperboctys

Let's define the full CCW rotation of s steps in $HS_1[g,h,i]$ the new hyperboctys

$$HS_2[g+s,h,i+s]$$

Then,

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}$$

$$[\mathrm{g}-\mathrm{C}+\mathrm{s},\mathrm{h},\mathrm{i}+\mathrm{C}+\mathrm{s}] \equiv Y_2[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}+s\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}$$

So,

$$Y_2[y] = Y_1[y] + \mathrm{s}y^2$$

There is an increase in coefficient a by s, remaining unchanged the other coefficients.

Example 1: rotate $HS[41,41,41]$ one step CCW will result in $HS[42,41,42]$

a		0	0	0	0	0	0	0	0	0	0	0
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		41	41	41	41	41	41	41	41	41	41	41
	10	-9	1	11	21	31	41	51	61	71	81	91
	9	-4	5	14	23	32	41	50	59	68	77	86
	8	1	9	17	25	33	41	49	57	65	73	81
	7	6	13	20	27	34	41	48	55	62	69	76
	6	11	17	23	29	35	41	47	53	59	65	71
	5	16	21	26	31	36	41	46	51	56	61	66
	4	21	25	29	33	37	41	45	49	53	57	61
	3	26	29	32	35	38	41	44	47	50	53	56
	2	31	33	35	37	39	41	43	45	47	49	51
Y[1]	1	36	37	38	39	40	41	42	43	44	45	46
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	46	45	44	43	42	41	40	39	38	37	36
	-2	51	49	47	45	43	41	39	37	35	33	31
	-3	56	53	50	47	44	41	38	35	32	29	26
	-4	61	57	53	49	45	41	37	33	29	25	21
	-5	66	61	56	51	46	41	36	31	26	21	16
	-6	71	65	59	53	47	41	35	29	23	17	11
	-7	76	69	62	55	48	41	34	27	20	13	6
	-8	81	73	65	57	49	41	33	25	17	9	1
	-9	86	77	68	59	50	41	32	23	14	5	-4
	-10	91	81	71	61	51	41	31	21	11	1	-9

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		41	41	41	41	41	41	41	41	41	41	41
	10	91	101	111	121	131	141	151	161	171	181	191
	9	77	86	95	104	113	122	131	140	149	158	167
	8	65	73	81	89	97	105	113	121	129	137	145
	7	55	62	69	76	83	90	97	104	111	118	125
	6	47	53	59	65	71	77	83	89	95	101	107
	5	41	46	51	56	61	66	71	76	81	86	91
	4	37	41	45	49	53	57	61	65	69	73	77
	3	35	38	41	44	47	50	53	56	59	62	65
	2	35	37	39	41	43	45	47	49	51	53	55
Y[1]	1	37	38	39	40	41	42	43	44	45	46	47
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	47	46	45	44	43	42	41	40	39	38	37
	-2	55	53	51	49	47	45	43	41	39	37	35
	-3	65	62	59	56	53	50	47	44	41	38	35
	-4	77	73	69	65	61	57	53	49	45	41	37
	-5	91	86	81	76	71	66	61	56	51	46	41
	-6	107	101	95	89	83	77	71	65	59	53	47
	-7	125	118	111	104	97	90	83	76	69	62	55
	-8	145	137	129	121	113	105	97	89	81	73	65
	-9	167	158	149	140	131	122	113	104	95	86	77
	-10	191	181	171	161	151	141	131	121	111	101	91

Example 2: rotate $HS[42,41,42]$ three step CCW will result in $HS[45,41,45]$

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		41	41	41	41	41	41	41	41	41	41	41
	10	91	101	111	121	131	141	151	161	171	181	191
	9	77	86	95	104	113	122	131	140	149	158	167
	8	65	73	81	89	97	105	113	121	129	137	145
	7	55	62	69	76	83	90	97	104	111	118	125
	6	47	53	59	65	71	77	83	89	95	101	107
	5	41	46	51	56	61	66	71	76	81	86	91
	4	37	41	45	49	53	57	61	65	69	73	77
	3	35	38	41	44	47	50	53	56	59	62	65
	2	35	37	39	41	43	45	47	49	51	53	55
Y[1]	1	37	38	39	40	41	42	43	44	45	46	47
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	47	46	45	44	43	42	41	40	39	38	37
	-2	55	53	51	49	47	45	43	41	39	37	35
	-3	65	62	59	56	53	50	47	44	41	38	35
	-4	77	73	69	65	61	57	53	49	45	41	37
	-5	91	86	81	76	71	66	61	56	51	46	41
	-6	107	101	95	89	83	77	71	65	59	53	47
	-7	125	118	111	104	97	90	83	76	69	62	55
	-8	145	137	129	121	113	105	97	89	81	73	65
	-9	167	158	149	140	131	122	113	104	95	86	77
	-10	191	181	171	161	151	141	131	121	111	101	91

a		4	4	4	4	4	4	4	4	4	4	4
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		41	41	41	41	41	41	41	41	41	41	41
	10	391	401	411	421	431	441	451	461	471	481	491
	9	320	329	338	347	356	365	374	383	392	401	410
	8	257	265	273	281	289	297	305	313	321	329	337
	7	202	209	216	223	230	237	244	251	258	265	272
	6	155	161	167	173	179	185	191	197	203	209	215
	5	116	121	126	131	136	141	146	151	156	161	166
	4	85	89	93	97	101	105	109	113	117	121	125
	3	62	65	68	71	74	77	80	83	86	89	92
	2	47	49	51	53	55	57	59	61	63	65	67
Y[1]	1	40	41	42	43	44	45	46	47	48	49	50
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	50	49	48	47	46	45	44	43	42	41	40
	-2	67	65	63	61	59	57	55	53	51	49	47
	-3	92	89	86	83	80	77	74	71	68	65	62
	-4	125	121	117	113	109	105	101	97	93	89	85
	-5	166	161	156	151	146	141	136	131	126	121	116
	-6	215	209	203	197	191	185	179	173	167	161	155
	-7	272	265	258	251	244	237	230	223	216	209	202
	-8	337	329	321	313	305	297	289	281	273	265	257
	-9	410	401	392	383	374	365	356	347	338	329	320
	-10	491	481	471	461	451	441	431	421	411	401	391

Variation of the sequence angles from $HS[n, h, n]$ to $HS[(n+1), h, (n+1)]$

Variation of sequence angles in full CCW rotation (in degrees)					
HS[n,h,n]			HS[(n+1),h,(n+1)]		
x	y	arctan(y/x)	x	y	arctan(y/x)
10	1	5,7105931	9	1	6,3401917
9	1	6,3401917	8	1	7,1250163
8	1	7,1250163	7	1	8,1301024
7	1	8,1301024	6	1	9,4623222
6	1	9,4623222	5	1	11,309932
5	1	11,309932	4	1	14,036243
4	1	14,036243	3	1	18,434949
3	1	18,434949	2	1	26,565051
2	1	26,565051	1	1	45
1	1	45	0	1	90
0	1	90	-1	1	135
-1	1	135	-2	1	153,43495
-2	1	153,43495	-3	1	161,56505
-3	1	161,56505	-4	1	165,96376
-4	1	165,96376	-5	1	168,69007
-5	1	168,69007	-6	1	170,53768
-6	1	170,53768	-7	2	171,8699
-7	1	171,8699	-8	3	172,87498
-8	1	172,87498	-9	4	173,65981
-9	1	173,65981	-10	5	174,28941
-10	1	174,28941	-11	6	185,19443

3.3.4Full CW rotation of s steps in hyperboctys

Let's define the full CW rotation of s steps in $HS_1[g,h,i]$ the new hyperboctys

$$HS_2[g-s,h,i-s]$$

Then,

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}$$

$$[\mathrm{g}-\mathrm{C}-\mathrm{s},\mathrm{h},\mathrm{i}+\mathrm{C}-\mathrm{s}] \equiv Y_2[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}-s\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}$$

So,

$$Y_2[y] = Y_1[y] - \mathrm{s}y^2$$

There is a decrease in coefficient a by s, remaining unchanged the other coefficients.

Example 1: rotate $HS[-13,-13,-13]$ one step CW will result in $HS[-14,-13,-14]$

a		0	0	0	0	0	0	0	0	0	0	0
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
	10	-63	-53	-43	-33	-23	-13	-3	7	17	27	37
	9	-58	-49	-40	-31	-22	-13	-4	5	14	23	32
	8	-53	-45	-37	-29	-21	-13	-5	3	11	19	27
	7	-48	-41	-34	-27	-20	-13	-6	1	8	15	22
	6	-43	-37	-31	-25	-19	-13	-7	-1	5	11	17
	5	-38	-33	-28	-23	-18	-13	-8	-3	2	7	12
	4	-33	-29	-25	-21	-17	-13	-9	-5	-1	3	7
	3	-28	-25	-22	-19	-16	-13	-10	-7	-4	-1	2
	2	-23	-21	-19	-17	-15	-13	-11	-9	-7	-5	-3
Y[1]	1	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8
Y[0]	0	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
Y[-1]	-1	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18
	-2	-3	-5	-7	-9	-11	-13	-15	-17	-19	-21	-23
	-3	2	-1	-4	-7	-10	-13	-16	-19	-22	-25	-28
	-4	7	3	-1	-5	-9	-13	-17	-21	-25	-29	-33
	-5	12	7	2	-3	-8	-13	-18	-23	-28	-33	-38
	-6	17	11	5	-1	-7	-13	-19	-25	-31	-37	-43
	-7	22	15	8	1	-6	-13	-20	-27	-34	-41	-48
	-8	27	19	11	3	-5	-13	-21	-29	-37	-45	-53
	-9	32	23	14	5	-4	-13	-22	-31	-40	-49	-58
	-10	37	27	17	7	-3	-13	-23	-33	-43	-53	-63

a		-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
	10	###	###	###	###	###	###	###	-93	-83	-73	-63
	9	###	###	###	###	###	-94	-85	-76	-67	-58	-49
	8	###	###	###	-93	-85	-77	-69	-61	-53	-45	-37
	7	-97	-90	-83	-76	-69	-62	-55	-48	-41	-34	-27
	6	-79	-73	-67	-61	-55	-49	-43	-37	-31	-25	-19
	5	-63	-58	-53	-48	-43	-38	-33	-28	-23	-18	-13
	4	-49	-45	-41	-37	-33	-29	-25	-21	-17	-13	-9
	3	-37	-34	-31	-28	-25	-22	-19	-16	-13	-10	-7
	2	-27	-25	-23	-21	-19	-17	-15	-13	-11	-9	-7
Y[1]	1	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9
Y[0]	0	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
Y[-1]	-1	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19
	-2	-7	-9	-11	-13	-15	-17	-19	-21	-23	-25	-27
	-3	-7	-10	-13	-16	-19	-22	-25	-28	-31	-34	-37
	-4	-9	-13	-17	-21	-25	-29	-33	-37	-41	-45	-49
	-5	-13	-18	-23	-28	-33	-38	-43	-48	-53	-58	-63
	-6	-19	-25	-31	-37	-43	-49	-55	-61	-67	-73	-79
	-7	-27	-34	-41	-48	-55	-62	-69	-76	-83	-90	-97
	-8	-37	-45	-53	-61	-69	-77	-85	-93	###	###	###
	-9	-49	-58	-67	-76	-85	-94	###	###	###	###	###
	-10	-63	-73	-83	-93	###	###	###	###	###	###	###

Example 2: rotate $HS[-14,-13,-14]$ one step CW will result in $HS[-15,-13,-15]$

a		-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
	10	###	###	###	###	###	###	###	-93	-83	-73	-63
	9	###	###	###	###	###	-94	-85	-76	-67	-58	-49
	8	###	###	###	-93	-85	-77	-69	-61	-53	-45	-37
	7	-97	-90	-83	-76	-69	-62	-55	-48	-41	-34	-27
	6	-79	-73	-67	-61	-55	-49	-43	-37	-31	-25	-19
	5	-63	-58	-53	-48	-43	-38	-33	-28	-23	-18	-13
	4	-49	-45	-41	-37	-33	-29	-25	-21	-17	-13	-9
	3	-37	-34	-31	-28	-25	-22	-19	-16	-13	-10	-7
	2	-27	-25	-23	-21	-19	-17	-15	-13	-11	-9	-7
Y[1]	1	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9
Y[0]	0	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
Y[-1]	-1	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19
	-2	-7	-9	-11	-13	-15	-17	-19	-21	-23	-25	-27
	-3	-7	-10	-13	-16	-19	-22	-25	-28	-31	-34	-37
	-4	-9	-13	-17	-21	-25	-29	-33	-37	-41	-45	-49
	-5	-13	-18	-23	-28	-33	-38	-43	-48	-53	-58	-63
	-6	-19	-25	-31	-37	-43	-49	-55	-61	-67	-73	-79
	-7	-27	-34	-41	-48	-55	-62	-69	-76	-83	-90	-97
	-8	-37	-45	-53	-61	-69	-77	-85	-93	###	###	###
	-9	-49	-58	-67	-76	-85	-94	###	###	###	###	###
	-10	-63	-73	-83	-93	###	###	###	###	###	###	###

a		-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
	10	###	###	###	###	###	###	###	###	###	###	###
	9	###	###	###	###	###	###	###	###	###	###	###
	8	###	###	###	###	###	###	###	###	###	###	###
	7	###	###	###	###	###	###	###	-97	-90	-83	-76
	6	###	###	###	-97	-91	-85	-79	-73	-67	-61	-55
	5	-88	-83	-78	-73	-68	-63	-58	-53	-48	-43	-38
	4	-65	-61	-57	-53	-49	-45	-41	-37	-33	-29	-25
	3	-46	-43	-40	-37	-34	-31	-28	-25	-22	-19	-16
	2	-31	-29	-27	-25	-23	-21	-19	-17	-15	-13	-11
Y[1]	1	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
Y[0]	0	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13
Y[-1]	-1	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20
	-2	-11	-13	-15	-17	-19	-21	-23	-25	-27	-29	-31
	-3	-16	-19	-22	-25	-28	-31	-34	-37	-40	-43	-46
	-4	-25	-29	-33	-37	-41	-45	-49	-53	-57	-61	-65
	-5	-38	-43	-48	-53	-58	-63	-68	-73	-78	-83	-88
	-6	-55	-61	-67	-73	-79	-85	-91	-97	###	###	###
	-7	-76	-83	-90	-97	###	###	###	###	###	###	###
	-8	###	###	###	###	###	###	###	###	###	###	###
	-9	###	###	###	###	###	###	###	###	###	###	###
	-10	###	###	###	###	###	###	###	###	###	###	###

3.3.3 Variation of the sequence angles from $HS[n, h, n]$ to $HS[(n-1), h, (n-1)]$

Variation of sequence angles in full CW rotation (in degrees)					
HS[n,h,n]			HS[(n-1),h,(n-1)]		
x	y	arctan(y/x)	x	y	arctan(y/x)
10	1	5,7105931	11	1	5,1944289
9	1	6,3401917	10	1	5,7105931
8	1	7,1250163	9	1	6,3401917
7	1	8,1301024	8	1	7,1250163
6	1	9,4623222	7	1	8,1301024
5	1	11,309932	6	1	9,4623222
4	1	14,036243	5	1	11,309932
3	1	18,434949	4	1	14,036243
2	1	26,565051	3	1	18,434949
1	1	45	2	1	26,565051
0	1	90	1	1	45
-1	1	135	0	1	90
-2	1	153,43495	-1	1	135
-3	1	161,56505	-2	1	153,43495
-4	1	165,96376	-3	1	161,56505
-5	1	168,69007	-4	1	165,96376
-6	1	170,53768	-5	2	168,69007
-7	1	171,8699	-6	3	170,53768
-8	1	172,87498	-7	4	171,8699
-9	1	173,65981	-8	5	172,87498
-10	1	174,28941	-9	6	173,65981

3.3.4 Half-step CCW Rotation with hyperboctys

Let's define the half CCW rotation in $HS_1[g,h,i]$ the new hyperboctys

$$HS_2[g,h,i+1]$$

Then,

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y + \mathrm{h}$$

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}+1] \equiv Y_2[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}+0.5\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C+0.5\right)y + \mathrm{h}$$

So,

$$Y_2[y] = Y_1[y] + (0.5\mathrm{y}^2 + 0.5y)$$

There is a change in both coefficients a and b by 0.5.

Or we can define the half-step CCW rotation of the new hyperboctys

$$HS_3[g+1,h,i]$$

Then,

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y + \mathrm{h}$$

$$[\mathrm{g}-\mathrm{C}+1,\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_2[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}+0.5\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C-0.5\right)y + \mathrm{h}$$

So,

$$Y_2[y] = Y_1[y] + (0.5\mathrm{y}^2 - 0.5y)$$

There is a change in both coefficients a and b by 0.5. In this case, the results are inverted concerning the former result.

Example 1: rotate $FMT = HS[0,0,0]$ half-step CCW will result in $HS[1,0,0]$

	a	0	0	0	0	0	0	0	0	0	0	0
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	0	0	0	0	0	0	0	0	0	0	0
	10	-50	-40	-30	-20	-10	0	10	20	30	40	50
	9	-45	-36	-27	-18	-9	0	9	18	27	36	45
	8	-40	-32	-24	-16	-8	0	8	16	24	32	40
	7	-35	-28	-21	-14	-7	0	7	14	21	28	35
	6	-30	-24	-18	-12	-6	0	6	12	18	24	30
	5	-25	-20	-15	-10	-5	0	5	10	15	20	25
	4	-20	-16	-12	-8	-4	0	4	8	12	16	20
	3	-15	-12	-9	-6	-3	0	3	6	9	12	15
	2	-10	-8	-6	-4	-2	0	2	4	6	8	10
Y[1]	1	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	5	4	3	2	1	0	-1	-2	-3	-4	-5
	-2	10	8	6	4	2	0	-2	-4	-6	-8	-10
	-3	15	12	9	6	3	0	-3	-6	-9	-12	-15
	-4	20	16	12	8	4	0	-4	-8	-12	-16	-20
	-5	25	20	15	10	5	0	-5	-10	-15	-20	-25
	-6	30	24	18	12	6	0	-6	-12	-18	-24	-30
	-7	35	28	21	14	7	0	-7	-14	-21	-28	-35
	-8	40	32	24	16	8	0	-8	-16	-24	-32	-40
	-9	45	36	27	18	9	0	-9	-18	-27	-36	-45
	-10	50	40	30	20	10	0	-10	-20	-30	-40	-50

	a	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
	b	-5,5	-4,5	-3,5	-2,5	-1,5	-0,5	0,5	1,5	2,5	3,5	4,5
	c	0	0	0	0	0	0	0	0	0	0	0
	10	-5	5	15	25	35	45	55	65	75	85	95
	9	-9	0	9	18	27	36	45	54	63	72	81
	8	-12	-4	4	12	20	28	36	44	52	60	68
	7	-14	-7	0	7	14	21	28	35	42	49	56
	6	-15	-9	-3	3	9	15	21	27	33	39	45
	5	-15	-10	-5	0	5	10	15	20	25	30	35
	4	-14	-10	-6	-2	2	6	10	14	18	22	26
	3	-12	-9	-6	-3	0	3	6	9	12	15	18
	2	-9	-7	-5	-3	-1	1	3	5	7	9	11
Y[1]	1	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	6	5	4	3	2	1	0	-1	-2	-3	-4
	-2	13	11	9	7	5	3	1	-1	-3	-5	-7
	-3	21	18	15	12	9	6	3	0	-3	-6	-9
	-4	30	26	22	18	14	10	6	2	-2	-6	-10
	-5	40	35	30	25	20	15	10	5	0	-5	-10
	-6	51	45	39	33	27	21	15	9	3	-3	-9
	-7	63	56	49	42	35	28	21	14	7	0	-7
	-8	76	68	60	52	44	36	28	20	12	4	-4
	-9	90	81	72	63	54	45	36	27	18	9	0
	-10	105	95	85	75	65	55	45	35	25	15	5

3.3.5Half-step CW Rotation with hyperboctys

Let's define the half CW rotation in $HS_1[g,h,i]$ the new hyperboctys

$$HS_2[g,h,i-1]$$

Then,

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}$$

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}-1] \equiv Y_2[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}-0.5\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C-0.5\right)y+\mathrm{h}$$

So,

$$Y_2[y] = Y_1[y] - (0.5\mathrm{y}^2 + 0.5y)$$

There is a change in both coefficients a and b by 0.5.

Example 1: rotate $HS[42,41,42]$ half-step CW will result in $HS[42,41,41]$

	a	1	1	1	1	1	1	1	1	1	1	1
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	41	41	41	41	41	41	41	41	41	41	41
	10	91	101	111	121	131	141	151	161	171	181	191
	9	77	86	95	104	113	122	131	140	149	158	167
	8	65	73	81	89	97	105	113	121	129	137	145
	7	55	62	69	76	83	90	97	104	111	118	125
	6	47	53	59	65	71	77	83	89	95	101	107
	5	41	46	51	56	61	66	71	76	81	86	91
	4	37	41	45	49	53	57	61	65	69	73	77
	3	35	38	41	44	47	50	53	56	59	62	65
	2	35	37	39	41	43	45	47	49	51	53	55
Y[1]	1	37	38	39	40	41	42	43	44	45	46	47
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	47	46	45	44	43	42	41	40	39	38	37
	-2	55	53	51	49	47	45	43	41	39	37	35
	-3	65	62	59	56	53	50	47	44	41	38	35
	-4	77	73	69	65	61	57	53	49	45	41	37
	-5	91	86	81	76	71	66	61	56	51	46	41
	-6	107	101	95	89	83	77	71	65	59	53	47
	-7	125	118	111	104	97	90	83	76	69	62	55
	-8	145	137	129	121	113	105	97	89	81	73	65
	-9	167	158	149	140	131	122	113	104	95	86	77
	-10	191	181	171	161	151	141	131	121	111	101	91

	a	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
	b	-5,5	-4,5	-3,5	-2,5	-1,5	-0,5	0,5	1,5	2,5	3,5	4,5
	c	41	41	41	41	41	41	41	41	41	41	41
	10	36	46	56	66	76	86	96	106	116	126	136
	9	32	41	50	59	68	77	86	95	104	113	122
	8	29	37	45	53	61	69	77	85	93	101	109
	7	27	34	41	48	55	62	69	76	83	90	97
	6	26	32	38	44	50	56	62	68	74	80	86
	5	26	31	36	41	46	51	56	61	66	71	76
	4	27	31	35	39	43	47	51	55	59	63	67
	3	29	32	35	38	41	44	47	50	53	56	59
	2	32	34	36	38	40	42	44	46	48	50	52
Y[1]	1	36	37	38	39	40	41	42	43	44	45	46
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	47	46	45	44	43	42	41	40	39	38	37
	-2	54	52	50	48	46	44	42	40	38	36	34
	-3	62	59	56	53	50	47	44	41	38	35	32
	-4	71	67	63	59	55	51	47	43	39	35	31
	-5	81	76	71	66	61	56	51	46	41	36	31
	-6	92	86	80	74	68	62	56	50	44	38	32
	-7	104	97	90	83	76	69	62	55	48	41	34
	-8	117	109	101	93	85	77	69	61	53	45	37
	-9	131	122	113	104	95	86	77	68	59	50	41
	-10	146	136	126	116	106	96	86	76	66	56	46

Or we can define the half-step CW rotation of the new hyperboctys

$$HS_3[g-1,h,i]$$

Then,

$$[\mathrm{g}-\mathrm{C},\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_1[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C\right)y+\mathrm{h}$$

$$[\mathrm{g}-\mathrm{C}-1,\mathrm{h},\mathrm{i}+\mathrm{C}] \equiv Y_2[y] = \left(\frac{\mathrm{g}-2\mathrm{h}+\mathrm{i}}{2}-0.5\right)y^2 + \left(\frac{\mathrm{i}-\mathrm{g}}{2}+C+0.5\right)y+\mathrm{h}$$

So,

$$Y_2[y] = Y_1[y] - (0.5\mathrm{y}^2 - 0.5y)$$

There is a change in both coefficients a and b by 0.5.

4 The theorem of the element Zero

The quadratic general equation is

(1) $x = ay^2 + by + c$

The roots will be given when $x = ay^2 + by + c = 0$.

Let's multiply both sides by (2^2a):

$$(2^2a)(ay^2 + by + c) = 0$$
$$(2ay)^2 + 2.2ay.b + 4ac = 0$$
$$(2ay)^2 + 2.2ay.b = -4ac$$
$$(2ay)^2 + 2.2ay.b + b^2 = b^2 - 4ac$$
$$(2ay + b)^2 = b^2 - 4ac$$
$$2ay + b = \pm\sqrt{b^2 - 4ac}$$
$$2ay = -b \pm \sqrt{b^2 - 4ac}$$

The two roots are given by:

(2) $y_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \in \mathbb{C}$

Then,

(3) $ay_1^2 + by_1 + c = 0$

And,

(4) $ay_2^2 + by_2 + c = 0$

4.1 Analysis for integer coefficients a, b, c

Considering the first root, let's analyze the difference (1)–(3):

$$x = a(y^2 - y_1^2) + b(y - y_1)$$
$$x = a(y + y_1)(y - y_1) + b(y - y_1)$$
$$x = (y - y_1)[a(y + y_1) + b]$$

There are 2 factors in this product. The multiplier is $(y - y_1)$. The multiplicand is $[a(y + y_1) + b]$.

Let's analyze the possibilities of the multiplier factor.

If $y - y_1 = 0$ then, $x = 0$ which is the element Zero in the sequence.

If $y - y_1 = 1$ then, $x = a(2y_1 + 1) + b$ which may be a positive or negative Prime number.

If $y - y_1 = -1$ then, $x = -a(2y_1 - 1) - b$ which may be a negative or positive Prime number.

For any $|y - y_1| > 1$ if the multiplicand is not Zero then, $x = composite$.

Let's analyze the possibilities of the second factor.

If $a(y + y_1) + b = 0$ then, $x = 0$ which is the element Zero in the sequence.

If $a(y + y_1) + b = 1$ then,

$$ay + ay_1 + b = 1$$
$$ay = -ay_1 - b + 1$$
$$y = -y_1 + \frac{-b + 1}{a}$$
$$x = -2y_1 - \frac{b - 1}{a}$$

which may be a positive or negative Prime number.

If $a(y + y_1) + b = -1$

$$ay + ay_1 + b = -1$$
$$ay = -ay_1 - b - 1$$
$$y = -y_1 + \frac{-b - 1}{a}$$
$$x = -2y_1 - \frac{b + 1}{a}$$

which may be a positive or negative Prime number.

For any $a(y + y_1) + b > 1$ if the multiplier is not Zero then, $x = composite$.

Considering the second root, let's analyze the difference (1)–(4):

$$x = a(y^2 - y_2^2) + b(y - y_2)$$
$$x = a(y + y_2)(y - y_2) + b(y - y_2)$$
$$x = (y - y_2)[a(y + y_2) + b]$$

There are 2 factors in this product.

Let's analyze the possibilities of the multiplier factor.

If $y - y_2 = 0$ then, $x = 0$ which is the element Zero in the sequence.

If $y - y_2 = 1$ then, $x = a(2y_2 + 1) + b$ which may be a positive or negative Prime number.

If $y - y_2 = -1$ then, $x = -a(2y_2 - 1) - b$ which may be a negative or positive Prime number.

For any $|y - y_2| > 1$ if the multiplicand is not Zero then, $x = composite$.

Let's analyze the possibilities of the second factor.

If $a(y + y_2) + b = 0$ then, $x = 0$ which is the element Zero in the sequence.

If $a(y + y_2) + b = 1$ then,

$$ay + ay_2 + b = 1$$
$$ay = -ay_2 - b + 1$$
$$y = -y_2 + \frac{-b + 1}{a}$$
$$x = -2y_2 - \frac{b - 1}{a}$$

which may be a positive or negative Prime number.

If $a(y + y_2) + b = -1$

$$ay + ay_2 + b = -1$$
$$ay = -ay_2 - b - 1$$
$$y = -y_2 + \frac{-b - 1}{a}$$
$$x = -2y_2 - \frac{b + 1}{a}$$

which may be a positive or negative Prime number.

For any $a(y + y_2) + b > 1$ if the multiplier is not Zero then, $x = composite$.

4.1.1 Conclusion for integer coefficients a, b, c

There will be a maximum of 2 Prime numbers for each element of value Zero in a quadratic sequence with Integer coefficients, next to the Zero.

4.2 Analysis for coefficients a, b equal to $\frac{Odd}{2}$ and integer c coefficient

Considering the first root, let's analyze the difference (1)–(3):

$$x = a(y^2 - y_1^2) + b(y - y_1)$$
$$x = a(y + y_1)(y - y_1) + b(y - y_1)$$
$$x = (y - y_1)[a(y + y_1) + b]$$

If a and b are $\frac{Odd}{2}$, then

$$a = \frac{2k+1}{2}$$
$$b = \frac{2h+1}{2}$$
$$x = (y - y_1)\left[\frac{2k+1}{2}(y + y_1) + \frac{2h+1}{2}\right]$$
$$x = \frac{1}{2}(y - y_1)[(2k+1)(y + y_1) + 2h + 1]$$

There are 2 ways to see the 2 factors in this product.

1st way: the multiplier is $(y - y_1)$. The multiplicand is $\left[\frac{(2k+1)(y+y_1)+2h+1}{2}\right]$ and

2nd way: the multiplier is $\frac{(y-y_1)}{2}$. The multiplicand is $[(2k+1)(y + y_1) + 2h + 1]$

Let's analyze the possibilities:

If $y - y_1 = 0$ then, $x = 0$ which is the element Zero in the sequence.

If $y - y_1 = 1$ then, $x = \frac{(2k+1)(2y_1+1)+2h+1}{2} = 2ky_1 + k + y_1 + h + 1$ which may be a positive or negative Prime number.

If $y - y_1 = -1$ then, $x = \frac{(2k+1)(2y_1-1)+2h+1}{2} = 2ky_1 + k + y_1 + h$ which may be a positive or negative Prime number.

If $y - y_1 = 2$ then, $x = (2k+1)(2y_1 + 2) + 2h + 1$ which may be a positive or negative Prime number.

If $y - y_1 = -2$ then, $x = (2k+1)(2y_1 - 2) + 2h + 1$ which may be a positive or negative Prime number.

For any $|y - y_1| > 2$ if the multiplicand is not Zero then, $x = composite$.

And so on.

4.2.1 Conclusion for coefficients a, b equal to $\frac{Odd}{2}$ and integer c coefficient:

There will be a maximum of 4 Prime numbers for each element of value Zero in a quadratic sequence with $\frac{Odd}{2}$ coefficients, two positives, and two negatives, next to the Zero.

5 Composite Generator

Let's define a *composite generator polynomial* (CG) as the polynomial that has at least one Zero number as an element. We represent them in the form:

$$CG[y] = a_n y^n + a_{n-1} y^{n-1} + \ldots + a_3 y^3 + a y^2 + b y$$
$$= y * (a_n y^{n-1} + a_{n-1} y^{n-2} + \ldots + a_3 * y^2 + a y + b)$$

So, the quadratics composite generators are of the form:

$$CG[y] = ay^2 + by = y(ay + b)$$

Because of the theorem of the Zero, these are the sequences that always have zero or a finite number of Prime numbers. Maximum of 2 Primes per each Zero number element in the quadratic with Integer coefficients. So, any quadratic composite generator with Integer coefficients has from 0 to a maximum of 4 primes.

In quadratics, given 3 consecutive elements of the sequence $(Y[y_1], Y[y_2], Y[y_3]) = (x_1, x_2, x_3)$, the simplest equation is

$$Y[y] = \left(\frac{x_1 - 2x_2 + x_3}{2}\right) y^2 + \left(\frac{x_3 - x_1}{2}\right) y + x_2$$

If we set $x_2 = 0$, all the composite generators in quadratics will be of the form

$$CG[y] = \left(\frac{x_1 + x_3}{2}\right) y^2 + \left(\frac{x_3 - x_1}{2}\right) y = \mathrm{y}\left(\left(\frac{x_1 + x_3}{2}\right) \mathrm{y} + \left(\frac{x_3 - x_1}{2}\right)\right)$$

We can detect a CG by the discriminant. Because $\Delta = b^2 - 4ac$, when we have $c = 0$ then, $\Delta = b^2$ and, $\sqrt{\Delta} = \pm b$.

In terms of the 3 consecutive elements:

$$\Delta = \frac{x_1^2 + (4x_2)^2 + x_3^2 - 2x_1(4x_2) - 2(4x_2)x_3 - 2x_1x_3}{4}$$

$$\Delta_{\mathrm{CG}} = \frac{x_1^2 + x_3^2 - 2x_1x_3}{4}$$

$$\sqrt{\Delta_{\mathrm{CG}}} = \pm \frac{x_3 - x_1}{2} = \pm b$$

This means that if $\sqrt{\Delta}$ is an Integer or an $\frac{Odd}{2}$ then, the quadratic is a CG with Integer elements. Note the case $\sqrt{\Delta} = \frac{odd}{2}$ cover the quadratics CG with Integer elements, but non-Integer coefficients.

5.1 Consequences of the Composite Generator

In quadratics, the maximum quantity of elements Zero is 2.

For each element Zero in a CG with Integer coefficients, it is possible to have a maximum of two Primes. All other elements will be Composite, Positive or negative One or another Zero.

For each element Zero in a CG with $\frac{Odd}{2}$ coefficients, it is possible to have a maximum of four Primes next to the Zero.

So, generically, any Composite Generator in quadratics with Integer elements has no Prime or a maximum of 8 (eight) Primes as elements.

5.2 Example of a Composite Generator

The quadratics $Y[y] = 9y^2 - 8y$ have one element Zero.

The sequence is

$$\{\dots, 265, 176, 105, 52, 17, 0, 1, 20, 57, 112, 185, \dots\}$$

The only Prime in this sequence is 17 next to 0.

6 The composite generators in the FMT

Because of the equation form of the hyperbolic grid, note that a polynomial curve of degree $n-1$ represents a polynomial of degree n in the FMT-Hyperbolic Lattice-Grid:

$$Y[y] = a_n y^n + a_{n-1} y^{n-1} + a_{n-2} y^{n-2} + \cdots + a_3 y^3 + a y^2 + b y + c$$

$$Y[y] = (a_n y^{n-1} + a_{n-1} y^{n-2} + \cdots + a_3 y^2 + a y + b) y + c$$

Then, for $c = 0$ we have

$$Y[y] = (a_n y^{n-1} + a_{n-1} y^{n-2} + \cdots + a_3 y^2 + a y + b) y$$

Because $Y[y] = xy$ then,

$$x = a_n y^{n-1} + a_{n-1} y^{n-2} + \cdots + a_3 y^2 + a y + b$$

So, the line curve in XY plane of the form $x = ay + b$ represents the quadratic $Y[y] = ay^2 + by + c$ polynomial, the quadratic curve in XY plane represents the cubic polynomial, the cubic curve in XY plane represents the quartic polynomial, and so on.

We will now find all the quadratic CG in the FMT.

6.1 The A000290 Square numbers in the FMT

Let's construct the quadratic The Square numbers A000290. The Square numbers are all positive numbers or all negative numbers.

The Square numbers A000290 and the Oblong numbers A002378 are the only two quadratic Composite Generator with coefficient $a = 1$ that generate all its elements with a positive sign or all with a negative sign.

All other quadratic Composite Generators with coefficient $a = 1$ always generate its elements with a positive sign and others with a negative sign.

In the FMT hyperbolic lattice-grid, we represent the Square numbers A000290 as

$$z = y^2 \text{ or } z = -y^2$$

Because $z = xy$, the Square numbers A000290 in the XY plane are the two lines of the form

$$x = y \text{ or } x = -y$$

They are the dots representing the positive or the negative Square numbers A000290 in our hyperbolic lattice grid given by the sequences:

$$\{\ldots, 9, 4, 1, 0, 1, 4, 9, \ldots\}$$

$$\{\ldots, -9, -4, -1, 0, -1, -4, -9, \ldots\}$$

6.2 The A002378 Oblong numbers in the FMT

Now we construct the second quadratic: The Oblong numbers A002378. They are all positive or all negative numbers.

In the FMT hyperbolic lattice-grid, we represent the Oblong numbers A002378 as

$$z = y^2 \pm y \text{ or } z = -y^2 \pm y$$

Because $z = xy$, the Oblong numbers A002378 in the XY plane are the four lines of the form

$$x = y \pm 1 \text{ or } x = -y \pm 1$$

They are the dots representing the positive or the negative Oblong numbers A002378 in our hyperbolic lattice grid given by the sequences

$$\{\ldots, 12, 6, 2, 0, 0, 2, 6, 12, \ldots\}$$

$$\{\dots,-12,-6,-2,0,0,-2,-6,-12,\dots\}$$

6.3 The A005563 (Square minus One) numbers in the FMT

Now we construct the next quadratic: The (Square minus One) numbers A005563. This sequence has positive and negative numbers.

In the FMT hyperbolic lattice-grid, we represent the (Square minus One) numbers A005563 as

$$z = y^2 \pm 2y \text{ or } z = -y^2 \pm 2y$$

Because $z = xy$, the (Square minus One) numbers A005563 in the XY plane are the four lines of the form

$$x = y \pm 2 \text{ or } x = -y \pm 2$$

They are the dots representing the sequences

$$\{\dots,15,8,3,0,-1,0,3,8,15,\dots\}$$
$$\{\dots,-15,-8,-3,0,1,0,-3,-8,-15,\dots\}$$

6.4 The A028552 (Oblong minus Two) numbers in the FMT

Now we construct the next quadratic: The (Oblong minus Two) numbers A028552. This sequence has positive and negative numbers.

In the FMT hyperbolic lattice-grid, we represent the (Oblong minus Two) numbers A028552 as

$$z = y^2 \pm 3y \text{ or } z = -y^2 \pm 3y$$

Because $z = xy$, the (Oblong minus Two) numbers A028552 in the XY plane are the four lines of the form

$$x = y \pm 3 \text{ or } x = -y \pm 3$$

They are the dots representing the sequences

$$\{\dots,18,10,4,0,-2,-2,0,4,10,18,\dots\}$$
$$\{\dots,-18,-10,-4,0,2,2,0,-4,-10,-18,\dots\}$$

6.5 The A028347 (Square minus Four) numbers in the FMT

Now we construct the next quadratic: The (Square minus Four) numbers A028347. This sequence has positive and negative numbers.

In the FMT hyperbolic lattice-grid, we represent the (Square minus Four) numbers A028347 as

$$z = y^2 \pm 4y \text{ or } z = -y^2 \pm 4y$$

Because $z = xy$, the (Square minus Four) numbers A028347 in the XY plane are the four lines of the form

$$x = y \pm 4 \text{ or } x = -y \pm 4$$

They are the dots representing the sequences

$$\{\dots,21,12,5,0,-3,-4,-3,0,5,12,21,\dots\}$$
$$\{\dots,-21,-12,-5,0,3,4,3,0,-5,-12,-21,\dots\}$$

6.6 The A028557 (Oblong minus Six) numbers in the FMT

Now we construct the next quadratic: The (Oblong minus Six) numbers A028557. This sequence has positive and negative numbers.

In the FMT hyperbolic lattice-grid, we represent the (Oblong minus Six) numbers A028557 as

$$z = y^2 \pm 5y \text{ or } z = -y^2 \pm 5y$$

Because $z = xy$, the (Oblong minus Six) numbers A028557 in the XY plane are the four lines of the form

$$x = y \pm 5 \text{ or } x = -y \pm 5$$

They are the dots representing the sequences:

$$\{\dots,24,14,6,0,-4,-6,-6,-4,0,6,14,24,\dots\}$$
$$\{\dots,-24,-14,-6,0,4,6,6,4,0,-6,-14,-24,\dots\}$$

6.7 The A028560 (Square minus Nine) numbers in the FMT

Now we construct the next quadratic: The (Square minus Nine) numbers A028560. This sequence has positive and negative numbers.

In the FMT hyperbolic lattice-grid, we represent the (Square minus Nine) numbers A028560 as

$$z = y^2 \pm 6y \text{ or } z = -y^2 \pm 6y$$

Because $z = xy$, the (Square minus Nine) numbers A028560 in the XY plane are the four lines of the form

$$x = y \pm 6 \text{ or } x = -y \pm 6$$

They are the dots representing the sequences:

$$\{\dots,27,16,7,0,-5,-8,-9,-8,-5,0,7,16,27,\dots\}$$
$$\{\dots,-27,-16,-7,0,5,8,9,8,5,0,-7,-16,-27,\dots\}$$

6.8 The A028563 (Oblong minus Twelve) numbers in the FMT

Now we construct the next quadratic: The (Oblong minus Twelve) numbers A028563. This sequence has positive and negative numbers.

In the FMT hyperbolic lattice-grid, we represent the (Oblong minus Twelve) numbers A028563 as

$$z = y^2 \pm 7y \text{ or } z = -y^2 \pm 7y$$

Because $z = xy$, the (Oblong minus Twelve) numbers A028563 in the XY plane are the four lines of the form

$$x = y \pm 7 \text{ or } x = -y \pm 7$$

They are the dots representing the sequences:

$$\{\dots,30,18,8,0,-6,-10,-12,-12,-10,-6,0,8,18,30,\dots\}$$
$$\{\dots,-30,-18,-8,0,6,10,12,12,10,6,0,-8,-18,-30,\dots\}$$

6.9 And so on

We keep constructing quadratics interleaving the forms:

- $(Square\ sequence\ minus\ (one\ Square\ element\ at\ a\ time))$ numbers, and
- $(Oblong\ sequence\ minus\ (one\ Oblong\ element\ at\ a\ time))$ numbers.

This algorithm will produce in FMT Hyperbolic Lattice-Grid all quadratics of the form $y(y \pm n)$.

See the initial diagonals in the first quadrant in the picture:

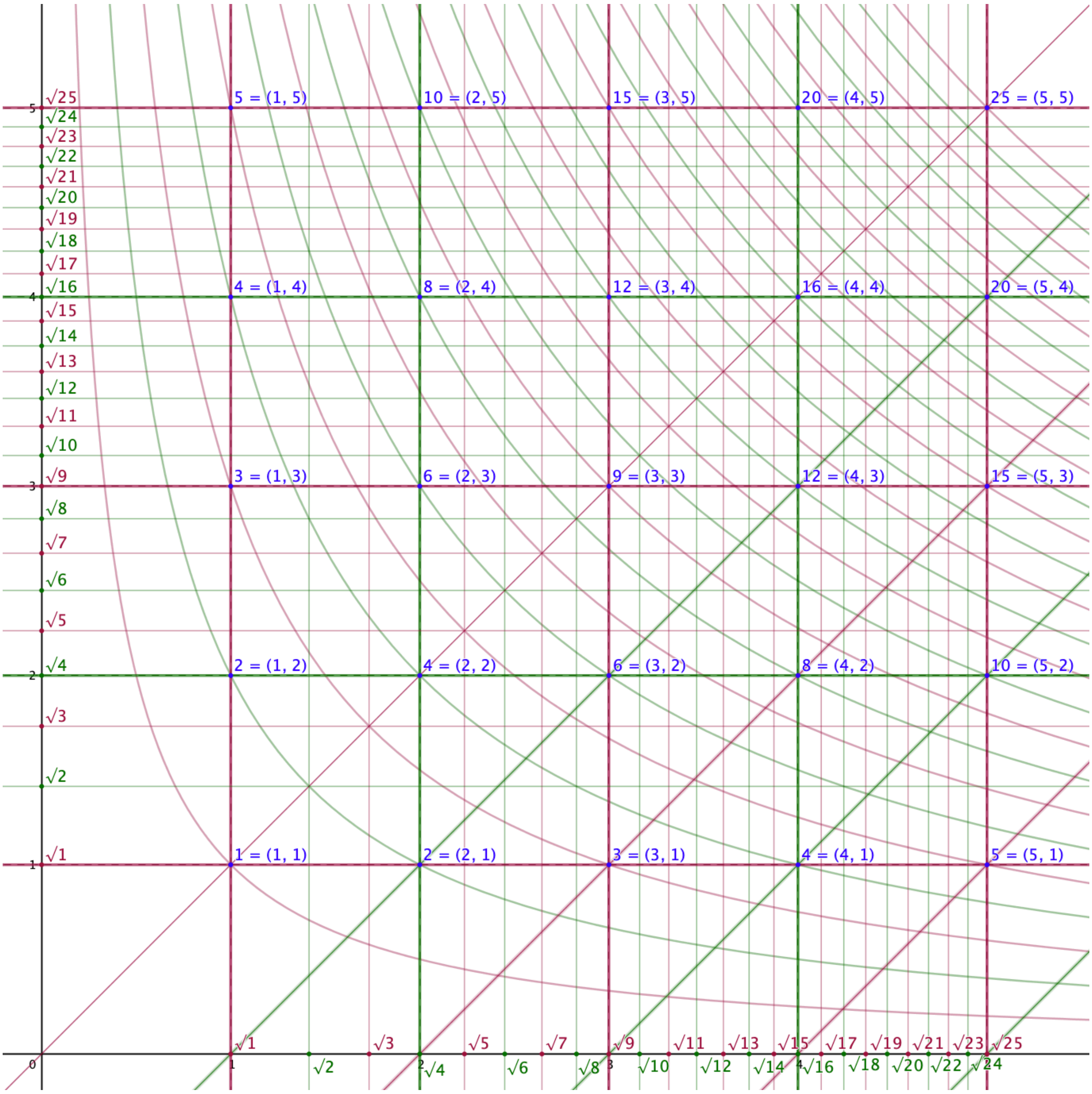

Figure 5. The FMT Hyperbolic Lattice-Grid with its diagonals representing the quadratics of the form $y(y \pm n)$.

Again, in the FMT Hyperbolic Lattice-Grid, all the dots have their value defined by $H[y] = xy$.

For example,

- The dot $(x, y) = (4,3)$ has a value of $4 * 3 = 12$.
- The dot $(x, y) = (6,2)$ has a value of $6 * 2 = 12$. And,
- The dot $(x, y) = (12,1)$ has a value of $12 * 1 = 12$.

They have the same value because the hyperbola $xy = 12$ tie them.

Each dot has a different divisor pair. Here, there are 3 different dots with value 12. This means there are only 3 quadratics of the form $n(n \pm k)$ which have 12 as an element.

If we look in the diagonals, only $n(n \pm 1) = 12$, $n(n \pm 4) = 12$, and $n(n \pm 11) = 12$ have the solution for 3 different values of the index n as an Integer.

If there is an Integer element common between any 2 quadratics, one hyperbola will link them passing through dots (x, y) where x and y are Integers. Otherwise, there is no common Integer element between the quadratics.

This idea is valid for any other function we may want to study. For example, the famous Erdos/Brocard's problem $Square\ minus\ One = Factorial$ or $n^2 - 1 = k!$.

Note that we have all the diagonals $\pm 45^{\circ}$ in HS[0,0,0] representing the quadratics in the form of $y(y \pm b)$. They represent the composite generators with coefficient $a = 1$.

$$Y[y] = CG = y^2 + \mathrm{b}y = \mathrm{y}(\mathrm{y} + \mathrm{b})$$

6.10 Summary of quadratics CG for |a|=1 in Hyperboctys HS[1,0,1]

	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x_ip	-56	-49	-42	-36	-30	-25	-20	-16	-12	-9	-6,3	-4	-2,3	-1	-0,3	0	-0,25	-1	-2,3	-4	-6,3	-9	-12	-16	-20	-25	-30	-36	-42	-49	-56
x_focus	-56	-49	-42	-36	-30	-25	-20	-16	-12	-8,8	-6	-3,8	-2	-0,8	0	0,25	0	-0,8	-2	-3,8	-6	-8,8	-12	-16	-20	-25	-30	-36	-42	-49	-56
LR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Δ	225	196	169	144	121	100	81	64	49	36	25	16	9	4	1	0	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
|√Δ|	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C. G.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Root1	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Root2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
Root2-Root1	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
Classif.	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES
y_ip	7,5	7	6,5	6	5,5	5	4,5	4	3,5	3	2,5	2	1,5	1	0,5	0	-0,5	-1	-1,5	-2	-2,5	-3	-3,5	-4	-4,5	-5	-5,5	-6	-6,5	-7	-7,5
f	7	7	6	6	5	5	4	4	3	3	2	2	1	1	0	0	-1	-1	-2	-2	-3	-3	-4	-4	-5	-5	-6	-6	-7	-7	-8
a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375	390	405	420	435	450
14	-14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350	364	378	392	406
13	-26	-13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325	338	351	364
12	-36	-24	-12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300	312	324
11	-44	-33	-22	-11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275	286
10	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
9	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216
8	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184
7	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154
6	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126
5	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	-44	-40	-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
3	-36	-33	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
2	-26	-24	-22	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
Y[1] 1	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Y[0] 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1] -1	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14
-2	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20	-22	-24	-26
-3	54	51	48	45	42	39	36	33	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	-30	-33	-36
-4	76	72	68	64	60	56	52	48	44	40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36	-40	-44
-5	100	95	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
-6	126	120	114	108	102	96	90	84	78	72	66	60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54
-7	154	147	140	133	126	119	112	105	98	91	84	77	70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56
-8	184	176	168	160	152	144	136	128	120	112	104	96	88	80	72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56
-9	216	207	198	189	180	171	162	153	144	135	126	117	108	99	90	81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54
-10	250	240	230	220	210	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	0	-10	-20	-30	-40	-50
-11	286	275	264	253	242	231	220	209	198	187	176	165	154	143	132	121	110	99	88	77	66	55	44	33	22	11	0	-11	-22	-33	-44
-12	324	312	300	288	276	264	252	240	228	216	204	192	180	168	156	144	132	120	108	96	84	72	60	48	36	24	12	0	-12	-24	-36
-13	364	351	338	325	312	299	286	273	260	247	234	221	208	195	182	169	156	143	130	117	104	91	78	65	52	39	26	13	0	-13	-26
-14	406	392	378	364	350	336	322	308	294	280	266	252	238	224	210	196	182	168	154	140	126	112	98	84	70	56	42	28	14	0	-14
-15	450	435	420	405	390	375	360	345	330	315	300	285	270	255	240	225	210	195	180	165	150	135	120	105	90	75	60	45	30	15	0
Root°1	8	7	7	6	6	5	5	4	4	3	3	2	2	1	1	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
Root°2	-7	-7	-6	-6	-5	-5	-4	-4	-3	-3	-2	-2	-1	-1	0	0	0	-1	-1	-2	-2	-3	-3	-4	-4	-5	-5	-6	-6	-7	-7
Root°2-Root°1	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
Classif.	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES	SUB	DES
y°_ip	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5	0	0,5
f°	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a°	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b°	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1	0	-1
c°	-56	-49	-42	-36	-30	-25	-20	-16	-12	-9	-6	-4	-2	-1	0	0	0	-1	-2	-4	-6	-9	-12	-16	-20	-25	-30	-36	-42	-49	-56
15	154	176	168	189	180	200	190	209	198	216	204	221	208	224	210	225	210	224	208	221	204	216	198	209	190	200	180	189	168	176	154
14	126	147	140	160	152	171	162	180	170	187	176	192	180	195	182	196	182	195	180	192	176	187	170	180	162	171	152	160	140	147	126
13	100	120	114	133	126	144	136	153	144	160	150	165	154	168	156	169	156	168	154	165	150	160	144	153	136	144	126	133	114	120	100
12	76	95	90	108	102	119	112	128	120	135	126	140	130	143	132	144	132	143	130	140	126	135	120	128	112	119	102	108	90	95	76
11	54	72	68	85	80	96	90	105	98	112	104	117	108	120	110	121	110	120	108	117	104	112	98	105	90	96	80	85	68	72	54
10	34	51	48	64	60	75	70	84	78	91	84	96	88	99	90	100	90	99	88	96	84	91	78	84	70	75	60	64	48	51	34
9	16	32	30	45	42	56	52	65	60	72	66	77	70	80	72	81	72	80	70	77	66	72	60	65	52	56	42	45	30	32	16
8	0	15	14	28	26	39	36	48	44	55	50	60	54	63	56	64	56	63	54	60	50	55	44	48	36	39	26	28	14	15	0
7	-14	0	0	13	12	24	22	33	30	40	36	45	40	48	42	49	42	48	40	45	36	40	30	33	22	24	12	13	0	0	-14
6	-26	-13	-12	0	0	11	10	20	18	27	24	32	28	35	30	36	30	35	28	32	24	27	18	20	10	11	0	0	-12	-13	-26
5	-36	-24	-22	-11	-10	0	0	9	8	16	14	21	18	24	20	25	20	24	18	21	14	16	8	9	0	0	-10	-11	-22	-24	-36
4	-44	-33	-30	-20	-18	-9	-8	0	0	7	6	12	10	15	12	16	12	15	10	12	6	7	0	0	-8	-9	-18	-20	-30	-33	-44
3	-50	-40	-36	-27	-24	-16	-14	-7	-6	0	0	5	4	8	6	9	6	8	4	5	0	0	-6	-7	-14	-16	-24	-27	-36	-40	-50
2	-54	-45	-40	-32	-28	-21	-18	-12	-10	-5	-4	0	0	3	2	4	2	3	0	0	-4	-5	-10	-12	-18	-21	-28	-32	-40	-45	-54
Y[1] 1	-56	-48	-42	-35	-30	-24	-20	-15	-12	-8	-6	-3	-2	0	0	1	0	0	-2	-3	-6	-8	-12	-15	-20	-24	-30	-35	-42	-48	-56
Y[0] 0	-56	-49	-42	-36	-30	-25	-20	-16	-12	-9	-6	-4	-2	-1	0	0	0	-1	-2	-4	-6	-9	-12	-16	-20	-25	-30	-36	-42	-49	-56
Y[-1] -1	-54	-48	-40	-35	-28	-24	-18	-15	-10	-8	-4	-3	0	0	2	1	2	0	0	-3	-4	-8	-10	-15	-18	-24	-28	-35	-40	-48	-54
-2	-50	-45	-36	-32	-24	-21	-14	-12	-6	-5	0	0	4	3	6	4	6	3	4	0	0	-5	-6	-12	-14	-21	-24	-32	-36	-45	-50
-3	-44	-40	-30	-27	-18	-16	-8	-7	0	0	6	5	10	8	12	9	12	8	10	5	6	0	0	-7	-8	-16	-18	-27	-30	-40	-44
-4	-36	-33	-22	-20	-10	-9	0	0	8	7	14	12	18	15	20	16	20	15	18	12	14	7	8	0	0	-9	-10	-20	-22	-33	-36
-5	-26	-24	-12	-11	0	0	10	9	18	16	24	21	28	24	30	25	30	24	28	21	24	16	18	9	10	0	0	-11	-12	-24	-26
-6	-14	-13	0	0	12	11	22	20	30	27	36	32	40	35	42	36	42	35	40	32	36	27	30	20	22	11	12	0	0	-13	-14
-7	0	0	14	13	26	24	36	33	44	40	50	45	54	48	56	49	56	48	54	45	50	40	44	33	36	24	26	13	14	0	0
-8	16	15	30	28	42	39	52	48	60	55	66	60	70	63	72	64	72	63	70	60	66	55	60	48	52	39	42	28	30	15	16
-9	34	32	48	45	60	56	70	65	78	72	84	77	88	80	90	81	90	80	88	77	84	72	78	65	70	56	60	45	48	32	34
-10	54	51	68	64	80	75	90	84	98	91	104	96	108	99	110	100	110	99	108	96	104	91	98	84	90	75	80	64	68	51	54
-11	76	72	90	85	102	96	112	105	120	112	126	117	130	120	132	121	132	120	130	117	126	112	120	105	112	96	102	85	90	72	76
-12	100	95	114	108	126	119	136	128	144	135	150	140	154	143	156	144	156	143	154	140	150	135	144	128	136	119	126	108	114	95	100
-13	126	120	140	133	152	144	162	153	170	160	176	165	180	168	182	169	182	168	180	165	176	160	170	153	162	144	152	133	140	120	126
-14	154	147	168	160	180	171	190	180	198	187	204	192	208	195	210	196	210	195	208	192	204	187	198	180	190	171	180	160	168	147	154
-15	184	176	198	189	210	200	220	209	228	216	234	221	238	224	240	225	240	224	238	221	234	216	228	209	220	200	210	189	198	176	184

Table 1. Second-Degree CG's with |a|=1 Summary for HS[1,0,1] in the top and all verticals in offset Zero in the bottom. See the Square and Oblong sequence A002620.

6.11 Summary of quadratics CG for |a|=1

Second-Degree CG's with |a|=1 Summary (from Square numbers sequence [1,0,1] - A000290)																				
Tally	f	2f	f^2	y (y-2f)	≡	with Offset	≡	Equation	b	y_ip	Offset	≡	Equation f=0	b°	y_ip	Offset	≡	Offset 0	≡	OEIS
1	0	0	0	y (y+0)	≡	[0 , 1 , 4]	≡	y^2+0y	0	0	0	≡	y^2-0	0	0	0	≡	[1 , 0 , 1]	≡	A000290
2	-1	-2	1	y (y+2)	≡	[0 , 3 , 8]	≡	y^2+2y	2	-1	-1	≡	y^2-1	0	0	0	≡	[0 , -1 , 0]	≡	A005563
3	-2	-4	4	y (y+4)	≡	[0 , 5 , 12]	≡	y^2+4y	4	-2	-2	≡	y^2-4	0	0	0	≡	[-3 , -4 , -3]	≡	A028347
4	-3	-6	9	y (y+6)	≡	[0 , 7 , 16]	≡	y^2+6y	6	-3	-3	≡	y^2-9	0	0	0	≡	[-8 , -9 , -8]	≡	A028560
5	-4	-8	16	y (y+8)	≡	[0 , 9 , 20]	≡	y^2+8y	8	-4	-4	≡	y^2-16	0	0	0	≡	[-15 , -16 , -15]	≡	A028566
6	-5	-10	25	y (y+10)	≡	[0 , 11 , 24]	≡	y^2+10y	10	-5	-5	≡	y^2-25	0	0	0	≡	[-24 , -25 , -24]	≡	A098603
7	-6	-12	36	y (y+12)	≡	[0 , 13 , 28]	≡	y^2+12y	12	-6	-6	≡	y^2-36	0	0	0	≡	[-35 , -36 , -35]	≡	A098847
8	-7	-14	49	y (y+14)	≡	[0 , 15 , 32]	≡	y^2+14y	14	-7	-7	≡	y^2-49	0	0	0	≡	[-48 , -49 , -48]	≡	A098848
9	-8	-16	64	y (y+16)	≡	[0 , 17 , 36]	≡	y^2+16y	16	-8	-8	≡	y^2-64	0	0	0	≡	[-63 , -64 , -63]	≡	A098849
10	-9	-18	81	y (y+18)	≡	[0 , 19 , 40]	≡	y^2+18y	18	-9	-9	≡	y^2-81	0	0	0	≡	[-80 , -81 , -80]	≡	A098850
11	-10	-20	100	y (y+20)	≡	[0 , 21 , 44]	≡	y^2+20y	20	-10	-10	≡	y^2-100	0	0	0	≡	[-99 , -100 , -99]	≡	A120071
12	-11	-22	121	y (y+22)	≡	[0 , 23 , 48]	≡	y^2+22y	22	-11	-11	≡	y^2-121	0	0	0	≡	[-120 , -121 , -120]	≡	A132764
13	-12	-24	144	y (y+24)	≡	[0 , 25 , 52]	≡	y^2+24y	24	-12	-12	≡	y^2-144	0	0	0	≡	[-143 , -144 , -143]	≡	A132766
14	-13	-26	169	y (y+26)	≡	[0 , 27 , 56]	≡	y^2+26y	26	-13	-13	≡	y^2-169	0	0	0	≡	[-168 , -169 , -168]	≡	A132768
15	-14	-28	196	y (y+28)	≡	[0 , 29 , 60]	≡	y^2+28y	28	-14	-14	≡	y^2-196	0	0	0	≡	[-195 , -196 , -195]	≡	A132770
16	-15	-30	225	y (y+30)	≡	[0 , 31 , 64]	≡	y^2+30y	30	-15	-15	≡	y^2-225	0	0	0	≡	[-224 , -225 , -224]	≡	A132772
17	-16	-32	256	y (y+32)	≡	[0 , 33 , 68]	≡	y^2+32y	32	-16	-16	≡	y^2-256	0	0	0	≡	[-255 , -256 , -255]	≡	A
18	-17	-34	289	y (y+34)	≡	[0 , 35 , 72]	≡	y^2+34y	34	-17	-17	≡	y^2-289	0	0	0	≡	[-288 , -289 , -288]	≡	A
19	-18	-36	324	y (y+36)	≡	[0 , 37 , 76]	≡	y^2+36y	36	-18	-18	≡	y^2-324	0	0	0	≡	[-323 , -324 , -323]	≡	A
20	-19	-38	361	y (y+38)	≡	[0 , 39 , 80]	≡	y^2+38y	38	-19	-19	≡	y^2-361	0	0	0	≡	[-360 , -361 , -360]	≡	A
21	-20	-40	400	y (y+40)	≡	[0 , 41 , 84]	≡	y^2+40y	40	-20	-20	≡	y^2-400	0	0	0	≡	[-399 , -400 , -399]	≡	A
22	-21	-42	441	y (y+42)	≡	[0 , 43 , 88]	≡	y^2+42y	42	-21	-21	≡	y^2-441	0	0	0	≡	[-440 , -441 , -440]	≡	A
23	-22	-44	484	y (y+44)	≡	[0 , 45 , 92]	≡	y^2+44y	44	-22	-22	≡	y^2-484	0	0	0	≡	[-483 , -484 , -483]	≡	A
24	-23	-46	529	y (y+46)	≡	[0 , 47 , 96]	≡	y^2+46y	46	-23	-23	≡	y^2-529	0	0	0	≡	[-528 , -529 , -528]	≡	A
25	-24	-48	576	y (y+48)	≡	[0 , 49 , 100]	≡	y^2+48y	48	-24	-24	≡	y^2-576	0	0	0	≡	[-575 , -576 , -575]	≡	A

Table 1. Second-Degree CG's with |a|=1 Summary (from Square numbers sequence [1,0,1] - A000290).

Second-Degree CG's with |a|=1 Summary (from Oblong numbers sequence [2,0,0] - A002378)																				
Tally	f	2f+1	f^2+f	y (y-(2f+1))	≡	with Offset	≡	Equation	b	y_ip	Offset	≡	Equation f=0	b°	y_ip	Offset	≡	Offset 0	≡	OEIS
1	-1	-1	0	y (y+1)	≡	[0 , 2 , 6]	≡	y^2+1y	1	-0,5	-1	≡	y^2-y-0	-1	0,5	0	≡	[2 , 0 , 2]	≡	A002378
2	-2	-3	2	y (y+3)	≡	[0 , 4 , 10]	≡	y^2+3y	3	-1,5	-2	≡	y^2-y-2	-1	0,5	0	≡	[0 , -2 , 0]	≡	A028552
3	-3	-5	6	y (y+5)	≡	[0 , 6 , 14]	≡	y^2+5y	5	-2,5	-3	≡	y^2-y-6	-1	0,5	0	≡	[-4 , -6 , -4]	≡	A028557
4	-4	-7	12	y (y+7)	≡	[0 , 8 , 18]	≡	y^2+7y	7	-3,5	-4	≡	y^2-y-12	-1	0,5	0	≡	[-10 , -12 , -10]	≡	A028563
5	-5	-9	20	y (y+9)	≡	[0 , 10 , 22]	≡	y^2+9y	9	-4,5	-5	≡	y^2-y-20	-1	0,5	0	≡	[-18 , -20 , -18]	≡	A028569
6	-6	-11	30	y (y+11)	≡	[0 , 12 , 26]	≡	y^2+11y	11	-5,5	-6	≡	y^2-y-30	-1	0,5	0	≡	[-28 , -30 , -28]	≡	A119412
7	-7	-13	42	y (y+13)	≡	[0 , 14 , 30]	≡	y^2+13y	13	-6,5	-7	≡	y^2-y-42	-1	0,5	0	≡	[-40 , -42 , -40]	≡	A132759
8	-8	-15	56	y (y+15)	≡	[0 , 16 , 34]	≡	y^2+15y	15	-7,5	-8	≡	y^2-y-56	-1	0,5	0	≡	[-54 , -56 , -54]	≡	A132760
9	-9	-17	72	y (y+17)	≡	[0 , 18 , 38]	≡	y^2+17y	17	-8,5	-9	≡	y^2-y-72	-1	0,5	0	≡	[-70 , -72 , -70]	≡	A132761
10	-10	-19	90	y (y+19)	≡	[0 , 20 , 42]	≡	y^2+19y	19	-9,5	-10	≡	y^2-y-90	-1	0,5	0	≡	[-88 , -90 , -88]	≡	A132762
11	-11	-21	110	y (y+21)	≡	[0 , 22 , 46]	≡	y^2+21y	21	-11	-11	≡	y^2-y-110	-1	0,5	0	≡	[-108 , -110 , -108]	≡	A132763
12	-12	-23	132	y (y+23)	≡	[0 , 24 , 50]	≡	y^2+23y	23	-12	-12	≡	y^2-y-132	-1	0,5	0	≡	[-130 , -132 , -130]	≡	A132765
13	-13	-25	156	y (y+25)	≡	[0 , 26 , 54]	≡	y^2+25y	25	-13	-13	≡	y^2-y-156	-1	0,5	0	≡	[-154 , -156 , -154]	≡	A132767
14	-14	-27	182	y (y+27)	≡	[0 , 28 , 58]	≡	y^2+27y	27	-14	-14	≡	y^2-y-182	-1	0,5	0	≡	[-180 , -182 , -180]	≡	A132769
15	-15	-29	210	y (y+29)	≡	[0 , 30 , 62]	≡	y^2+29y	29	-15	-15	≡	y^2-y-210	-1	0,5	0	≡	[-208 , -210 , -208]	≡	A132771
16	-16	-31	240	y (y+31)	≡	[0 , 32 , 66]	≡	y^2+31y	31	-16	-16	≡	y^2-y-240	-1	0,5	0	≡	[-238 , -240 , -238]	≡	A132773
17	-17	-33	272	y (y+33)	≡	[0 , 34 , 70]	≡	y^2+33y	33	-17	-17	≡	y^2-y-272	-1	0,5	0	≡	[-270 , -272 , -270]	≡	A
18	-18	-35	306	y (y+35)	≡	[0 , 36 , 74]	≡	y^2+35y	35	-18	-18	≡	y^2-y-306	-1	0,5	0	≡	[-304 , -306 , -304]	≡	A
19	-19	-37	342	y (y+37)	≡	[0 , 38 , 78]	≡	y^2+37y	37	-19	-19	≡	y^2-y-342	-1	0,5	0	≡	[-340 , -342 , -340]	≡	A
20	-20	-39	380	y (y+39)	≡	[0 , 40 , 82]	≡	y^2+39y	39	-20	-20	≡	y^2-y-380	-1	0,5	0	≡	[-378 , -380 , -378]	≡	A
21	-21	-41	420	y (y+41)	≡	[0 , 42 , 86]	≡	y^2+41y	41	-21	-21	≡	y^2-y-420	-1	0,5	0	≡	[-418 , -420 , -418]	≡	A
22	-22	-43	462	y (y+43)	≡	[0 , 44 , 90]	≡	y^2+43y	43	-22	-22	≡	y^2-y-462	-1	0,5	0	≡	[-460 , -462 , -460]	≡	A
23	-23	-45	506	y (y+45)	≡	[0 , 46 , 94]	≡	y^2+45y	45	-23	-23	≡	y^2-y-506	-1	0,5	0	≡	[-504 , -506 , -504]	≡	A
24	-24	-47	552	y (y+47)	≡	[0 , 48 , 98]	≡	y^2+47y	47	-24	-24	≡	y^2-y-552	-1	0,5	0	≡	[-550 , -552 , -550]	≡	A
25	-25	-49	600	y (y+49)	≡	[0 , 50 , 102]	≡	y^2+49y	49	-25	-25	≡	y^2-y-600	-1	0,5	0	≡	[-598 , -600 , -598]	≡	A

Table 1. Second-Degree CG's with |a|=1 Summary (from Oblong numbers sequence [2,0,0] - A002378).

There is the separation between even and odd or between Square numbers sequence A000290 and Oblong numbers sequence A002378 because for |a|=1 there are 2 consecutive values of coefficient b to change offset. As we saw in the Offset study, for each value of |a| there are b=|2a| values in the same offset.

Second-Degree CG's with |a|=1 Summary (from Square and Oblong alternatively numbers sequence - A002620)

Tally	f	A002620		Equation	≡	with Offset	≡	Equation	b	y_ip	Offset	≡	Equation f=0	b°	y_ip	Offset	≡	Offset 0	≡	OEIS
1	0	0	0	y (y+0)	≡	[0 , 1 , 4]	≡	y^2+0y	0	0	0	≡	y^2-0	0	0	0	≡	[1 , 0 , 1]	≡	A000290
2	-1	-1	0	y (y+1)	≡	[0 , 2 , 6]	≡	y^2+1y	1	-0,5	-1	≡	y^2-y-0	-1	0,5	0	≡	[2 , 0 , 2]	≡	A002378
3	-1	-2	1	y (y+2)	≡	[0 , 3 , 8]	≡	y^2+2y	2	-1	-1	≡	y^2-1	0	0	0	≡	[0 , -1 , 0]	≡	A005563
4	-2	-3	2	y (y+3)	≡	[0 , 4 , 10]	≡	y^2+3y	3	-1,5	-2	≡	y^2-y-2	-1	0,5	0	≡	[0 , -2 , 0]	≡	A028552
5	-2	-4	4	y (y+4)	≡	[0 , 5 , 12]	≡	y^2+4y	4	-2	-2	≡	y^2-4	0	0	0	≡	[-3 , -4 , -3]	≡	A028347
6	-3	-5	6	y (y+5)	≡	[0 , 6 , 14]	≡	y^2+5y	5	-2,5	-3	≡	y^2-y-6	-1	0,5	0	≡	[-4 , -6 , -4]	≡	A028557
7	-3	-6	9	y (y+6)	≡	[0 , 7 , 16]	≡	y^2+6y	6	-3	-3	≡	y^2-9	0	0	0	≡	[-8 , -9 , -8]	≡	A028560
8	-4	-7	12	y (y+7)	≡	[0 , 8 , 18]	≡	y^2+7y	7	-3,5	-4	≡	y^2-y-12	-1	0,5	0	≡	[-10 , -12 , -10]	≡	A028563
9	-4	-8	16	y (y+8)	≡	[0 , 9 , 20]	≡	y^2+8y	8	-4	-4	≡	y^2-16	0	0	0	≡	[-15 , -16 , -15]	≡	A028566
10	-5	-9	20	y (y+9)	≡	[0 , 10 , 22]	≡	y^2+9y	9	-4,5	-5	≡	y^2-y-20	-1	0,5	0	≡	[-18 , -20 , -18]	≡	A028569
11	-5	-10	25	y (y+10)	≡	[0 , 11 , 24]	≡	y^2+10y	10	-5	-5	≡	y^2-25	0	0	0	≡	[-24 , -25 , -24]	≡	A098603
12	-6	-11	30	y (y+11)	≡	[0 , 12 , 26]	≡	y^2+11y	11	-5,5	-6	≡	y^2-y-30	-1	0,5	0	≡	[-28 , -30 , -28]	≡	A119412
13	-6	-12	36	y (y+12)	≡	[0 , 13 , 28]	≡	y^2+12y	12	-6	-6	≡	y^2-36	0	0	0	≡	[-35 , -36 , -35]	≡	A098847
14	-7	-13	42	y (y+13)	≡	[0 , 14 , 30]	≡	y^2+13y	13	-6,5	-7	≡	y^2-y-42	-1	0,5	0	≡	[-40 , -42 , -40]	≡	A132759
15	-7	-14	49	y (y+14)	≡	[0 , 15 , 32]	≡	y^2+14y	14	-7	-7	≡	y^2-49	0	0	0	≡	[-48 , -49 , -48]	≡	A098848
16	-8	-15	56	y (y+15)	≡	[0 , 16 , 34]	≡	y^2+15y	15	-7,5	-8	≡	y^2-y-56	-1	0,5	0	≡	[-54 , -56 , -54]	≡	A132760
17	-8	-16	64	y (y+16)	≡	[0 , 17 , 36]	≡	y^2+16y	16	-8	-8	≡	y^2-64	0	0	0	≡	[-63 , -64 , -63]	≡	A098849
18	-9	-17	72	y (y+17)	≡	[0 , 18 , 38]	≡	y^2+17y	17	-8,5	-9	≡	y^2-y-72	-1	0,5	0	≡	[-70 , -72 , -70]	≡	A132761
19	-9	-18	81	y (y+18)	≡	[0 , 19 , 40]	≡	y^2+18y	18	-9	-9	≡	y^2-81	0	0	0	≡	[-80 , -81 , -80]	≡	A098850
20	-10	-19	90	y (y+19)	≡	[0 , 20 , 42]	≡	y^2+19y	19	-9,5	-10	≡	y^2-y-90	-1	0,5	0	≡	[-88 , -90 , -88]	≡	A132762
21	-10	-20	100	y (y+20)	≡	[0 , 21 , 44]	≡	y^2+20y	20	-10	-10	≡	y^2-100	0	0	0	≡	[-99 , -100 , -99]	≡	A120071
22	-11	-21	110	y (y+21)	≡	[0 , 22 , 46]	≡	y^2+21y	21	-11	-11	≡	y^2-y-110	-1	0,5	0	≡	[-108 , -110 , -108]	≡	A132763
23	-11	-22	121	y (y+22)	≡	[0 , 23 , 48]	≡	y^2+22y	22	-11	-11	≡	y^2-121	0	0	0	≡	[-120 , -121 , -120]	≡	A132764
24	-12	-23	132	y (y+23)	≡	[0 , 24 , 50]	≡	y^2+23y	23	-12	-12	≡	y^2-y-132	-1	0,5	0	≡	[-130 , -132 , -130]	≡	A132765
25	-12	-24	144	y (y+24)	≡	[0 , 25 , 52]	≡	y^2+24y	24	-12	-12	≡	y^2-144	0	0	0	≡	[-143 , -144 , -143]	≡	A132766
26	-13	-25	156	y (y+25)	≡	[0 , 26 , 54]	≡	y^2+25y	25	-13	-13	≡	y^2-y-156	-1	0,5	0	≡	[-154 , -156 , -154]	≡	A132767
27	-13	-26	169	y (y+26)	≡	[0 , 27 , 56]	≡	y^2+26y	26	-13	-13	≡	y^2-169	0	0	0	≡	[-168 , -169 , -168]	≡	A132768
28	-14	-27	182	y (y+27)	≡	[0 , 28 , 58]	≡	y^2+27y	27	-14	-14	≡	y^2-y-182	-1	0,5	0	≡	[-180 , -182 , -180]	≡	A132769
29	-14	-28	196	y (y+28)	≡	[0 , 29 , 60]	≡	y^2+28y	28	-14	-14	≡	y^2-196	0	0	0	≡	[-195 , -196 , -195]	≡	A132770
30	-15	-29	210	y (y+29)	≡	[0 , 30 , 62]	≡	y^2+29y	29	-15	-15	≡	y^2-y-210	-1	0,5	0	≡	[-208 , -210 , -208]	≡	A132771
31	-15	-30	225	y (y+30)	≡	[0 , 31 , 64]	≡	y^2+30y	30	-15	-15	≡	y^2-225	0	0	0	≡	[-224 , -225 , -224]	≡	A132772
32	-16	-31	240	y (y+31)	≡	[0 , 32 , 66]	≡	y^2+31y	31	-16	-16	≡	y^2-y-240	-1	0,5	0	≡	[-238 , -240 , -238]	≡	A132773
33	-16	-32	256	y (y+32)	≡	[0 , 33 , 68]	≡	y^2+32y	32	-16	-16	≡	y^2-256	0	0	0	≡	[-255 , -256 , -255]	≡	A
34	-17	-33	272	y (y+33)	≡	[0 , 34 , 70]	≡	y^2+33y	33	-17	-17	≡	y^2-y-272	-1	0,5	0	≡	[-270 , -272 , -270]	≡	A
35	-17	-34	289	y (y+34)	≡	[0 , 35 , 72]	≡	y^2+34y	34	-17	-17	≡	y^2-289	0	0	0	≡	[-288 , -289 , -288]	≡	A
36	-18	-35	306	y (y+35)	≡	[0 , 36 , 74]	≡	y^2+35y	35	-18	-18	≡	y^2-y-306	-1	0,5	0	≡	[-304 , -306 , -304]	≡	A
37	-18	-36	324	y (y+36)	≡	[0 , 37 , 76]	≡	y^2+36y	36	-18	-18	≡	y^2-324	0	0	0	≡	[-323 , -324 , -323]	≡	A
38	-19	-37	342	y (y+37)	≡	[0 , 38 , 78]	≡	y^2+37y	37	-19	-19	≡	y^2-y-342	-1	0,5	0	≡	[-340 , -342 , -340]	≡	A
39	-19	-38	361	y (y+38)	≡	[0 , 39 , 80]	≡	y^2+38y	38	-19	-19	≡	y^2-361	0	0	0	≡	[-360 , -361 , -360]	≡	A
40	-20	-39	380	y (y+39)	≡	[0 , 40 , 82]	≡	y^2+39y	39	-20	-20	≡	y^2-y-380	-1	0,5	0	≡	[-378 , -380 , -378]	≡	A
41	-20	-40	400	y (y+40)	≡	[0 , 41 , 84]	≡	y^2+40y	40	-20	-20	≡	y^2-400	0	0	0	≡	[-399 , -400 , -399]	≡	A
42	-21	-41	420	y (y+41)	≡	[0 , 42 , 86]	≡	y^2+41y	41	-21	-21	≡	y^2-y-420	-1	0,5	0	≡	[-418 , -420 , -418]	≡	A
43	-21	-42	441	y (y+42)	≡	[0 , 43 , 88]	≡	y^2+42y	42	-21	-21	≡	y^2-441	0	0	0	≡	[-440 , -441 , -440]	≡	A
44	-22	-43	462	y (y+43)	≡	[0 , 44 , 90]	≡	y^2+43y	43	-22	-22	≡	y^2-y-462	-1	0,5	0	≡	[-460 , -462 , -460]	≡	A
45	-22	-44	484	y (y+44)	≡	[0 , 45 , 92]	≡	y^2+44y	44	-22	-22	≡	y^2-484	0	0	0	≡	[-483 , -484 , -483]	≡	A
46	-23	-45	506	y (y+45)	≡	[0 , 46 , 94]	≡	y^2+45y	45	-23	-23	≡	y^2-y-506	-1	0,5	0	≡	[-504 , -506 , -504]	≡	A
47	-23	-46	529	y (y+46)	≡	[0 , 47 , 96]	≡	y^2+46y	46	-23	-23	≡	y^2-529	0	0	0	≡	[-528 , -529 , -528]	≡	A
48	-24	-47	552	y (y+47)	≡	[0 , 48 , 98]	≡	y^2+47y	47	-24	-24	≡	y^2-y-552	-1	0,5	0	≡	[-550 , -552 , -550]	≡	A
49	-24	-48	576	y (y+48)	≡	[0 , 49 , 100]	≡	y^2+48y	48	-24	-24	≡	y^2-576	0	0	0	≡	[-575 , -576 , -575]	≡	A
50	-25	-49	600	y (y+49)	≡	[0 , 50 , 102]	≡	y^2+49y	49	-25	-25	≡	y^2-y-600	-1	0,5	0	≡	[-598 , -600 , -598]	≡	A

Table 1. Second-Degree CG's with |a|=1 Summary (from Square and Oblong alternatively numbers sequence - A002620).

7 Rotations of the FMT

Now we saw hyperboctys operations and composite generator, let's see the behavior of FMT in rotations.

7.1 The first CCW rotation of the Full Multiplication Table is the HS[1,0,1]

In this way, if we make a one-step counterclockwise rotation of FMT Hyperboctys HS[0,0,0] we will get Hyperboctys HS[1,0,1].

A counterclockwise rotation, transforms the diagonals with a 45° inclination of the HS[0,0,0] in the vertical position in the HS[1,0,1], and transforms the verticals of the HS[0,0,0] into diagonal slopes 135°(or −45°) in the HS[1,0,1].

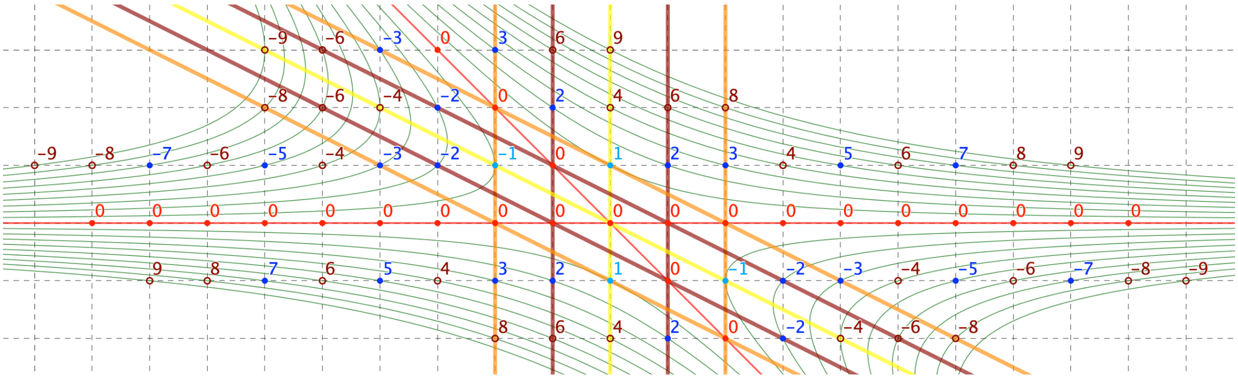

Figure 1. The hyperboctys HS[1,0,1] in XY plane, hyperbolas $xy + y^2 = Integer$

a		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b		-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
c		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500	520	540	560	580	600
	19	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475	494	513	532	551
	18	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450	468	486	504
	17	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425	442	459
	16	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400	416
	15	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
	14	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336
	13	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264
	11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231
	10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
	9	-9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
	8	-16	-8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144
	7	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119
	6	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96
	5	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
	4	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56
	3	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30	33	36	39
	2	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22	24
Y[1]	1	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
	-2	24	22	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16
	-3	39	36	33	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21
	-4	56	52	48	44	40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24
	-5	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25
	-6	96	90	84	78	72	66	60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24
	-7	119	112	105	98	91	84	77	70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21
	-8	144	136	128	120	112	104	96	88	80	72	64	56	48	40	32	24	16	8	0	-8	-16
	-9	171	162	153	144	135	126	117	108	99	90	81	72	63	54	45	36	27	18	9	0	-9
	-10	200	190	180	170	160	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	0
	-11	231	220	209	198	187	176	165	154	143	132	121	110	99	88	77	66	55	44	33	22	11
	-12	264	252	240	228	216	204	192	180	168	156	144	132	120	108	96	84	72	60	48	36	24
	-13	299	286	273	260	247	234	221	208	195	182	169	156	143	130	117	104	91	78	65	52	39
	-14	336	322	308	294	280	266	252	238	224	210	196	182	168	154	140	126	112	98	84	70	56
	-15	375	360	345	330	315	300	285	270	255	240	225	210	195	180	165	150	135	120	105	90	75
	-16	416	400	384	368	352	336	320	304	288	272	256	240	224	208	192	176	160	144	128	112	96
	-17	459	442	425	408	391	374	357	340	323	306	289	272	255	238	221	204	187	170	153	136	119
	-18	504	486	468	450	432	414	396	378	360	342	324	306	288	270	252	234	216	198	180	162	144
	-19	551	532	513	494	475	456	437	418	399	380	361	342	323	304	285	266	247	228	209	190	171
	-20	600	580	560	540	520	500	480	460	440	420	400	380	360	340	320	300	280	260	240	220	200

Table 1. The hyperboctys HS[1,0,1] in table format

Now, the three basic elements $\left[Y[-1], Y[0], Y[1]\right] = [1,0,1]$ are the base of this new hyperbolic grid structure. In the verticals, we have the quadratic function with the coefficient $a = 1$.

The lattice grid remains hyperbolic, now with equations $y^2 + xy = n$.

Note that the construction algorithm for the lines remains the same.

Any element in row 1 results from adding 1 to the previous element, any element in row 2 results from adding 2 to the previous element, any element in row 3 results from adding 3 to the previous element, and so on.

Following the variation of the sequence angles from $HS[n, h, n]$ to $HS[(n+1), h, (n+1)]$ above, all the sequences in FMT $HS[0,0,0]$ appear now in $HS[1,0,1]$ rotated counterclockwise.

This ensures the continuity of the hyperbolic grid.

7.2 The second CCW rotation of the Full Multiplication Table is the HS[2,0,2]

If we rotate counterclockwise HS[1,0,1] we will get HS[2,0,2].

Here, the verticals remain quadratic functions, but now with coefficient $a = 2$.

Lattice-grid remains hyperbolic with equation $Y(y) = 2y^2 + xy = n$.

We rotate counterclockwise all the sequences.

And again, the construction algorithm for the lines remains the same.

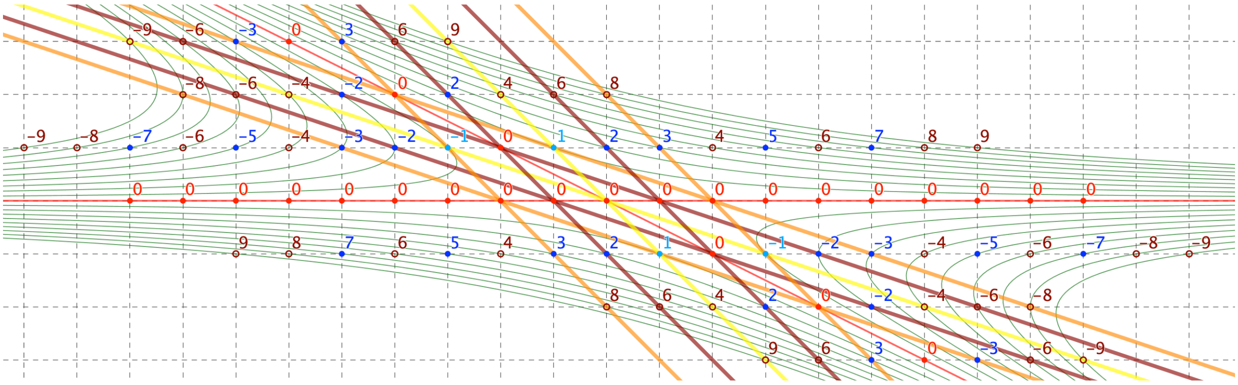

Figure 1. The hyperboctys HS[2,0,2] in XY plane, hyperbolas $xy + 2y^2 = Integer$

a		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
b		-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
c		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	600	620	640	660	680	700	720	740	760	780	800	820	840	860	880	900	920	940	960	980	1000
	19	532	551	570	589	608	627	646	665	684	703	722	741	760	779	798	817	836	855	874	893	912
	18	468	486	504	522	540	558	576	594	612	630	648	666	684	702	720	738	756	774	792	810	828
	17	408	425	442	459	476	493	510	527	544	561	578	595	612	629	646	663	680	697	714	731	748
	16	352	368	384	400	416	432	448	464	480	496	512	528	544	560	576	592	608	624	640	656	672
	15	300	315	330	345	360	375	390	405	420	435	450	465	480	495	510	525	540	555	570	585	600
	14	252	266	280	294	308	322	336	350	364	378	392	406	420	434	448	462	476	490	504	518	532
	13	208	221	234	247	260	273	286	299	312	325	338	351	364	377	390	403	416	429	442	455	468
	12	168	180	192	204	216	228	240	252	264	276	288	300	312	324	336	348	360	372	384	396	408
	11	132	143	154	165	176	187	198	209	220	231	242	253	264	275	286	297	308	319	330	341	352
	10	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300
	9	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225	234	243	252
	8	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200	208
	7	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168
	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132
	5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
	4	-8	-4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72
	3	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48
	2	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28
Y[1]	1	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8
	-2	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12
	-3	48	45	42	39	36	33	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12
	-4	72	68	64	60	56	52	48	44	40	36	32	28	24	20	16	12	8	4	0	-4	-8
	-5	100	95	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
	-6	132	126	120	114	108	102	96	90	84	78	72	66	60	54	48	42	36	30	24	18	12
	-7	168	161	154	147	140	133	126	119	112	105	98	91	84	77	70	63	56	49	42	35	28
	-8	208	200	192	184	176	168	160	152	144	136	128	120	112	104	96	88	80	72	64	56	48
	-9	252	243	234	225	216	207	198	189	180	171	162	153	144	135	126	117	108	99	90	81	72
	-10	300	290	280	270	260	250	240	230	220	210	200	190	180	170	160	150	140	130	120	110	100
	-11	352	341	330	319	308	297	286	275	264	253	242	231	220	209	198	187	176	165	154	143	132
	-12	408	396	384	372	360	348	336	324	312	300	288	276	264	252	240	228	216	204	192	180	168
	-13	468	455	442	429	416	403	390	377	364	351	338	325	312	299	286	273	260	247	234	221	208
	-14	532	518	504	490	476	462	448	434	420	406	392	378	364	350	336	322	308	294	280	266	252
	-15	600	585	570	555	540	525	510	495	480	465	450	435	420	405	390	375	360	345	330	315	300
	-16	672	656	640	624	608	592	576	560	544	528	512	496	480	464	448	432	416	400	384	368	352
	-17	748	731	714	697	680	663	646	629	612	595	578	561	544	527	510	493	476	459	442	425	408
	-18	828	810	792	774	756	738	720	702	684	666	648	630	612	594	576	558	540	522	504	486	468
	-19	912	893	874	855	836	817	798	779	760	741	722	703	684	665	646	627	608	589	570	551	532
	-20	1000	980	960	940	920	900	880	860	840	820	800	780	760	740	720	700	680	660	640	620	600

Table 1. The hyperboctys HS[2,0,2] in table format

7.3 The generic rotation of the Full Multiplication Table is the HS[a,0,a]

This CCW rotation procedure is performed indefinitely. Here a is the rotation value and the coefficient of 2nd degree of the vertical quadratics. They are all quadratic composite generators.

$$ay^2 + xy = n$$

	a	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	b	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	1000	1020	1040	1060	1080	1100	1120	1140	1160	1180	1200	1220	1240	1260	1280	1300	1320	1340	1360	1380	1400
	19	893	912	931	950	969	988	1007	1026	1045	1064	1083	1102	1121	1140	1159	1178	1197	1216	1235	1254	1273
	18	792	810	828	846	864	882	900	918	936	954	972	990	1008	1026	1044	1062	1080	1098	1116	1134	1152
	17	697	714	731	748	765	782	799	816	833	850	867	884	901	918	935	952	969	986	1003	1020	1037
	16	608	624	640	656	672	688	704	720	736	752	768	784	800	816	832	848	864	880	896	912	928
	15	525	540	555	570	585	600	615	630	645	660	675	690	705	720	735	750	765	780	795	810	825
	14	448	462	476	490	504	518	532	546	560	574	588	602	616	630	644	658	672	686	700	714	728
	13	377	390	403	416	429	442	455	468	481	494	507	520	533	546	559	572	585	598	611	624	637
	12	312	324	336	348	360	372	384	396	408	420	432	444	456	468	480	492	504	516	528	540	552
	11	253	264	275	286	297	308	319	330	341	352	363	374	385	396	407	418	429	440	451	462	473
	10	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	360	370	380	390	400
	9	153	162	171	180	189	198	207	216	225	234	243	252	261	270	279	288	297	306	315	324	333
	8	112	120	128	136	144	152	160	168	176	184	192	200	208	216	224	232	240	248	256	264	272
	7	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175	182	189	196	203	210	217
	6	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150	156	162	168
	5	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88
	3	-3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
	2	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
Y[1]	1	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7
	-2	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8
	-3	57	54	51	48	45	42	39	36	33	30	27	24	21	18	15	12	9	6	3	0	-3
	-4	88	84	80	76	72	68	64	60	56	52	48	44	40	36	32	28	24	20	16	12	8
	-5	125	120	115	110	105	100	95	90	85	80	75	70	65	60	55	50	45	40	35	30	25
	-6	168	162	156	150	144	138	132	126	120	114	108	102	96	90	84	78	72	66	60	54	48
	-7	217	210	203	196	189	182	175	168	161	154	147	140	133	126	119	112	105	98	91	84	77
	-8	272	264	256	248	240	232	224	216	208	200	192	184	176	168	160	152	144	136	128	120	112
	-9	333	324	315	306	297	288	279	270	261	252	243	234	225	216	207	198	189	180	171	162	153
	-10	400	390	380	370	360	350	340	330	320	310	300	290	280	270	260	250	240	230	220	210	200
	-11	473	462	451	440	429	418	407	396	385	374	363	352	341	330	319	308	297	286	275	264	253
	-12	552	540	528	516	504	492	480	468	456	444	432	420	408	396	384	372	360	348	336	324	312
	-13	637	624	611	598	585	572	559	546	533	520	507	494	481	468	455	442	429	416	403	390	377
	-14	728	714	700	686	672	658	644	630	616	602	588	574	560	546	532	518	504	490	476	462	448
	-15	825	810	795	780	765	750	735	720	705	690	675	660	645	630	615	600	585	570	555	540	525
	-16	928	912	896	880	864	848	832	816	800	784	768	752	736	720	704	688	672	656	640	624	608
	-17	1037	1020	1003	986	969	952	935	918	901	884	867	850	833	816	799	782	765	748	731	714	697
	-18	1152	1134	1116	1098	1080	1062	1044	1026	1008	990	972	954	936	918	900	882	864	846	828	810	792
	-19	1273	1254	1235	1216	1197	1178	1159	1140	1121	1102	1083	1064	1045	1026	1007	988	969	950	931	912	893
	-20	1400	1380	1360	1340	1320	1300	1280	1260	1240	1220	1200	1180	1160	1140	1120	1100	1080	1060	1040	1020	1000

Table 1. The hyperboctys HS[3,0,3] in table format

	a	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	b	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	1400	1420	1440	1460	1480	1500	1520	1540	1560	1580	1600	1620	1640	1660	1680	1700	1720	1740	1760	1780	1800
	19	1254	1273	1292	1311	1330	1349	1368	1387	1406	1425	1444	1463	1482	1501	1520	1539	1558	1577	1596	1615	1634
	18	1116	1134	1152	1170	1188	1206	1224	1242	1260	1278	1296	1314	1332	1350	1368	1386	1404	1422	1440	1458	1476
	17	986	1003	1020	1037	1054	1071	1088	1105	1122	1139	1156	1173	1190	1207	1224	1241	1258	1275	1292	1309	1326
	16	864	880	896	912	928	944	960	976	992	1008	1024	1040	1056	1072	1088	1104	1120	1136	1152	1168	1184
	15	750	765	780	795	810	825	840	855	870	885	900	915	930	945	960	975	990	1005	1020	1035	1050
	14	644	658	672	686	700	714	728	742	756	770	784	798	812	826	840	854	868	882	896	910	924
	13	546	559	572	585	598	611	624	637	650	663	676	689	702	715	728	741	754	767	780	793	806
	12	456	468	480	492	504	516	528	540	552	564	576	588	600	612	624	636	648	660	672	684	696
	11	374	385	396	407	418	429	440	451	462	473	484	495	506	517	528	539	550	561	572	583	594
	10	300	310	320	330	340	350	360	370	380	390	400	410	420	430	440	450	460	470	480	490	500
	9	234	243	252	261	270	279	288	297	306	315	324	333	342	351	360	369	378	387	396	405	414
	8	176	184	192	200	208	216	224	232	240	248	256	264	272	280	288	296	304	312	320	328	336
	7	126	133	140	147	154	161	168	175	182	189	196	203	210	217	224	231	238	245	252	259	266
	6	84	90	96	102	108	114	120	126	132	138	144	150	156	162	168	174	180	186	192	198	204
	5	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150
	4	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100	104
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66
	2	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36
Y[1]	1	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
	-2	36	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	-2	-4
	-3	66	63	60	57	54	51	48	45	42	39	36	33	30	27	24	21	18	15	12	9	6
	-4	104	100	96	92	88	84	80	76	72	68	64	60	56	52	48	44	40	36	32	28	24
	-5	150	145	140	135	130	125	120	115	110	105	100	95	90	85	80	75	70	65	60	55	50
	-6	204	198	192	186	180	174	168	162	156	150	144	138	132	126	120	114	108	102	96	90	84
	-7	266	259	252	245	238	231	224	217	210	203	196	189	182	175	168	161	154	147	140	133	126
	-8	336	328	320	312	304	296	288	280	272	264	256	248	240	232	224	216	208	200	192	184	176
	-9	414	405	396	387	378	369	360	351	342	333	324	315	306	297	288	279	270	261	252	243	234
	-10	500	490	480	470	460	450	440	430	420	410	400	390	380	370	360	350	340	330	320	310	300
	-11	594	583	572	561	550	539	528	517	506	495	484	473	462	451	440	429	418	407	396	385	374
	-12	696	684	672	660	648	636	624	612	600	588	576	564	552	540	528	516	504	492	480	468	456
	-13	806	793	780	767	754	741	728	715	702	689	676	663	650	637	624	611	598	585	572	559	546
	-14	924	910	896	882	868	854	840	826	812	798	784	770	756	742	728	714	700	686	672	658	644
	-15	1050	1035	1020	1005	990	975	960	945	930	915	900	885	870	855	840	825	810	795	780	765	750
	-16	1184	1168	1152	1136	1120	1104	1088	1072	1056	1040	1024	1008	992	976	960	944	928	912	896	880	864
	-17	1326	1309	1292	1275	1258	1241	1224	1207	1190	1173	1156	1139	1122	1105	1088	1071	1054	1037	1020	1003	986
	-18	1476	1458	1440	1422	1404	1386	1368	1350	1332	1314	1296	1278	1260	1242	1224	1206	1188	1170	1152	1134	1116
	-19	1634	1615	1596	1577	1558	1539	1520	1501	1482	1463	1444	1425	1406	1387	1368	1349	1330	1311	1292	1273	1254
	-20	1800	1780	1760	1740	1720	1700	1680	1660	1640	1620	1600	1580	1560	1540	1520	1500	1480	1460	1440	1420	1400

Table 1. The hyperboctys HS[4,0,4] in table format

	a	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	b	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	1800	1820	1840	1860	1880	1900	1920	1940	1960	1980	2000	2020	2040	2060	2080	2100	2120	2140	2160	2180	2200
	19	1615	1634	1653	1672	1691	1710	1729	1748	1767	1786	1805	1824	1843	1862	1881	1900	1919	1938	1957	1976	1995
	18	1440	1458	1476	1494	1512	1530	1548	1566	1584	1602	1620	1638	1656	1674	1692	1710	1728	1746	1764	1782	1800
	17	1275	1292	1309	1326	1343	1360	1377	1394	1411	1428	1445	1462	1479	1496	1513	1530	1547	1564	1581	1598	1615
	16	1120	1136	1152	1168	1184	1200	1216	1232	1248	1264	1280	1296	1312	1328	1344	1360	1376	1392	1408	1424	1440
	15	975	990	1005	1020	1035	1050	1065	1080	1095	1110	1125	1140	1155	1170	1185	1200	1215	1230	1245	1260	1275
	14	840	854	868	882	896	910	924	938	952	966	980	994	1008	1022	1036	1050	1064	1078	1092	1106	1120
	13	715	728	741	754	767	780	793	806	819	832	845	858	871	884	897	910	923	936	949	962	975
	12	600	612	624	636	648	660	672	684	696	708	720	732	744	756	768	780	792	804	816	828	840
	11	495	506	517	528	539	550	561	572	583	594	605	616	627	638	649	660	671	682	693	704	715
	10	400	410	420	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590	600
	9	315	324	333	342	351	360	369	378	387	396	405	414	423	432	441	450	459	468	477	486	495
	8	240	248	256	264	272	280	288	296	304	312	320	328	336	344	352	360	368	376	384	392	400
	7	175	182	189	196	203	210	217	224	231	238	245	252	259	266	273	280	287	294	301	308	315
	6	120	126	132	138	144	150	156	162	168	174	180	186	192	198	204	210	216	222	228	234	240
	5	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175
	4	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116	120
	3	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
	2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
Y[1]	1	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5
	-2	40	38	36	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0
	-3	75	72	69	66	63	60	57	54	51	48	45	42	39	36	33	30	27	24	21	18	15
	-4	120	116	112	108	104	100	96	92	88	84	80	76	72	68	64	60	56	52	48	44	40
	-5	175	170	165	160	155	150	145	140	135	130	125	120	115	110	105	100	95	90	85	80	75
	-6	240	234	228	222	216	210	204	198	192	186	180	174	168	162	156	150	144	138	132	126	120
	-7	315	308	301	294	287	280	273	266	259	252	245	238	231	224	217	210	203	196	189	182	175
	-8	400	392	384	376	368	360	352	344	336	328	320	312	304	296	288	280	272	264	256	248	240
	-9	495	486	477	468	459	450	441	432	423	414	405	396	387	378	369	360	351	342	333	324	315
	-10	600	590	580	570	560	550	540	530	520	510	500	490	480	470	460	450	440	430	420	410	400
	-11	715	704	693	682	671	660	649	638	627	616	605	594	583	572	561	550	539	528	517	506	495
	-12	840	828	816	804	792	780	768	756	744	732	720	708	696	684	672	660	648	636	624	612	600
	-13	975	962	949	936	923	910	897	884	871	858	845	832	819	806	793	780	767	754	741	728	715
	-14	1120	1106	1092	1078	1064	1050	1036	1022	1008	994	980	966	952	938	924	910	896	882	868	854	840
	-15	1275	1260	1245	1230	1215	1200	1185	1170	1155	1140	1125	1110	1095	1080	1065	1050	1035	1020	1005	990	975
	-16	1440	1424	1408	1392	1376	1360	1344	1328	1312	1296	1280	1264	1248	1232	1216	1200	1184	1168	1152	1136	1120
	-17	1615	1598	1581	1564	1547	1530	1513	1496	1479	1462	1445	1428	1411	1394	1377	1360	1343	1326	1309	1292	1275
	-18	1800	1782	1764	1746	1728	1710	1692	1674	1656	1638	1620	1602	1584	1566	1548	1530	1512	1494	1476	1458	1440
	-19	1995	1976	1957	1938	1919	1900	1881	1862	1843	1824	1805	1786	1767	1748	1729	1710	1691	1672	1653	1634	1615
	-20	2200	2180	2160	2140	2120	2100	2080	2060	2040	2020	2000	1980	1960	1940	1920	1900	1880	1860	1840	1820	1800

Table 1. The hyperboctys HS[5,0,5] in table format

	a	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
	b	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	2200	2220	2240	2260	2280	2300	2320	2340	2360	2380	2400	2420	2440	2460	2480	2500	2520	2540	2560	2580	2600
	19	1976	1995	2014	2033	2052	2071	2090	2109	2128	2147	2166	2185	2204	2223	2242	2261	2280	2299	2318	2337	2356
	18	1764	1782	1800	1818	1836	1854	1872	1890	1908	1926	1944	1962	1980	1998	2016	2034	2052	2070	2088	2106	2124
	17	1564	1581	1598	1615	1632	1649	1666	1683	1700	1717	1734	1751	1768	1785	1802	1819	1836	1853	1870	1887	1904
	16	1376	1392	1408	1424	1440	1456	1472	1488	1504	1520	1536	1552	1568	1584	1600	1616	1632	1648	1664	1680	1696
	15	1200	1215	1230	1245	1260	1275	1290	1305	1320	1335	1350	1365	1380	1395	1410	1425	1440	1455	1470	1485	1500
	14	1036	1050	1064	1078	1092	1106	1120	1134	1148	1162	1176	1190	1204	1218	1232	1246	1260	1274	1288	1302	1316
	13	884	897	910	923	936	949	962	975	988	1001	1014	1027	1040	1053	1066	1079	1092	1105	1118	1131	1144
	12	744	756	768	780	792	804	816	828	840	852	864	876	888	900	912	924	936	948	960	972	984
	11	616	627	638	649	660	671	682	693	704	715	726	737	748	759	770	781	792	803	814	825	836
	10	500	510	520	530	540	550	560	570	580	590	600	610	620	630	640	650	660	670	680	690	700
	9	396	405	414	423	432	441	450	459	468	477	486	495	504	513	522	531	540	549	558	567	576
	8	304	312	320	328	336	344	352	360	368	376	384	392	400	408	416	424	432	440	448	456	464
	7	224	231	238	245	252	259	266	273	280	287	294	301	308	315	322	329	336	343	350	357	364
	6	156	162	168	174	180	186	192	198	204	210	216	222	228	234	240	246	252	258	264	270	276
	5	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180	185	190	195	200
	4	56	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116	120	124	128	132	136
	3	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84
	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44
Y[1]	1	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4
	-2	44	42	40	38	36	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4
	-3	84	81	78	75	72	69	66	63	60	57	54	51	48	45	42	39	36	33	30	27	24
	-4	136	132	128	124	120	116	112	108	104	100	96	92	88	84	80	76	72	68	64	60	56
	-5	200	195	190	185	180	175	170	165	160	155	150	145	140	135	130	125	120	115	110	105	100
	-6	276	270	264	258	252	246	240	234	228	222	216	210	204	198	192	186	180	174	168	162	156
	-7	364	357	350	343	336	329	322	315	308	301	294	287	280	273	266	259	252	245	238	231	224
	-8	464	456	448	440	432	424	416	408	400	392	384	376	368	360	352	344	336	328	320	312	304
	-9	576	567	558	549	540	531	522	513	504	495	486	477	468	459	450	441	432	423	414	405	396
	-10	700	690	680	670	660	650	640	630	620	610	600	590	580	570	560	550	540	530	520	510	500
	-11	836	825	814	803	792	781	770	759	748	737	726	715	704	693	682	671	660	649	638	627	616
	-12	984	972	960	948	936	924	912	900	888	876	864	852	840	828	816	804	792	780	768	756	744
	-13	1144	1131	1118	1105	1092	1079	1066	1053	1040	1027	1014	1001	988	975	962	949	936	923	910	897	884
	-14	1316	1302	1288	1274	1260	1246	1232	1218	1204	1190	1176	1162	1148	1134	1120	1106	1092	1078	1064	1050	1036
	-15	1500	1485	1470	1455	1440	1425	1410	1395	1380	1365	1350	1335	1320	1305	1290	1275	1260	1245	1230	1215	1200
	-16	1696	1680	1664	1648	1632	1616	1600	1584	1568	1552	1536	1520	1504	1488	1472	1456	1440	1424	1408	1392	1376
	-17	1904	1887	1870	1853	1836	1819	1802	1785	1768	1751	1734	1717	1700	1683	1666	1649	1632	1615	1598	1581	1564
	-18	2124	2106	2088	2070	2052	2034	2016	1998	1980	1962	1944	1926	1908	1890	1872	1854	1836	1818	1800	1782	1764
	-19	2356	2337	2318	2299	2280	2261	2242	2223	2204	2185	2166	2147	2128	2109	2090	2071	2052	2033	2014	1995	1976
	-20	2600	2580	2560	2540	2520	2500	2480	2460	2440	2420	2400	2380	2360	2340	2320	2300	2280	2260	2240	2220	2200

Table 1. The hyperboctys HS[6,0,6] in table format

8 Hyperboctys Rotation Characteristics

To maintain the Integers in the hyperbolic structure, there are two possibilities of hyperboctys rotation:

- shift the same number of steps the two rows Y[1] and Y[-1] in opposite directions;
  - In this case, if the coefficients of the quadratic equations of the hyperboctys are Integers, then the resulting hyperboctys will also have Integer coefficients.
  - If the coefficients of the quadratic equations of the hyperboctys are $\frac{Odd}{2}$, then the resulting hyperboctys will have $\frac{Odd}{2}$ coefficients.
- shift one step only one of the two rows Y[1] and Y[-1] in any direction.
  - In this case, if the coefficients of the quadratic equations of the hyperboctys are Integers, then the resulting hyperboctys will have $\frac{Odd}{2}$ coefficients.
  - If the coefficients of the quadratic equations of the hyperboctys are $\frac{Odd}{2}$, then the resulting hyperboctys will have Integer coefficients.

Each counterclockwise (CCW) rotation increases the coefficient "a". Each CW rotation decreases the coefficient "a".

Each hyperboctys rotation generates a new tessellation.

In the special case of the FMT=HS[0,0,0] that has the Y[0] row always with Zeros, as we do the complete rotations steps, we obtain all the quadratic CG tessellations for each coefficient "a".

The rotation of the hyperboctys does not alter the hyperbolic properties of the lattice-grid. We just deform the hyperbolic curves. This procedure is equivalent to the deformation of one triangle into another triangle. All triangle properties remain unchanged.

This means that the properties of the prime numbers appearing in the FMT remain unchanged whatever the rotation.

Whatever the hyperboctys with Integer coefficients equations the distribution of prime numbers will always obey the following rules:

- Zeroes with no Prime number next to it.
- Zeroes with one Prime number next to it.
- Zeroes with two Prime numbers, one on each side next to it.

Whatever the hyperboctys with $\frac{Odd}{2}$ coefficients equations the distribution of prime numbers will always obey the following rules:

- Zeroes with no Prime number next to it.
- Zeroes with one Prime number next to it.
- Zeroes with two Prime numbers, one on each side next to it.
- Zeroes with three Prime numbers: one on one side and the other two on the other side next to it.
- Zeroes with four Prime numbers, two on each side next to it.

9 The polynomial sequences of repeated composites in the FMT

When we look at the hyperbolic lattice-grid of the FMT, it draws attention to the diagonal 45° (1: 1 slope) of distinct Square numbers and other diagonals of repeated Square numbers at (1: 4), (1: 9), (1: 16), …, $(1{:}\, n^2)$.

As we are in a hyperbolic lattice-grid, each one of these lines represents a quadratic.

0	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
0	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
0	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
0	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
0	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 1. The distinct Square numbers in 1: 1 line and the repeated Squares distributed in several polynomial lines

By continuing to focus only on Square numbers, we immediately realize that The Square numbers sequences on diagonals $(1{:}\, n^2)$ cannot cover all repeated Square numbers.

See that repeated Squares form several other polynomial sequences of higher degree than quadratics.

Also, it is possible to see that this formation has no end.

No finite number of polynomials will be enough to cover all the repeated Square numbers.

The same occurs for all other repeated composites.

In this introductory study of hyperboctys, we will only show some quadratics and quartics formations of the repeated composites, leaving for the next study a more complete solution.

9.1 The quadratics sequences of repeated composites in the FMT

9.1.1Repeated composites generated by the A256958 The Integer numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A256958	A256958	A000290	A256958	A256958	A002378	A256958	A256958	A005563	A256958	A256958	A028552	A256958	A256958	A028347	A256958	A256958	A028557	A256958	A256958	A028560
y_ip		0	0	0	1	0	0,5	2	0	1	3	0	1,5	4	0	2	5	0	2,5	6	0	3
offset = f		0	0	0	1	0	0	2	0	1	3	0	1	4	0	2	5	0	2	6	0	3
a = 1				1			1			1			1			1			1			1
b = -d		1	1	0	1	1	-1	1	1	-2	1	1	-3	1	1	-4	1	1	-5	1	1	-6
c = 0		0	0	0	-1	0	0	-2	0	0	-3	0	0	-4	0	0	-5	0	0	-6	0	0
	10	10	10	100	9	10	90	8	10	80	7	10	70	6	10	60	5	10	50	4	10	40
	9	9	9	81	8	9	72	7	9	63	6	9	54	5	9	45	4	9	36	3	9	27
	8	8	8	64	7	8	56	6	8	48	5	8	40	4	8	32	3	8	24	2	8	16
	7	7	7	49	6	7	42	5	7	35	4	7	28	3	7	21	2	7	14	1	7	7
	6	6	6	36	5	6	30	4	6	24	3	6	18	2	6	12	1	6	6	0	6	0
	5	5	5	25	4	5	20	3	5	15	2	5	10	1	5	5	0	5	0	-1	5	-5
	4	4	4	16	3	4	12	2	4	8	1	4	4	0	4	0	-1	4	-4	-2	4	-8
	3	3	3	9	2	3	6	1	3	3	0	3	0	-1	3	-3	-2	3	-6	-3	3	-9
	2	2	2	4	1	2	2	0	2	0	-1	2	-2	-2	2	-4	-3	2	-6	-4	2	-8
Y[1]	1	1	1	1	0	1	0	-1	1	-1	-2	1	-2	-3	1	-3	-4	1	-4	-5	1	-5
Y[0]	0	0	0	0	-1	0	0	-2	0	0	-3	0	0	-4	0	0	-5	0	0	-6	0	0
Y[-1]	-1	-1	-1	1	-2	-1	2	-3	-1	3	-4	-1	4	-5	-1	5	-6	-1	6	-7	-1	7
	-2	-2	-2	4	-3	-2	6	-4	-2	8	-5	-2	10	-6	-2	12	-7	-2	14	-8	-2	16
	-3	-3	-3	9	-4	-3	12	-5	-3	15	-6	-3	18	-7	-3	21	-8	-3	24	-9	-3	27
	-4	-4	-4	16	-5	-4	20	-6	-4	24	-7	-4	28	-8	-4	32	-9	-4	36	-10	-4	40
	-5	-5	-5	25	-6	-5	30	-7	-5	35	-8	-5	40	-9	-5	45	-10	-5	50	-11	-5	55
	-6	-6	-6	36	-7	-6	42	-8	-6	48	-9	-6	54	-10	-6	60	-11	-6	66	-12	-6	72
	-7	-7	-7	49	-8	-7	56	-9	-7	63	-10	-7	70	-11	-7	77	-12	-7	84	-13	-7	91
	-8	-8	-8	64	-9	-8	72	-10	-8	80	-11	-8	88	-12	-8	96	-13	-8	104	-14	-8	112
	-9	-9	-9	81	-10	-9	90	-11	-9	99	-12	-9	108	-13	-9	117	-14	-9	126	-15	-9	135
	-10	-10	-10	100	-11	-10	110	-12	-10	120	-13	-10	130	-14	-10	140	-15	-10	150	-16	-10	160

Table 1. Linear functions y and $(y-d)$ producing the quadratics y^2-dy. Both linear functions are determined by the same two elements [0,1].

Each factor y and $(y-d)$ produce the same sequence A256958 The Integer numbers.

The two multiplications $y(y-d)$ and $(y-d)y$ generate the quadratics $Y[y]=y^2-dy$ in the FMT.

All these quadratics are a line with the equation $x=y-d, -\infty \leq d \leq \infty$ in the XY plane with 1:1 slope line.

For $d=0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. There is no difference between the factors $(y-0)$ and $(y+0)$.

All these quadratics for $-\infty \leq d \leq \infty$ cover all elements of FMT. Therefore, they are the equations of the formation of the FMT. So, we cannot consider any of them as equations of the Quadratic sequences of repeated composites.

9.1.2. Repeated composites generated by the A005843 The Even numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A005843	A005843	A016742	A005843	A005843	A033996	A005843	A005843	A134582	A005843	A005843	A332519	A005843	A005843	A	A005843	A005843	A277108	A005843	A005843	A
y_ip		0	0	0	1	0	0,5	2	0	1	3	0	1,5	4	0	2	5	0	2,5	6	0	3
offset = f		0	0	0	1	0	0	2	0	1	3	0	1	4	0	2	5	0	2	6	0	3
a = 1				4			4			4			4			4			4			4
b = -4d		2	2	0	2	2	-4	2	2	-8	2	2	-12	2	2	-16	2	2	-20	2	2	-24
c = 0		0	0	0	-2	0	0	-4	0	0	-6	0	0	-8	0	0	-10	0	0	-12	0	0
	10	20	20	400	18	20	360	16	20	320	14	20	280	12	20	240	10	20	200	8	20	160
	9	18	18	324	16	18	288	14	18	252	12	18	216	10	18	180	8	18	144	6	18	108
	8	16	16	256	14	16	224	12	16	192	10	16	160	8	16	128	6	16	96	4	16	64
	7	14	14	196	12	14	168	10	14	140	8	14	112	6	14	84	4	14	56	2	14	28
	6	12	12	144	10	12	120	8	12	96	6	12	72	4	12	48	2	12	24	0	12	0
	5	10	10	100	8	10	80	6	10	60	4	10	40	2	10	20	0	10	0	-2	10	-20
	4	8	8	64	6	8	48	4	8	32	2	8	16	0	8	0	-2	8	-16	-4	8	-32
	3	6	6	36	4	6	24	2	6	12	0	6	0	-2	6	-12	-4	6	-24	-6	6	-36
	2	4	4	16	2	4	8	0	4	0	-2	4	-8	-4	4	-16	-6	4	-24	-8	4	-32
Y[1]	1	2	2	4	0	2	0	-2	2	-4	-4	2	-8	-6	2	-12	-8	2	-16	-10	2	-20
Y[0]	0	0	0	0	-2	0	0	-4	0	0	-6	0	0	-8	0	0	-10	0	0	-12	0	0
Y[-1]	-1	-2	-2	4	-4	-2	8	-6	-2	12	-8	-2	16	-10	-2	20	-12	-2	24	-14	-2	28
	-2	-4	-4	16	-6	-4	24	-8	-4	32	-10	-4	40	-12	-4	48	-14	-4	56	-16	-4	64
	-3	-6	-6	36	-8	-6	48	-10	-6	60	-12	-6	72	-14	-6	84	-16	-6	96	-18	-6	108
	-4	-8	-8	64	-10	-8	80	-12	-8	96	-14	-8	112	-16	-8	128	-18	-8	144	-20	-8	160
	-5	-10	-10	100	-12	-10	120	-14	-10	140	-16	-10	160	-18	-10	180	-20	-10	200	-22	-10	220
	-6	-12	-12	144	-14	-12	168	-16	-12	192	-18	-12	216	-20	-12	240	-22	-12	264	-24	-12	288
	-7	-14	-14	196	-16	-14	224	-18	-14	252	-20	-14	280	-22	-14	308	-24	-14	336	-26	-14	364
	-8	-16	-16	256	-18	-16	288	-20	-16	320	-22	-16	352	-24	-16	384	-26	-16	416	-28	-16	448
	-9	-18	-18	324	-20	-18	360	-22	-18	396	-24	-18	432	-26	-18	468	-28	-18	504	-30	-18	540
	-10	-20	-20	400	-22	-20	440	-24	-20	480	-26	-20	520	-28	-20	560	-30	-20	600	-32	-20	640

Table 1. Linear functions $2y$ and $(2y - 2d)$ producing the quadratics $4y^2 - 4dy$. Both linear functions are determined by the same two elements [0,2].

Each factor $2y$ and $(2y - 2d)$ produce the same sequence A005843 The Even numbers.

The two multiplications $2y(2y - 2d)$ and $(2y - 2d)2y$ generate the quadratics $Y[y] = 4y^2 - 4dy$ in the FMT.

All these quadratics are a line with the equation $x = 4y - 4d, -\infty \leq d \leq \infty$ in the XY plane with 1:4 slope lines.

9.1.3Repeated composites generated by the A008585 The Multiples of 3

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A008585	A008585	A016766	A008585	A008585	A163758	A008585	A008585	A147651	A008585	A008585	A	A008585	A008585	A	A008585	A008585	A	A008585	A008585	A
y_ip		0	0	0	1	0	0,5	2	0	1	3	0	1,5	4	0	2	5	0	2,5	6	0	3
offset = f		0	0	0	1	0	0	2	0	1	3	0	1	4	0	2	5	0	2	6	0	3
a = 9				9			9			9			9			9			9			9
b = -9d		3	3	0	3	3	-9	3	3	-18	3	3	-27	3	3	-36	3	3	-45	3	3	-54
c = 0		0	0	0	-3	0	0	-6	0	0	-9	0	0	-12	0	0	-15	0	0	-18	0	0
	10	30	30	900	27	30	810	24	30	720	21	30	630	18	30	540	15	30	450	12	30	360
	9	27	27	729	24	27	648	21	27	567	18	27	486	15	27	405	12	27	324	9	27	243
	8	24	24	576	21	24	504	18	24	432	15	24	360	12	24	288	9	24	216	6	24	144
	7	21	21	441	18	21	378	15	21	315	12	21	252	9	21	189	6	21	126	3	21	63
	6	18	18	324	15	18	270	12	18	216	9	18	162	6	18	108	3	18	54	0	18	0
	5	15	15	225	12	15	180	9	15	135	6	15	90	3	15	45	0	15	0	-3	15	-45
	4	12	12	144	9	12	108	6	12	72	3	12	36	0	12	0	-3	12	-36	-6	12	-72
	3	9	9	81	6	9	54	3	9	27	0	9	0	-3	9	-27	-6	9	-54	-9	9	-81
	2	6	6	36	3	6	18	0	6	0	-3	6	-18	-6	6	-36	-9	6	-54	-12	6	-72
Y[1]	1	3	3	9	0	3	0	-3	3	-9	-6	3	-18	-9	3	-27	-12	3	-36	-15	3	-45
Y[0]	0	0	0	0	-3	0	0	-6	0	0	-9	0	0	-12	0	0	-15	0	0	-18	0	0
Y[-1]	-1	-3	-3	9	-6	-3	18	-9	-3	27	-12	-3	36	-15	-3	45	-18	-3	54	-21	-3	63
	-2	-6	-6	36	-9	-6	54	-12	-6	72	-15	-6	90	-18	-6	108	-21	-6	126	-24	-6	144
	-3	-9	-9	81	-12	-9	108	-15	-9	135	-18	-9	162	-21	-9	189	-24	-9	216	-27	-9	243
	-4	-12	-12	144	-15	-12	180	-18	-12	216	-21	-12	252	-24	-12	288	-27	-12	324	-30	-12	360
	-5	-15	-15	225	-18	-15	270	-21	-15	315	-24	-15	360	-27	-15	405	-30	-15	450	-33	-15	495
	-6	-18	-18	324	-21	-18	378	-24	-18	432	-27	-18	486	-30	-18	540	-33	-18	594	-36	-18	648
	-7	-21	-21	441	-24	-21	504	-27	-21	567	-30	-21	630	-33	-21	693	-36	-21	756	-39	-21	819
	-8	-24	-24	576	-27	-24	648	-30	-24	720	-33	-24	792	-36	-24	864	-39	-24	936	-42	-24	1008
	-9	-27	-27	729	-30	-27	810	-33	-27	891	-36	-27	972	-39	-27	1053	-42	-27	1134	-45	-27	1215
	-10	-30	-30	900	-33	-30	990	-36	-30	1080	-39	-30	1170	-42	-30	1260	-45	-30	1350	-48	-30	1440

Table 1. Linear functions $3y$ and $(3y - 3d)$ producing the quadratics $9y^2 - 9dy$. Both linear functions are determined by the same two elements [0,3].

Each factor $3y$ and $(3y - 3d)$ produce the same sequence A008585 The Multiples of 3.

The two multiplications $3y(3y - 3d)$ and $(3y - 3d)3y$ generate the quadratics $Y[y] = 9y^2 - 9dy$ in the FMT.

All these quadratics are a line with the equation $x = 9y - 9d, -\infty \leq d \leq \infty$ in the XY plane with 1:9 slope lines.

9.1.4Repeated composites generated by the A008586 The Multiples of 4

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A008586	A008586	A016802	A008586	A008586	A	A008586	A008586	A114444	A008586	A008586	A	A008586	A008586	A	A008586	A008586	A	A008586	A008586	A
y_ip		0	0	0	1	0	0,5	2	0	1	3	0	1,5	4	0	2	5	0	2,5	6	0	3
offset = f		0	0	0	1	0	0	2	0	1	3	0	1	4	0	2	5	0	2	6	0	3
a = 16				16			16			16			16			16			16			16
b = -16d		4	4	0	4	4	-16	4	4	-32	4	4	-48	4	4	-64	4	4	-80	4	4	-96
c = 0		0	0	0	-4	0	0	-8	0	0	-12	0	0	-16	0	0	-20	0	0	-24	0	0
	10	40	40	1600	36	40	1440	32	40	1280	28	40	1120	24	40	960	20	40	800	16	40	640
	9	36	36	1296	32	36	1152	28	36	1008	24	36	864	20	36	720	16	36	576	12	36	432
	8	32	32	1024	28	32	896	24	32	768	20	32	640	16	32	512	12	32	384	8	32	256
	7	28	28	784	24	28	672	20	28	560	16	28	448	12	28	336	8	28	224	4	28	112
	6	24	24	576	20	24	480	16	24	384	12	24	288	8	24	192	4	24	96	0	24	0
	5	20	20	400	16	20	320	12	20	240	8	20	160	4	20	80	0	20	0	-4	20	-80
	4	16	16	256	12	16	192	8	16	128	4	16	64	0	16	0	-4	16	-64	-8	16	-128
	3	12	12	144	8	12	96	4	12	48	0	12	0	-4	12	-48	-8	12	-96	-12	12	-144
	2	8	8	64	4	8	32	0	8	0	-4	8	-32	-8	8	-64	-12	8	-96	-16	8	-128
Y[1]	1	4	4	16	0	4	0	-4	4	-16	-8	4	-32	-12	4	-48	-16	4	-64	-20	4	-80
Y[0]	0	0	0	0	-4	0	0	-8	0	0	-12	0	0	-16	0	0	-20	0	0	-24	0	0
Y[-1]	-1	-4	-4	16	-8	-4	32	-12	-4	48	-16	-4	64	-20	-4	80	-24	-4	96	-28	-4	112
	-2	-8	-8	64	-12	-8	96	-16	-8	128	-20	-8	160	-24	-8	192	-28	-8	224	-32	-8	256
	-3	-12	-12	144	-16	-12	192	-20	-12	240	-24	-12	288	-28	-12	336	-32	-12	384	-36	-12	432
	-4	-16	-16	256	-20	-16	320	-24	-16	384	-28	-16	448	-32	-16	512	-36	-16	576	-40	-16	640
	-5	-20	-20	400	-24	-20	480	-28	-20	560	-32	-20	640	-36	-20	720	-40	-20	800	-44	-20	880
	-6	-24	-24	576	-28	-24	672	-32	-24	768	-36	-24	864	-40	-24	960	-44	-24	1056	-48	-24	1152
	-7	-28	-28	784	-32	-28	896	-36	-28	1008	-40	-28	1120	-44	-28	1232	-48	-28	1344	-52	-28	1456
	-8	-32	-32	1024	-36	-32	1152	-40	-32	1280	-44	-32	1408	-48	-32	1536	-52	-32	1664	-56	-32	1792
	-9	-36	-36	1296	-40	-36	1440	-44	-36	1584	-48	-36	1728	-52	-36	1872	-56	-36	2016	-60	-36	2160
	-10	-40	-40	1600	-44	-40	1760	-48	-40	1920	-52	-40	2080	-56	-40	2240	-60	-40	2400	-64	-40	2560

Table 1. Linear functions $4y$ and $(4y - 4d)$ producing the quadratics $16y^2 - 16dy$. Both linear functions are determined by the same two elements [0,4].

Each factor $4y$ and $(4y - 4d)$ produce the same sequence A008586 The Multiples of 4.

The two multiplications $4y(4y - 4d)$ and $(4y - 4d)4y$ generate the quadratics $Y[y] = 16y^2 - 16dy$ in the FMT.

All these quadratics are a line with the equation $x = 16y - 16d, -\infty \leq d \leq \infty$ in the XY plane with 1:16 slope lines.

9.1.5Summary of repeated composites generated by linear $Y[y] = sy$

$$Y_{2y}[y] = 4y^2 - 4dy$$
$$Y_{3y}[y] = 9y^2 - 9dy$$
$$Y_{4y}[y] = 16y^2 - 16dy$$
$$\dots$$
$$Y_{sy}[y] = s^2y^2 - s^2dy$$

9.2 The quartic sequences of repeated composites in the FMT

Because of the hyperbolic lattice-grid of the FMT:

- all vertical columns are linear;
- all horizontal rows are linear;
- all diagonal lines are quadratic.

Since the first diagonal is the symmetry diagonal of A000290 The Square numbers, then the first repeated composites to appear repeatedly in FMT will be the composite elements of the Square number sequence.

This means that the entire region between the 45˚ diagonal of symmetry A000290 The Square numbers and the line formed by the first Square numbers repeated sequence is an area free of repeated composites that only have distinct composites.

We will call this region of FMT free of repeated composites as "*distinct-area*". We color the distinct-area with green in the figure below.

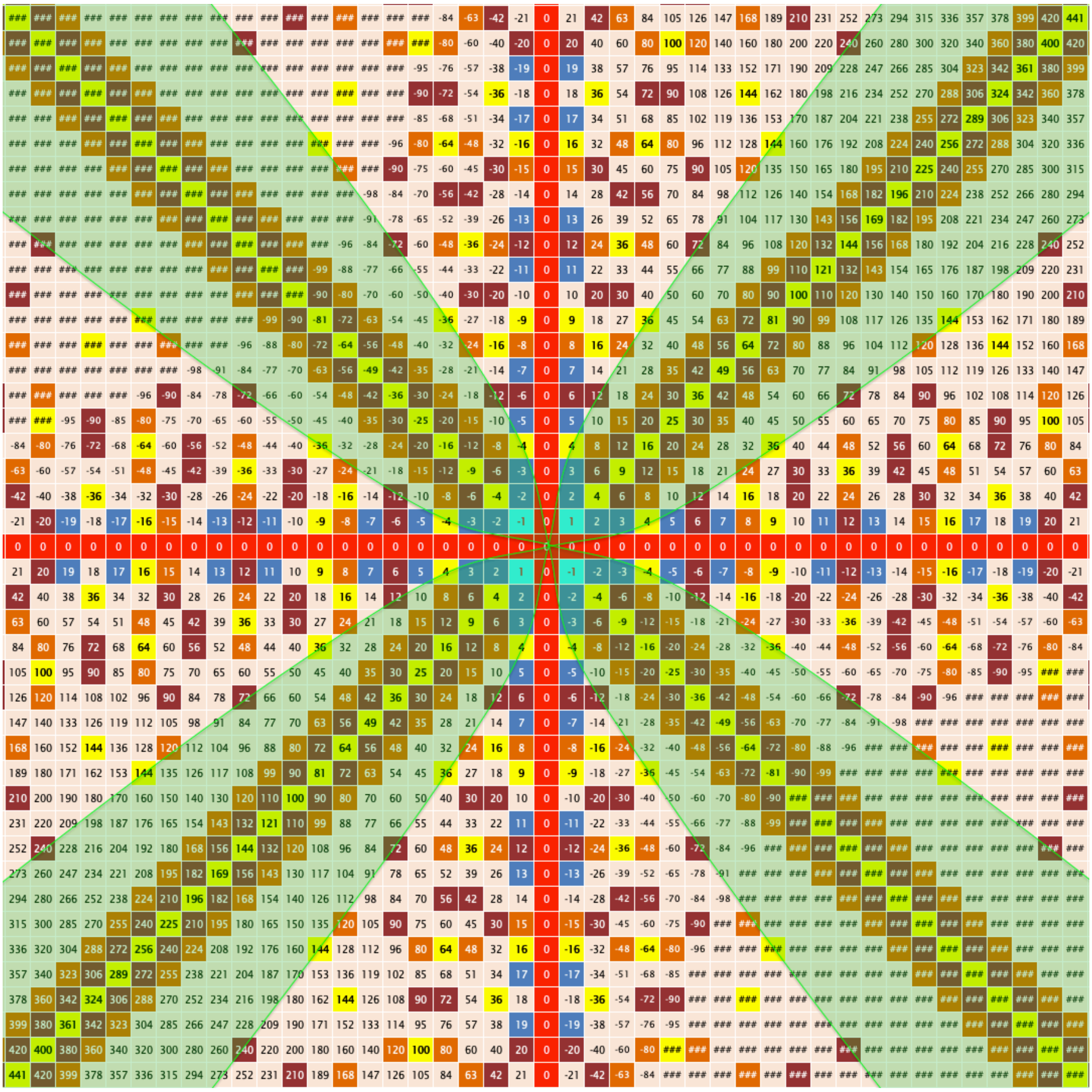

Figure 1. The green region is the distinct-area in FMT.

9.2.1 Property of Oblong and Square numbers

The product of two consecutive Square numbers results in a Square number.

$$n^2(n-1)^2 = \big(n(n-1)\big)^2 = (Oblong)^2 \equiv Square$$

The product of two consecutive Oblong numbers results in an Oblong number.

$$\big(n(n-1)\big) * \big((n+1)n\big) = n^2(n^2-1) = Square(Square-1) \equiv Oblong$$

They start all repeated Composites in the multiplication table from the hyperbole lines.

9.2.2 The first quartic sequence of repeated composites

The two edges of the distinct-area are the symmetrical sequence

$$\{\dots, 5184, 3136, 1764, 900, 400, 144, 36, 4, 0, 0, 4, 36, 144, 400, 900, 1764, 3136, 5184, \dots\}$$

This is the sequence A035287 The Oblong numbers squared.

The equation is

$$\big(n(n-1)\big)^2 = n^2(n-1)^2 = n^2(n^2-2n+1) = n^4 - 2n^3 + n^2$$

Because each element is an Oblong number squared, this means that these repeated composites result from the product of two sequences A000290 The Square numbers one offset by one step concerning the other. So, we can write for one edge:

$$A035287[n] = A000290[n] * A000290\,[n-1]$$

And for the other edge:

$$A035287[n] = A000290[n-1] * A000290\,[n]$$

	A000290	A000290	A035287	A000290	A000290	A035287
	y^2	(y-1)^2	(y(y-1))^2	(y-1)^2	y^2	((y-1)y)^2
10	100	121	12100	121	100	12100
9	81	100	8100	100	81	8100
8	64	81	5184	81	64	5184
7	49	64	3136	64	49	3136
6	36	49	1764	49	36	1764
5	25	36	900	36	25	900
4	16	25	400	25	16	400
3	9	16	144	16	9	144
2	4	9	36	9	4	36
1	1	4	4	4	1	4
0	0	1	0	1	0	0
-1	1	0	0	0	1	0
-2	4	1	4	1	4	4
-3	9	4	36	4	9	36
-4	16	9	144	9	16	144
-5	25	16	400	16	25	400
-6	36	25	900	25	36	900
-7	49	36	1764	36	49	1764
-8	64	49	3136	49	64	3136
-9	81	64	5184	64	81	5184
-10	100	81	8100	81	100	8100

Table 1. The quartic sequences that form the two edges of the distinct-area result from the product of two sequences A000290 Square numbers with a difference 1 between their offset.

The product of two Square number sequences forms the 4th-degree sequence which delimits the distinct area.

So, in hyperboctys, it is a 3rd-degree curve polynomial which delimits the distinct-area in XY-plane.

9.2.3The second quartic sequence of repeated composites

The second quartic sequence of the repeated composites is Oblong number of the form

$$Square[n] * (Square[n] - 1)$$

The equation is

$$n^2(n^2 - 1) = n^4 - n^2$$

This is the sequence A047928 Oblong of the form $Square[n] * (Square[n] - 1)$:
{…, 50400, 38220, 28392, 20592, 14520, 9900, 6480, 4032, 2352, 1260, 600, 240, 72, 12, 0, 0, 0, 12, 72, 240, 600, 1260, 2352, 4032, 6480, 9900, 14520, 20592, 28392, 38220, 50400…}

9.2.3.1 The edge between repeated composites and distinct composites: A307182

Except for the sequences A000290 Square numbers and A002378 Oblong numbers, all other sequences of the form $n(n \pm k)$ have negative elements.

Thus, we will define as the dividing line between the region that has repeated composites of the region free of repeated composites as the sequence A307182.

The sequence A307182 results from the interlacing of A035287 $Oblong\ squared = Square[n] * Square[n-1] = Oblong[n] * Oblong[n] = Square\ number$ and A047928 $Square[n] * (Square[n] - 1) = Oblong\ number$.

$$A307182[y] = A047928[y = Even] + A035287[y = Odd]$$

$A047928[y = Even]$ is based on the sequence $[12,0,0,0,12] = n^2(n^2-1) = n^4 - n^2$

$$y = Even = 2n$$

$$n = \frac{y}{2}$$

$$A047928[y = Even] = \left(\frac{y}{2}\right)^4 - \left(\frac{y}{2}\right)^2 = \frac{y^4}{16} - \frac{y^2}{4} = \frac{y^4 - 4y^2}{16}$$

$A035287[y = Odd]$ is based on the sequence $[36,4,0,0,4] = n^2(n-1)^2 = n^4 - 2n^3 + n^2$

$$y = Odd = 2n - 1$$

$$n = \frac{y+1}{2}$$

$$A035287[y = Odd] = \left(\frac{y+1}{2}\right)^4 - 2\left(\frac{y+1}{2}\right)^3 + \left(\frac{y+1}{2}\right)^2$$

$$= \frac{y^4 + 4y^3 + 6y^2 + 4y + 1}{16} - 2\left(\frac{y^3 + 3y^2 + 3y + 1}{8}\right) + \frac{y^2 + 2y + 1}{4} =$$

$$= \frac{y^4 + 4y^3 + 6y^2 + 4y + 1}{16} - \frac{y^3 + 3y^2 + 3y + 1}{4} + \frac{y^2 + 2y + 1}{4} =$$

$$= \frac{y^4 + 4y^3 + 6y^2 + 4y + 1 - 4y^3 - 12y^2 - 12y - 4 + 4y^2 + 8y + 4}{16} =$$

$$= \frac{y^4 + 4y^3 + 6y^2 + 4y + 1 - 4y^3 - 12y^2 - 12y - 4 + 4y^2 + 8y + 4}{16} =$$

$$= \frac{y^4 - 2y^2 + 1}{16}$$

$$A047928[y = Even] = \frac{y^4 - 4y^2}{16}$$

$$A035287[y = Odd] = \frac{y^4 - 2y^2 + 1}{16}$$

$$A307182[y] = \frac{y^4 - (3y^2 + y^2(-1)^y) + (0.5 - 0.5(-1)^y)}{16}$$

$$A307182[y] = \frac{y^4 - 3y^2 + 0.5 - (y^2 + 0.5)(-1)^y}{16}$$

$$A307182[y] = \frac{2y^4 - 6y^2 + 1 - (2y^2 + 1)(-1)^y}{32}$$

The sequence $A307182[y] = A047928[y = Even] + A035287[y = Odd]$ is: {…, 9900, 8100, 6480, 5184, 4032, 3136, 2352, 1764, 1260, 900, 600, 400, 240, 144, 72, 36, 12, 4, 0, 0, 0, 0, 0, 4, 12, 36, 72, 144, 240, 400, 600, 900, 1260, 1764, 2352, 3136, 4032, 5184, 6480, 8100, 9900, …}.

Summary:

OEIS	A047928[y]	A035287[y]	A047928[y=Even]	A035287[y=Odd]	A307182[y]= A047928[y=Even]+ A035287[y=Odd]
a_4	1	1	0,0625	0,0625	
a_3	0	-2	0	0	
a_2	-1	1	-0,25	-0,125	
a_1	0	0	0	0	
a_0	0	0	0	0,0625	
15	50400	44100	3107,8125	3136	3136
14	38220	33124	2352	2376,5625	2352
13	28392	24336	1742,8125	1764	1764
12	20592	17424	1260	1278,0625	1260
11	14520	12100	884,8125	900	900
10	9900	8100	600	612,5625	600
9	6480	5184	389,8125	400	400
8	4032	3136	240	248,0625	240
7	2352	1764	137,8125	144	144
6	1260	900	72	76,5625	72
5	600	400	32,8125	36	36
4	240	144	12	14,0625	12
3	72	36	2,8125	4	4
2	12	4	0	0,5625	0
1	0	0	-0,1875	0	0
0	0	0	0	0,0625	0
-1	0	4	-0,1875	0	0
-2	12	36	0	0,5625	0
-3	72	144	2,8125	4	4
-4	240	400	12	14,0625	12
-5	600	900	32,8125	36	36
-6	1260	1764	72	76,5625	72
-7	2352	3136	137,8125	144	144
-8	4032	5184	240	248,0625	240
-9	6480	8100	389,8125	400	400
-10	9900	12100	600	612,5625	600
-11	14520	17424	884,8125	900	900
-12	20592	24336	1260	1278,0625	1260
-13	28392	33124	1742,8125	1764	1764
-14	38220	44100	2352	2376,5625	2352
-15	50400	57600	3107,8125	3136	3136

9.2.4 The third quartic sequence of repeated composites

The third quartic sequence of the repeated composites is

$$Square\ minus\ One[n-1] * Square\ minus\ One[n]$$

And

$$Square\ minus\ One[n] * Square\ minus\ One[n-1]$$

The equation is

$$(n^2-1)((n-1)^2-1) = (n-1)(n+1)(n^2-2n+1-1) = (n-1)(n+1)(n^2-2n)$$
$$= (n-2)(n-1)n(n+1) = Oblong[n-1] * Oblong[n+1]$$

This is the sequence A052762:

$\{0, 24, 120, 360, 840, 1680, 3024, 5040, 7920, 11880, 17160, 24024, 32760, \dots\}$

9.2.5 And so on…

This phenomenon of the appearance of a repeated composite is an endless recursive algorithm.

The sequences of the repeated composites sequences will also generate new repeated composites that will form sequences of a higher degree, and so on.

So, see in the next item the tables of the quadratic repeated composites.

9.3 Repeated composites generated by a Square sequence minus a Square number

9.3.1Repeated composites generated by the A000290 Square numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A000290	A000290	A000583	A000290	A000290	A035287	A000290	A000290	A099761	A000290	A000290	A	A000290	A000290	A	A000290	A000290	A	A000290	A000290	A
y_ip		0	0	0	0	1	0,5	0	2	1	0	3	1,5	0	4	2	0	5	2,5	0	6	3
offset = f		0	0	0	0	1	0	0	2	1	0	3	1	0	4	2	0	5	2	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				0			-2			-4			-6			-8			-10			-12
a = d^2		1	1	0	1	1	1	1	1	4	1	1	9	1	1	16	1	1	25	1	1	36
b = 0		0	0	0	0	-2	0	0	-4	0	0	-6	0	0	-8	0	0	-10	0	0	-12	0
c = 0		0	0	0	0	1	0	0	4	0	0	9	0	0	16	0	0	25	0	0	36	0
	10	100	100	10000	100	81	8100	100	64	6400	100	49	4900	100	36	3600	100	25	2500	100	16	1600
	9	81	81	6561	81	64	5184	81	49	3969	81	36	2916	81	25	2025	81	16	1296	81	9	729
	8	64	64	4096	64	49	3136	64	36	2304	64	25	1600	64	16	1024	64	9	576	64	4	256
	7	49	49	2401	49	36	1764	49	25	1225	49	16	784	49	9	441	49	4	196	49	1	49
	6	36	36	1296	36	25	900	36	16	576	36	9	324	36	4	144	36	1	36	36	0	0
	5	25	25	625	25	16	400	25	9	225	25	4	100	25	1	25	25	0	0	25	1	25
	4	16	16	256	16	9	144	16	4	64	16	1	16	16	0	0	16	1	16	16	4	64
	3	9	9	81	9	4	36	9	1	9	9	0	0	9	1	9	9	4	36	9	9	81
Y[2]	2	4	4	16	4	1	4	4	0	0	4	1	4	4	4	16	4	9	36	4	16	64
Y[1]	1	1	1	1	1	0	0	1	1	1	1	4	4	1	9	9	1	16	16	1	25	25
Y[0]	0	0	0	0	0	1	0	0	4	0	0	9	0	0	16	0	0	25	0	0	36	0
Y[-1]	-1	1	1	1	1	4	4	1	9	9	1	16	16	1	25	25	1	36	36	1	49	49
Y[-2]	-2	4	4	16	4	9	36	4	16	64	4	25	100	4	36	144	4	49	196	4	64	256
	-3	9	9	81	9	16	144	9	25	225	9	36	324	9	49	441	9	64	576	9	81	729
	-4	16	16	256	16	25	400	16	36	576	16	49	784	16	64	1024	16	81	1296	16	100	1600
	-5	25	25	625	25	36	900	25	49	1225	25	64	1600	25	81	2025	25	100	2500	25	121	3025
	-6	36	36	1296	36	49	1764	36	64	2304	36	81	2916	36	100	3600	36	121	4356	36	144	5184
	-7	49	49	2401	49	64	3136	49	81	3969	49	100	4900	49	121	5929	49	144	7056	49	169	8281
	-8	64	64	4096	64	81	5184	64	100	6400	64	121	7744	64	144	9216	64	169	10816	64	196	12544
	-9	81	81	6561	81	100	8100	81	121	9801	81	144	11664	81	169	13689	81	196	15876	81	225	18225
	-10	100	100	10000	100	121	12100	100	144	14400	100	169	16900	100	196	19600	100	225	22500	100	256	25600

Table 1. Quadratic sequences y^2 and $(y-d)^2$ producing the quartics $y^4 - 2dy^3 + d^2y^2$.

Each factor y^2 and $(y-d)^2$ produces the same quadratic sequence A000290 Square numbers. Both quadratics are determined by the same three consecutive elements [1,0,1].

The two multiplications $y^2(y-d)^2$ and $(y-d)^2y^2$ generate the quartics $Y[y] = y^4 - 2dy^3 + d^2y^2$ in the FMT.

All these quartics are a 3rd-degree curve with the equation $x = y^3 - 2dy^2 + d^2y, -\infty \leq d \leq \infty$ in the XY plane.

For $d = 0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. There is no difference between the factors $(y-0)^2$ and $(y+0)^2$. The multiplier is equal to the multiplicand.

9.3.2Repeated composites generated by the A005563 (Square minus One) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A005563	A005563	A099761	A005563	A005563	A052762	A005563	A005563	A	A005563	A005563	A	A005563	A005563	A190577	A005563	A005563	A	A005563	A005563	A
y_ip		0	0	0	0	1	0,5	0	2	1	0	3	1,5	0	4	2	0	5	2,5	0	6	3
offset = f		0	0	0	0	1	0	0	2	1	0	3	1	0	4	2	0	5	2	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				0			-2			-4			-6			-8			-10			-12
a = d^2-2		1	1	-2	1	1	-1	1	1	2	1	1	7	1	1	14	1	1	23	1	1	34
b=2d		0	0	0	0	-2	2	0	-4	4	0	-6	6	0	-8	8	0	-10	10	0	-12	12
c=-d^2+1		-1	-1	1	-1	0	0	-1	3	-3	-1	8	-8	-1	15	-15	-1	24	-24	-1	35	-35
	10	99	99	9801	99	80	7920	99	63	6237	99	48	4752	99	35	3465	99	24	2376	99	15	1485
	9	80	80	6400	80	63	5040	80	48	3840	80	35	2800	80	24	1920	80	15	1200	80	8	640
	8	63	63	3969	63	48	3024	63	35	2205	63	24	1512	63	15	945	63	8	504	63	3	189
	7	48	48	2304	48	35	1680	48	24	1152	48	15	720	48	8	384	48	3	144	48	0	0
	6	35	35	1225	35	24	840	35	15	525	35	8	280	35	3	105	35	0	0	35	-1	-35
	5	24	24	576	24	15	360	24	8	192	24	3	72	24	0	0	24	-1	-24	24	0	0
	4	15	15	225	15	8	120	15	3	45	15	0	0	15	-1	-15	15	0	0	15	3	45
	3	8	8	64	8	3	24	8	0	0	8	-1	-8	8	0	0	8	3	24	8	8	64
Y[2]	2	3	3	9	3	0	0	3	-1	-3	3	0	0	3	3	9	3	8	24	3	15	45
Y[1]	1	0	0	0	0	-1	0	0	0	0	0	3	0	0	8	0	0	15	0	0	24	0
Y[0]	0	-1	-1	1	-1	0	0	-1	3	-3	-1	8	-8	-1	15	-15	-1	24	-24	-1	35	-35
Y[-1]	-1	0	0	0	0	3	0	0	8	0	0	15	0	0	24	0	0	35	0	0	48	0
Y[-2]	-2	3	3	9	3	8	24	3	15	45	3	24	72	3	35	105	3	48	144	3	63	189
	-3	8	8	64	8	15	120	8	24	192	8	35	280	8	48	384	8	63	504	8	80	640
	-4	15	15	225	15	24	360	15	35	525	15	48	720	15	63	945	15	80	1200	15	99	1485
	-5	24	24	576	24	35	840	24	48	1152	24	63	1512	24	80	1920	24	99	2376	24	120	2880
	-6	35	35	1225	35	48	1680	35	63	2205	35	80	2800	35	99	3465	35	120	4200	35	143	5005
	-7	48	48	2304	48	63	3024	48	80	3840	48	99	4752	48	120	5760	48	143	6864	48	168	8064
	-8	63	63	3969	63	80	5040	63	99	6237	63	120	7560	63	143	9009	63	168	10584	63	195	12285
	-9	80	80	6400	80	99	7920	80	120	9600	80	143	11440	80	168	13440	80	195	15600	80	224	17920
	-10	99	99	9801	99	120	11880	99	143	14157	99	168	16632	99	195	19305	99	224	22176	99	255	25245

Table 1. Quadratic sequences (y^2-1) and $((y-d)^2-1)$ producing the quartics $y^4-2dy^3+(d^2-2)y^2+2dy-(d^2-1)$.

Each factor (y^2-1) and $((y-d)^2-1)$ produces the same quadratic sequence A005563 (Square minus One) numbers. Both quadratics are determined by the same three consecutive elements $[0,-1,0]$.

The two multiplications $(y^2-1)((y-d)^2-1)$ and $((y-d)^2-1)(y^2-1)$ generate the quartics $Y[y]=y^4-2dy^3+(d^2-2)y^2+2dy-(d^2-1)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d=0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.3.3Repeated composites generated by the A028347 (Square minus Four) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A028347	A028347	A	A028347	A028347	A	A028347	A028347	A	A028347	A028347	A	A028347	A028347	A	A028347	A028347	A	A028347	A028347	A
y_ip		0	0	0	0	1	0,5	0	2	1	0	3	1,5	0	4	2	0	5	2,5	0	6	3
offset = f		0	0	0	0	1	0	0	2	1	0	3	1	0	4	2	0	5	2	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				0			-2			-4			-6			-8			-10			-12
a = d^2-8		1	1	-8	1	1	-7	1	1	-4	1	1	1	1	1	8	1	1	17	1	1	28
b = 8d		0	0	0	0	-2	8	0	-4	16	0	-6	24	0	-8	32	0	-10	40	0	-12	48
c = -4d^2+16		-4	-4	16	-4	-3	12	-4	0	0	-4	5	-20	-4	12	-48	-4	21	-84	-4	32	-128
	10	96	96	9216	96	77	7392	96	60	5760	96	45	4320	96	32	3072	96	21	2016	96	12	1152
	9	77	77	5929	77	60	4620	77	45	3465	77	32	2464	77	21	1617	77	12	924	77	5	385
	8	60	60	3600	60	45	2700	60	32	1920	60	21	1260	60	12	720	60	5	300	60	0	0
	7	45	45	2025	45	32	1440	45	21	945	45	12	540	45	5	225	45	0	0	45	-3	-135
	6	32	32	1024	32	21	672	32	12	384	32	5	160	32	0	0	32	-3	-96	32	-4	-128
	5	21	21	441	21	12	252	21	5	105	21	0	0	21	-3	-63	21	-4	-84	21	-3	-63
	4	12	12	144	12	5	60	12	0	0	12	-3	-36	12	-4	-48	12	-3	-36	12	0	0
	3	5	5	25	5	0	0	5	-3	-15	5	-4	-20	5	-3	-15	5	0	0	5	5	25
Y[2]	2	0	0	0	0	-3	0	0	-4	0	0	-3	0	0	0	0	0	5	0	0	12	0
Y[1]	1	-3	-3	9	-3	-4	12	-3	-3	9	-3	0	0	-3	5	-15	-3	12	-36	-3	21	-63
Y[0]	0	-4	-4	16	-4	-3	12	-4	0	0	-4	5	-20	-4	12	-48	-4	21	-84	-4	32	-128
Y[-1]	-1	-3	-3	9	-3	0	0	-3	5	-15	-3	12	-36	-3	21	-63	-3	32	-96	-3	45	-135
Y[-2]	-2	0	0	0	0	5	0	0	12	0	0	21	0	0	32	0	0	45	0	0	60	0
	-3	5	5	25	5	12	60	5	21	105	5	32	160	5	45	225	5	60	300	5	77	385
	-4	12	12	144	12	21	252	12	32	384	12	45	540	12	60	720	12	77	924	12	96	1152
	-5	21	21	441	21	32	672	21	45	945	21	60	1260	21	77	1617	21	96	2016	21	117	2457
	-6	32	32	1024	32	45	1440	32	60	1920	32	77	2464	32	96	3072	32	117	3744	32	140	4480
	-7	45	45	2025	45	60	2700	45	77	3465	45	96	4320	45	117	5265	45	140	6300	45	165	7425
	-8	60	60	3600	60	77	4620	60	96	5760	60	117	7020	60	140	8400	60	165	9900	60	192	11520
	-9	77	77	5929	77	96	7392	77	117	9009	77	140	10780	77	165	12705	77	192	14784	77	221	17017
	-10	96	96	9216	96	117	11232	96	140	13440	96	165	15840	96	192	18432	96	221	21216	96	252	24192

Table 1. Quadratic sequences (y^2-4) and $((y-d)^2-4)$ producing the quartics $y^4-2dy^3+(d^2-8)y^2+8dy-(4d^2-16)$.

Each factor (y^2-4) and $((y-d)^2-4)$ produces the same quadratic sequence A028347 (Square minus Four) numbers. Both quadratics are determined by the same three consecutive elements $[-3,-4,-3]$.

The two multiplications $(y^2-4)((y-d)^2-4)$ and $((y-d)^2-4)(y^2-4)$ generate the quartics $Y[y]=y^4-2dy^3+(d^2-8)y^2+8dy-(4d^2-16)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d=0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.3.4 Repeated composites generated by the A028560 (Square minus Nine) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A028560	A028560	A	A028560	A028560	A	A028560	A028560	A	A028560	A028560	A	A028560	A028560	A	A028560	A028560	A	A028560	A028560	A
y_ip		0	0	0	0	1	0,5	0	2	1	0	3	1,5	0	4	2	0	5	2,5	0	6	3
offset = f		0	0	0	0	1	0	0	2	1	0	3	1	0	4	2	0	5	2	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				0			-2			-4			-6			-8			-10			-12
a = d^2-18		1	1	-18	1	1	-17	1	1	-14	1	1	-9	1	1	-2	1	1	7	1	1	18
b = 18d		0	0	0	0	-2	18	0	-4	36	0	-6	54	0	-8	72	0	-10	90	0	-12	108
c = -9d^2+81		-9	-9	81	-9	-8	72	-9	-5	45	-9	0	0	-9	7	-63	-9	16	-144	-9	27	-243
	10	91	91	8281	91	72	6552	91	55	5005	91	40	3640	91	27	2457	91	16	1456	91	7	637
	9	72	72	5184	72	55	3960	72	40	2880	72	27	1944	72	16	1152	72	7	504	72	0	0
	8	55	55	3025	55	40	2200	55	27	1485	55	16	880	55	7	385	55	0	0	55	-5	-275
	7	40	40	1600	40	27	1080	40	16	640	40	7	280	40	0	0	40	-5	-200	40	-8	-320
	6	27	27	729	27	16	432	27	7	189	27	0	0	27	-5	-135	27	-8	-216	27	-9	-243
	5	16	16	256	16	7	112	16	0	0	16	-5	-80	16	-8	-128	16	-9	-144	16	-8	-128
	4	7	7	49	7	0	0	7	-5	-35	7	-8	-56	7	-9	-63	7	-8	-56	7	-5	-35
	3	0	0	0	0	-5	0	0	-8	0	0	-9	0	0	-8	0	0	-5	0	0	0	0
Y[2]	2	-5	-5	25	-5	-8	40	-5	-9	45	-5	-8	40	-5	-5	25	-5	0	0	-5	7	-35
Y[1]	1	-8	-8	64	-8	-9	72	-8	-8	64	-8	-5	40	-8	0	0	-8	7	-56	-8	16	-128
Y[0]	0	-9	-9	81	-9	-8	72	-9	-5	45	-9	0	0	-9	7	-63	-9	16	-144	-9	27	-243
Y[-1]	-1	-8	-8	64	-8	-5	40	-8	0	0	-8	7	-56	-8	16	-128	-8	27	-216	-8	40	-320
Y[-2]	-2	-5	-5	25	-5	0	0	-5	7	-35	-5	16	-80	-5	27	-135	-5	40	-200	-5	55	-275
	-3	0	0	0	0	7	0	0	16	0	0	27	0	0	40	0	0	55	0	0	72	0
	-4	7	7	49	7	16	112	7	27	189	7	40	280	7	55	385	7	72	504	7	91	637
	-5	16	16	256	16	27	432	16	40	640	16	55	880	16	72	1152	16	91	1456	16	112	1792
	-6	27	27	729	27	40	1080	27	55	1485	27	72	1944	27	91	2457	27	112	3024	27	135	3645
	-7	40	40	1600	40	55	2200	40	72	2880	40	91	3640	40	112	4480	40	135	5400	40	160	6400
	-8	55	55	3025	55	72	3960	55	91	5005	55	112	6160	55	135	7425	55	160	8800	55	187	10285
	-9	72	72	5184	72	91	6552	72	112	8064	72	135	9720	72	160	11520	72	187	13464	72	216	15552
	-10	91	91	8281	91	112	10192	91	135	12285	91	160	14560	91	187	17017	91	216	19656	91	247	22477

Table 1. Quadratic sequences (y^2-9) and $((y-d)^2-9)$ producing the quartics $y^4-2dy^3+(d^2-18)y^2+18dy-(9d^2-81)$.

Each factor (y^2-9) and $((y-d)^2-9)$ produces the same quadratic sequence A028560 (Square minus Nine) numbers. Both quadratics are determined by the same three consecutive elements $[-8,-9,-8]$.

The two multiplications $(y^2-9)((y-d)^2-9)$ and $((y-d)^2-9)(y^2-9)$ generate the quartics $Y[y]=y^4-2dy^3+(d^2-18)y^2+18dy-(9d^2-81)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d=0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.3.5Repeated composites generated by the A028566 (Square minus Sixteen) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A028566	A028566	A	A028566	A028566	A	A028566	A028566	A	A028566	A028566	A	A028566	A028566	A	A028566	A028566	A	A028566	A028566	A
y_ip		0	0	0	0	1	0,5	0	2	1	0	3	1,5	0	4	2	0	5	2,5	0	6	3
offset = f		0	0	0	0	1	0	0	2	1	0	3	1	0	4	2	0	5	2	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				0			-2			-4			-6			-8			-10			-12
a = d^2-32		1	1	-32	1	1	-31	1	1	-28	1	1	-23	1	1	-16	1	1	-7	1	1	4
b = 32d		0	0	0	0	-2	32	0	-4	64	0	-6	96	0	-8	128	0	-10	160	0	-12	192
c=-16d^2+256		-16	-16	256	-16	-15	240	-16	-12	192	-16	-7	112	-16	0	0	-16	9	-144	-16	20	-320
	10	84	84	7056	84	65	5460	84	48	4032	84	33	2772	84	20	1680	84	9	756	84	0	0
	9	65	65	4225	65	48	3120	65	33	2145	65	20	1300	65	9	585	65	0	0	65	-7	-455
	8	48	48	2304	48	33	1584	48	20	960	48	9	432	48	0	0	48	-7	-336	48	-12	-576
	7	33	33	1089	33	20	660	33	9	297	33	0	0	33	-7	-231	33	-12	-396	33	-15	-495
	6	20	20	400	20	9	180	20	0	0	20	-7	-140	20	-12	-240	20	-15	-300	20	-16	-320
	5	9	9	81	9	0	0	9	-7	-63	9	-12	-108	9	-15	-135	9	-16	-144	9	-15	-135
	4	0	0	0	0	-7	0	0	-12	0	0	-15	0	0	-16	0	0	-15	0	0	-12	0
	3	-7	-7	49	-7	-12	84	-7	-15	105	-7	-16	112	-7	-15	105	-7	-12	84	-7	-7	49
Y[2]	2	-12	-12	144	-12	-15	180	-12	-16	192	-12	-15	180	-12	-12	144	-12	-7	84	-12	0	0
Y[1]	1	-15	-15	225	-15	-16	240	-15	-15	225	-15	-12	180	-15	-7	105	-15	0	0	-15	9	-135
Y[0]	0	-16	-16	256	-16	-15	240	-16	-12	192	-16	-7	112	-16	0	0	-16	9	-144	-16	20	-320
Y[-1]	-1	-15	-15	225	-15	-12	180	-15	-7	105	-15	0	0	-15	9	-135	-15	20	-300	-15	33	-495
Y[-2]	-2	-12	-12	144	-12	-7	84	-12	0	0	-12	9	-108	-12	20	-240	-12	33	-396	-12	48	-576
	-3	-7	-7	49	-7	0	0	-7	9	-63	-7	20	-140	-7	33	-231	-7	48	-336	-7	65	-455
	-4	0	0	0	0	9	0	0	20	0	0	33	0	0	48	0	0	65	0	0	84	0
	-5	9	9	81	9	20	180	9	33	297	9	48	432	9	65	585	9	84	756	9	105	945
	-6	20	20	400	20	33	660	20	48	960	20	65	1300	20	84	1680	20	105	2100	20	128	2560
	-7	33	33	1089	33	48	1584	33	65	2145	33	84	2772	33	105	3465	33	128	4224	33	153	5049
	-8	48	48	2304	48	65	3120	48	84	4032	48	105	5040	48	128	6144	48	153	7344	48	180	8640
	-9	65	65	4225	65	84	5460	65	105	6825	65	128	8320	65	153	9945	65	180	11700	65	209	13585
	-10	84	84	7056	84	105	8820	84	128	10752	84	153	12852	84	180	15120	84	209	17556	84	240	20160

Table 1. Quadratic sequences $(y^2 - 16)$ and $((y-d)^2 - 16)$ producing the quartics $y^4 - 2dy^3 + (d^2 - 32)y^2 + 32dy - (16d^2 - 256)$.

Each factor $(y^2 - 16)$ and $((y-d)^2 - 16)$ produces the same quadratic sequence A028566 (Square minus Sixteen) numbers. Both quadratics are determined by the same three consecutive elements $[-15, -16, -15]$.

The two multiplications $(y^2 - 16)((y-d)^2 - 16)$ and $((y-d)^2 - 16)(y^2 - 16)$ generate the quartics $Y[y] = y^4 - 2dy^3 + (d^2 - 32)y^2 + 32dy - (16d^2 - 256)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d = 0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.3.6Summary for Repeated composites generated by the $Y[y] = y^2 - s^2$

$$Y_{y^2-0}[y] = y^4 - 2dy^3 + d^2y^2$$
$$Y_{y^2-1}[y] = y^4 - 2dy^3 + (d^2 - 2)y^2 + 2dy - (d^2 - 1)$$
$$Y_{y^2-4}[y] = y^4 - 2dy^3 + (d^2 - 8)y^2 + 8dy - (4d^2 - 16)$$
$$Y_{y^2-9}[y] = y^4 - 2dy^3 + (d^2 - 18)y^2 + 18dy - (9d^2 - 81)$$
$$Y_{y^2-16}[y] = y^4 - 2dy^3 + (d^2 - 32)y^2 + 32dy - (16d^2 - 256)$$
…
$$Y_{y^2-s^2}[y] = y^4 - 2dy^3 + (d^2 - 2s^2)y^2 + 2s^2dy - (s^2d^2 - s^4)$$

9.3 Repeated composites generated by an Oblong sequence minus an Oblong number

9.4.1Repeated composites generated by the A002378 Oblong numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A002378	A002378	A035287	A002378	A002378	A047928	A002378	A002378	A052762	A002378	A002378	A	A002378	A002378	A	A002378	A002378	A	A002378	A002378	A
y_ip		0,5	0,5	0,5	0,5	1,5	1	0,5	2,5	1,5	0,5	3,5	2	0,5	4,5	2,5	0,5	5,5	3	0,5	6,5	3,5
offset = f		0	0	0	0	1	1	0	2	1	0	3	2	0	4	2	0	5	3	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				-2			-4			-6			-8			-10			-12			-14
a=d^2+3d+1		1	1	1	1	1	5	1	1	11	1	1	19	1	1	29	1	1	41	1	1	55
b=-d^2-d		-1	-1	0	-1	-3	-2	-1	-5	-6	-1	-7	-12	-1	-9	-20	-1	-11	-30	-1	-13	-42
c = 0		0	0	0	0	2	0	0	6	0	0	12	0	0	20	0	0	30	0	0	42	0
	10	90	90	8100	90	72	6480	90	56	5040	90	42	3780	90	30	2700	90	20	1800	90	12	1080
	9	72	72	5184	72	56	4032	72	42	3024	72	30	2160	72	20	1440	72	12	864	72	6	432
	8	56	56	3136	56	42	2352	56	30	1680	56	20	1120	56	12	672	56	6	336	56	2	112
	7	42	42	1764	42	30	1260	42	20	840	42	12	504	42	6	252	42	2	84	42	0	0
	6	30	30	900	30	20	600	30	12	360	30	6	180	30	2	60	30	0	0	30	0	0
	5	20	20	400	20	12	240	20	6	120	20	2	40	20	0	0	20	0	0	20	2	40
	4	12	12	144	12	6	72	12	2	24	12	0	0	12	0	0	12	2	24	12	6	72
	3	6	6	36	6	2	12	6	0	0	6	0	0	6	2	12	6	6	36	6	12	72
Y[2]	2	2	2	4	2	0	0	2	0	0	2	2	4	2	6	12	2	12	24	2	20	40
Y[1]	1	0	0	0	0	0	0	0	2	0	0	6	0	0	12	0	0	20	0	0	30	0
Y[0]	0	0	0	0	0	2	0	0	6	0	0	12	0	0	20	0	0	30	0	0	42	0
Y[-1]	-1	2	2	4	2	6	12	2	12	24	2	20	40	2	30	60	2	42	84	2	56	112
Y[-2]	-2	6	6	36	6	12	72	6	20	120	6	30	180	6	42	252	6	56	336	6	72	432
	-3	12	12	144	12	20	240	12	30	360	12	42	504	12	56	672	12	72	864	12	90	1080
	-4	20	20	400	20	30	600	20	42	840	20	56	1120	20	72	1440	20	90	1800	20	110	2200
	-5	30	30	900	30	42	1260	30	56	1680	30	72	2160	30	90	2700	30	110	3300	30	132	3960
	-6	42	42	1764	42	56	2352	42	72	3024	42	90	3780	42	110	4620	42	132	5544	42	156	6552
	-7	56	56	3136	56	72	4032	56	90	5040	56	110	6160	56	132	7392	56	156	8736	56	182	10192
	-8	72	72	5184	72	90	6480	72	110	7920	72	132	9504	72	156	11232	72	182	13104	72	210	15120
	-9	90	90	8100	90	110	9900	90	132	11880	90	156	14040	90	182	16380	90	210	18900	90	240	21600
	-10	110	110	12100	110	132	14520	110	156	17160	110	182	20020	110	210	23100	110	240	26400	110	272	29920

Table 1. Quadratic sequences $(y^2 - y)$ and $\left((y-d)^2 - (y-d)\right)$ producing the quartics $y^4 - 2dy^3 + (d^2 + 3d + 1)y^2 + (-d^2 - d)y$.

Each factor $(y^2 - y)$ and $((y-d)^2 - (y-d))$ produces the same quadratic sequence A002378 Oblong numbers. Both quadratics are determined by the same three consecutive elements [2,0,0].

The two multiplications $(y^2 - y)\left((y-d)^2 - (y-d)\right)$ and $\left((y-d)^2 - (y-d)\right)(y\text{^}2 - y)$ generate the quartics $Y[y] = y^4 - 2dy^3 + (d^2 + 3d + 1)y^2 + (-d^2 - d)y$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d = 0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.4.2Repeated composites generated by the A028552 (Oblong minus Two) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A028552	A028552	A	A028552	A028552	A	A028552	A028552	A	A028552	A028552	A	A028552	A028552	A	A028552	A028552	A	A028552	A028552	A
y_ip		0,5	0,5	0,5	0,5	1,5	1	0,5	2,5	1,5	0,5	3,5	2	0,5	4,5	2,5	0,5	5,5	3	0,5	6,5	3,5
offset = f		0	0	0	0	1	1	0	2	1	0	3	2	0	4	2	0	5	3	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				-2			-4			-6			-8			-10			-12			-14
a=d^2+3d-3		1	1	-3	1	1	1	1	1	7	1	1	15	1	1	25	1	1	37	1	1	51
b=-d^2+3d+4		-1	-1	4	-1	-3	6	-1	-5	6	-1	-7	4	-1	-9	0	-1	-11	-6	-1	-13	-14
c=-2d^2-2d+4		-2	-2	4	-2	0	0	-2	4	-8	-2	10	-20	-2	18	-36	-2	28	-56	-2	40	-80
	10	88	88	7744	88	70	6160	88	54	4752	88	40	3520	88	28	2464	88	18	1584	88	10	880
	9	70	70	4900	70	54	3780	70	40	2800	70	28	1960	70	18	1260	70	10	700	70	4	280
	8	54	54	2916	54	40	2160	54	28	1512	54	18	972	54	10	540	54	4	216	54	0	0
	7	40	40	1600	40	28	1120	40	18	720	40	10	400	40	4	160	40	0	0	40	-2	-80
	6	28	28	784	28	18	504	28	10	280	28	4	112	28	0	0	28	-2	-56	28	-2	-56
	5	18	18	324	18	10	180	18	4	72	18	0	0	18	-2	-36	18	-2	-36	18	0	0
	4	10	10	100	10	4	40	10	0	0	10	-2	-20	10	-2	-20	10	0	0	10	4	40
	3	4	4	16	4	0	0	4	-2	-8	4	-2	-8	4	0	0	4	4	16	4	10	40
Y[2]	2	0	0	0	0	-2	0	0	-2	0	0	0	0	0	4	0	0	10	0	0	18	0
Y[1]	1	-2	-2	4	-2	-2	4	-2	0	0	-2	4	-8	-2	10	-20	-2	18	-36	-2	28	-56
Y[0]	0	-2	-2	4	-2	0	0	-2	4	-8	-2	10	-20	-2	18	-36	-2	28	-56	-2	40	-80
Y[-1]	-1	0	0	0	0	4	0	0	10	0	0	18	0	0	28	0	0	40	0	0	54	0
Y[-2]	-2	4	4	16	4	10	40	4	18	72	4	28	112	4	40	160	4	54	216	4	70	280
	-3	10	10	100	10	18	180	10	28	280	10	40	400	10	54	540	10	70	700	10	88	880
	-4	18	18	324	18	28	504	18	40	720	18	54	972	18	70	1260	18	88	1584	18	108	1944
	-5	28	28	784	28	40	1120	28	54	1512	28	70	1960	28	88	2464	28	108	3024	28	130	3640
	-6	40	40	1600	40	54	2160	40	70	2800	40	88	3520	40	108	4320	40	130	5200	40	154	6160
	-7	54	54	2916	54	70	3780	54	88	4752	54	108	5832	54	130	7020	54	154	8316	54	180	9720
	-8	70	70	4900	70	88	6160	70	108	7560	70	130	9100	70	154	10780	70	180	12600	70	208	14560
	-9	88	88	7744	88	108	9504	88	130	11440	88	154	13552	88	180	15840	88	208	18304	88	238	20944
	-10	108	108	11664	108	130	14040	108	154	16632	108	180	19440	108	208	22464	108	238	25704	108	270	29160

Table 1. Quadratic sequences $((y^2-y)-2)$ and $\left(((y-d)^2-(y-d))-2\right)$ producing the quartics $y^4-2dy^3+(d^2+3d-3)y^2+(-d^2+3d+4)y+(-2d^2-2d+4)$.

Each factor $((y^2-y)-2)$ and $\left(((y-d)^2-(y-d))-2\right)$ produces the same quadratic sequence A028552 (Oblong minus Two) numbers. Both quadratics are determined by the same three consecutive elements $[0,-2,-2]$.

The two multiplications $((y^2-y)-2)\left(((y-d)^2-(y-d))-2\right)$ and $\left(((y-d)^2-(y-d))-2\right)((y^2-y)-2)$ generate the quartics $Y[y]=y^4-2dy^3+(d^2+3d-3)y^2+(-d^2+3d+4)y+(-2d^2-2d+4)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d=0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.4.3 Repeated composites generated by the A028557 (Oblong minus Six) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A028557	A028557	A	A028557	A028557	A	A028557	A028557	A	A028557	A028557	A	A028557	A028557	A	A028557	A028557	A	A028557	A028557	A
y_ip		0,5	0,5	0,5	0,5	1,5	1	0,5	2,5	1,5	0,5	3,5	2	0,5	4,5	2,5	0,5	5,5	3	0,5	6,5	3,5
offset = f		0	0	0	0	1	1	0	2	1	0	3	2	0	4	2	0	5	3	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				-2			-4			-6			-8			-10			-12			-14
a=d^2+3d-11		1	1	-11	1	1	-7	1	1	-1	1	1	7	1	1	17	1	1	29	1	1	43
b=-d^2+11d+12		-1	-1	12	-1	-3	22	-1	-5	30	-1	-7	36	-1	-9	40	-1	-11	42	-1	-13	42
c=-6d^2-6d+36		-6	-6	36	-6	-4	24	-6	0	0	-6	6	-36	-6	14	-84	-6	24	-144	-6	36	-216
	10	84	84	7056	84	66	5544	84	50	4200	84	36	3024	84	24	2016	84	14	1176	84	6	504
	9	66	66	4356	66	50	3300	66	36	2376	66	24	1584	66	14	924	66	6	396	66	0	0
	8	50	50	2500	50	36	1800	50	24	1200	50	14	700	50	6	300	50	0	0	50	-4	-200
	7	36	36	1296	36	24	864	36	14	504	36	6	216	36	0	0	36	-4	-144	36	-6	-216
	6	24	24	576	24	14	336	24	6	144	24	0	0	24	-4	-96	24	-6	-144	24	-6	-144
	5	14	14	196	14	6	84	14	0	0	14	-4	-56	14	-6	-84	14	-6	-84	14	-4	-56
	4	6	6	36	6	0	0	6	-4	-24	6	-6	-36	6	-6	-36	6	-4	-24	6	0	0
	3	0	0	0	0	-4	0	0	-6	0	0	-6	0	0	-4	0	0	0	0	0	6	0
Y[2]	2	-4	-4	16	-4	-6	24	-4	-6	24	-4	-4	16	-4	0	0	-4	6	-24	-4	14	-56
Y[1]	1	-6	-6	36	-6	-6	36	-6	-4	24	-6	0	0	-6	6	-36	-6	14	-84	-6	24	-144
Y[0]	0	-6	-6	36	-6	-4	24	-6	0	0	-6	6	-36	-6	14	-84	-6	24	-144	-6	36	-216
Y[-1]	-1	-4	-4	16	-4	0	0	-4	6	-24	-4	14	-56	-4	24	-96	-4	36	-144	-4	50	-200
Y[-2]	-2	0	0	0	0	6	0	0	14	0	0	24	0	0	36	0	0	50	0	0	66	0
	-3	6	6	36	6	14	84	6	24	144	6	36	216	6	50	300	6	66	396	6	84	504
	-4	14	14	196	14	24	336	14	36	504	14	50	700	14	66	924	14	84	1176	14	104	1456
	-5	24	24	576	24	36	864	24	50	1200	24	66	1584	24	84	2016	24	104	2496	24	126	3024
	-6	36	36	1296	36	50	1800	36	66	2376	36	84	3024	36	104	3744	36	126	4536	36	150	5400
	-7	50	50	2500	50	66	3300	50	84	4200	50	104	5200	50	126	6300	50	150	7500	50	176	8800
	-8	66	66	4356	66	84	5544	66	104	6864	66	126	8316	66	150	9900	66	176	11616	66	204	13464
	-9	84	84	7056	84	104	8736	84	126	10584	84	150	12600	84	176	14784	84	204	17136	84	234	19656
	-10	104	104	10816	104	126	13104	104	150	15600	104	176	18304	104	204	21216	104	234	24336	104	266	27664

Table 1. Quadratic sequences $\left((y^2-y)-6\right)$ and $\left(((y-d)^2-(y-d))-6\right)$ producing the quartics $y^4-2dy^3+(d^2+3d-11)y^2+(-d^2+11d+12)y+(-6d^2-6d+36)$.

Each factor $\left((y^2-y)-6\right)$ and $\left(((y-d)^2-(y-d))-6\right)$ produces the same quadratic sequence A028557 (Oblong minus Six) numbers. Both quadratics are determined by the same three consecutive elements $[-4,-6,-6]$.

The two multiplications $\left((y^2-y)-6\right)\left(((y-d)^2-(y-d))-6\right)$ and $\left(((y-d)^2-(y-d))-6\right)\left((y^2-y)-6\right)$ generate the quartics $Y[y]=y^4-2dy^3+(d^2+3d-11)y^2+(-d^2+11d+12)y+(-6d^2-6d+36)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d=0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.4.4Repeated composites generated by the A028563 (Oblong minus Twelve) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A028563	A028563	A	A028563	A028563	A	A028563	A028563	A	A028563	A028563	A	A028563	A028563	A	A028563	A028563	A	A028563	A028563	A
y_ip		0,5	0,5	0,5	0,5	1,5	1	0,5	2,5	1,5	0,5	3,5	2	0,5	4,5	2,5	0,5	5,5	3	0,5	6,5	3,5
offset = f		0	0	0	0	1	1	0	2	1	0	3	2	0	4	2	0	5	3	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				-2			-4			-6			-8			-10			-12			-14
a=d^2+3d-23		1	1	-23	1	1	-19	1	1	-13	1	1	-5	1	1	5	1	1	17	1	1	31
b=-d^2+23d+24		-1	-1	24	-1	-3	46	-1	-5	66	-1	-7	84	-1	-9	100	-1	-11	114	-1	-13	126
c=-12d^2-12d+144		-12	-12	144	-12	-10	120	-12	-6	72	-12	0	0	-12	8	-96	-12	18	-216	-12	30	-360
	10	78	78	6084	78	60	4680	78	44	3432	78	30	2340	78	18	1404	78	8	624	78	0	0
	9	60	60	3600	60	44	2640	60	30	1800	60	18	1080	60	8	480	60	0	0	60	-6	-360
	8	44	44	1936	44	30	1320	44	18	792	44	8	352	44	0	0	44	-6	-264	44	-10	-440
	7	30	30	900	30	18	540	30	8	240	30	0	0	30	-6	-180	30	-10	-300	30	-12	-360
	6	18	18	324	18	8	144	18	0	0	18	-6	-108	18	-10	-180	18	-12	-216	18	-12	-216
	5	8	8	64	8	0	0	8	-6	-48	8	-10	-80	8	-12	-96	8	-12	-96	8	-10	-80
	4	0	0	0	0	-6	0	0	-10	0	0	-12	0	0	-12	0	0	-10	0	0	-6	0
	3	-6	-6	36	-6	-10	60	-6	-12	72	-6	-12	72	-6	-10	60	-6	-6	36	-6	0	0
Y[2]	2	-10	-10	100	-10	-12	120	-10	-12	120	-10	-10	100	-10	-6	60	-10	0	0	-10	8	-80
Y[1]	1	-12	-12	144	-12	-12	144	-12	-10	120	-12	-6	72	-12	0	0	-12	8	-96	-12	18	-216
Y[0]	0	-12	-12	144	-12	-10	120	-12	-6	72	-12	0	0	-12	8	-96	-12	18	-216	-12	30	-360
Y[-1]	-1	-10	-10	100	-10	-6	60	-10	0	0	-10	8	-80	-10	18	-180	-10	30	-300	-10	44	-440
Y[-2]	-2	-6	-6	36	-6	0	0	-6	8	-48	-6	18	-108	-6	30	-180	-6	44	-264	-6	60	-360
	-3	0	0	0	0	8	0	0	18	0	0	30	0	0	44	0	0	60	0	0	78	0
	-4	8	8	64	8	18	144	8	30	240	8	44	352	8	60	480	8	78	624	8	98	784
	-5	18	18	324	18	30	540	18	44	792	18	60	1080	18	78	1404	18	98	1764	18	120	2160
	-6	30	30	900	30	44	1320	30	60	1800	30	78	2340	30	98	2940	30	120	3600	30	144	4320
	-7	44	44	1936	44	60	2640	44	78	3432	44	98	4312	44	120	5280	44	144	6336	44	170	7480
	-8	60	60	3600	60	78	4680	60	98	5880	60	120	7200	60	144	8640	60	170	10200	60	198	11880
	-9	78	78	6084	78	98	7644	78	120	9360	78	144	11232	78	170	13260	78	198	15444	78	228	17784
	-10	98	98	9604	98	120	11760	98	144	14112	98	170	16660	98	198	19404	98	228	22344	98	260	25480

Table 1. Quadratic sequences $\left((y^2-y)-12\right)$ and $\left(\left((y-d)^2-(y-d)\right)-12\right)$ producing the quartics $y^4-2dy^3+(d^2+3d-23)y^2+(-d^2+23d+24)y+(-12d^2-12d+144)$.

Each factor $\left((y^2-y)-12\right)$ and $\left(\left((y-d)^2-(y-d)\right)-12\right)$ produces the same quadratic sequence A028563 (Oblong minus Twelve) numbers. Both quadratics are determined by the same three consecutive elements $[-10,-12,-12]$.

The two multiplications $\left((y^2-y)-12\right)\left(\left((y-d)^2-(y-d)\right)-12\right)$ and $\left(\left((y-d)^2-(y-d)\right)-12\right)\left((y^2-y)-12\right)$ generate the quartics $Y[y]=y^4-2dy^3+(d^2+3d-23)y^2+(-d^2+23d+24)y+(-12d^2-12d+144)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d=0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.4.5Repeated composites generated by the A028569 (Oblong minus Twenty) numbers

Δ offset = d		Δ offset = d = 0			Δ offset = d = 1			Δ offset = d = 2			Δ offset = d = 3			Δ offset = d = 4			Δ offset = d = 5			Δ offset = d = 6		
OEIS		A028569	A028569	A	A028569	A028569	A	A028569	A028569	A	A028569	A028569	A	A028569	A028569	A	A028569	A028569	A	A028569	A028569	A
y_ip		0,5	0,5	0,5	0,5	1,5	1	0,5	2,5	1,5	0,5	3,5	2	0,5	4,5	2,5	0,5	5,5	3	0,5	6,5	3,5
offset = f		0	0	0	0	1	1	0	2	1	0	3	2	0	4	2	0	5	3	0	6	3
a_4 = 1				1			1			1			1			1			1			1
a_3 = -2d				-2			-4			-6			-8			-10			-12			-14
a=d^2+3d-39		1	1	-39	1	1	-35	1	1	-29	1	1	-21	1	1	-11	1	1	1	1	1	15
b=-d^2+39d+40		-1	-1	40	-1	-3	78	-1	-5	114	-1	-7	148	-1	-9	180	-1	-11	210	-1	-13	238
c=-20d^2-20d+400		-20	-20	400	-20	-18	360	-20	-14	280	-20	-8	160	-20	0	0	-20	10	-200	-20	22	-440
	10	70	70	4900	70	52	3640	70	36	2520	70	22	1540	70	10	700	70	0	0	70	-8	-560
	9	52	52	2704	52	36	1872	52	22	1144	52	10	520	52	0	0	52	-8	-416	52	-14	-728
	8	36	36	1296	36	22	792	36	10	360	36	0	0	36	-8	-288	36	-14	-504	36	-18	-648
	7	22	22	484	22	10	220	22	0	0	22	-8	-176	22	-14	-308	22	-18	-396	22	-20	-440
	6	10	10	100	10	0	0	10	-8	-80	10	-14	-140	10	-18	-180	10	-20	-200	10	-20	-200
	5	0	0	0	0	-8	0	0	-14	0	0	-18	0	0	-20	0	0	-20	0	0	-18	0
	4	-8	-8	64	-8	-14	112	-8	-18	144	-8	-20	160	-8	-20	160	-8	-18	144	-8	-14	112
	3	-14	-14	196	-14	-18	252	-14	-20	280	-14	-20	280	-14	-18	252	-14	-14	196	-14	-8	112
Y[2]	2	-18	-18	324	-18	-20	360	-18	-20	360	-18	-18	324	-18	-14	252	-18	-8	144	-18	0	0
Y[1]	1	-20	-20	400	-20	-20	400	-20	-18	360	-20	-14	280	-20	-8	160	-20	0	0	-20	10	-200
Y[0]	0	-20	-20	400	-20	-18	360	-20	-14	280	-20	-8	160	-20	0	0	-20	10	-200	-20	22	-440
Y[-1]	-1	-18	-18	324	-18	-14	252	-18	-8	144	-18	0	0	-18	10	-180	-18	22	-396	-18	36	-648
Y[-2]	-2	-14	-14	196	-14	-8	112	-14	0	0	-14	10	-140	-14	22	-308	-14	36	-504	-14	52	-728
	-3	-8	-8	64	-8	0	0	-8	10	-80	-8	22	-176	-8	36	-288	-8	52	-416	-8	70	-560
	-4	0	0	0	0	10	0	0	22	0	0	36	0	0	52	0	0	70	0	0	90	0
	-5	10	10	100	10	22	220	10	36	360	10	52	520	10	70	700	10	90	900	10	112	1120
	-6	22	22	484	22	36	792	22	52	1144	22	70	1540	22	90	1980	22	112	2464	22	136	2992
	-7	36	36	1296	36	52	1872	36	70	2520	36	90	3240	36	112	4032	36	136	4896	36	162	5832
	-8	52	52	2704	52	70	3640	52	90	4680	52	112	5824	52	136	7072	52	162	8424	52	190	9880
	-9	70	70	4900	70	90	6300	70	112	7840	70	136	9520	70	162	11340	70	190	13300	70	220	15400
	-10	90	90	8100	90	112	10080	90	136	12240	90	162	14580	90	190	17100	90	220	19800	90	252	22680

Table 1. Quadratic sequences $\left((y^2 - y) - 20\right)$ and $\left(((y-d)^2 - (y-d)) - 20\right)$ producing the quartics $y^4 - 2dy^3 + (d^2 + 3d - 39)y^2 + (-d^2 + 39d + 40)y + (-20d^2 - 20d + 400)$.

Each factor $\left((y^2 - y) - 20\right)$ and $\left(((y-d)^2 - (y-d)) - 20\right)$ produces the same quadratic sequence A028569 (Oblong minus Twenty) numbers. Both quadratics are determined by the same three consecutive elements $[-18, -20, -20]$.

The two multiplications $\left((y^2 - y) - 20\right)\left(((y-d)^2 - (y-d)) - 20\right)$ and $\left(((y-d)^2 - (y-d)) - 20\right)\left((y^2 - y) - 20\right)$ generate the quartics $Y[y] = y^4 - 2dy^3 + (d^2 + 3d - 39)y^2 + (-d^2 + 39d + 40)y + (-20d^2 - 20d + 400)$ in the FMT.

All these quartics are a 3rd-degree curve in the XY plane.

For $d = 0$ it is the only case where there is no duplication because it is the FMT symmetrical axis. The multiplier is equal to the multiplicand.

9.4.6Summary for Repeated composites generated by the $Y[y] = (y^2 - y) - (s^2 - s)$

$$Y_{s=0}[y] = y^4 - 2dy^3 + (d^2 + 3d + 1)y^2 + (-d^2 - d)y$$
$$Y_{s=1}[y] = y^4 - 2dy^3 + (d^2 + 3d - 3)y^2 + (-d^2 + 3d + 4)y + (-2d^2 - 2d + 4)$$
$$Y_{s=2}[y] = y^4 - 2dy^3 + (d^2 + 3d - 11)y^2 + (-d^2 + 11d + 12)y + (-6d^2 - 6d + 36)$$
$$Y_{s=3}[y] = y^4 - 2dy^3 + (d^2 + 3d - 23)y^2 + (-d^2 + 23d + 24)y + (-12d^2 - 12d + 144)$$
$$Y_{s=4}[y] = y^4 - 2dy^3 + (d^2 + 3d - 39)y^2 + (-d^2 + 39d + 40)y + (-20d^2 - 20d + 400)$$
$$\dots$$
$$\begin{aligned} Y_{s[y]} = y^4 - 2dy^3 &+ (d^2 + 3d - 2s^2 - 2s + 1)y^2 \\ &+ \left(-d^2 + (-2s^2 - 2s + 1)d + (2s^2 + 2s)\right)y \\ &+ (-(s^2 + s)d^2 - (s^2 + s)d + (s^2 + s)^2) \end{aligned}$$

10 Where are the sequences of Prime numbers?

The FMT - Full Multiplication Table is the hyperboctys HS[0,0,0].

Because of the theorem of the Zero, we saw in such cases that the Primes will only appear next to the Zeroes, not elsewhere. So, every diagonal sequence in FMT has a finite number of Prime numbers, if it has one.

The only sequences that can have prime numbers in HS[0.0.0] are in the sequences of Integer numbers that appear in the horizontal for $y = \pm 1$ and in the verticals for $x = \pm 1$.

Then, no sequence of prime numbers appears that is different from the sequence of only 2 elements formed by the pair of prime 2 and prime 3. All other prime numbers are isolated and never in sequence.

All these properties extend at all rotations of the FMT. That is, any HS[a,0,a] has the same density of Primes and Composites.

Thus, the only possibility of prime number sequences with more than two elements is to add some integer in the multiplication table:

$$HS[0,0,0] \pm n, n\ any\ Integer \neq 0$$

As we have seen, in infinity, 75% of the elements of the FMT are even and 25% are odd. This way, to have a higher density of Odd Prime numbers, we have to add an Odd number to the FMT.

Note that whatever the Hyperboctys, the amount, shape, and equations in the XY plane of the repeated elements is the same as the FMT with the equivalent hyperbolic structure. Thus, there is always a hyperbolic line linking two elements of equal value in different quadratics sequences with the same coefficients "a" and "c".

10.1 Example for $a = 0$

See in the figure below $HS[0,0,0] + 1 = HS[1,1,1]$ and $HS[0,0,0] + 2 = HS[2,2,2]$. See how the density of odd Prime numbers in H[1,1,1] is greater than H[2,2,2].

a		0	0	0	0	0	0	0	0	0	0	0
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		1	1	1	1	1	1	1	1	1	1	1
	10	-49	-39	-29	-19	-9	1	11	21	31	41	51
	9	-44	-35	-26	-17	-8	1	10	19	28	37	46
	8	-39	-31	-23	-15	-7	1	9	17	25	33	41
	7	-34	-27	-20	-13	-6	1	8	15	22	29	36
	6	-29	-23	-17	-11	-5	1	7	13	19	25	31
	5	-24	-19	-14	-9	-4	1	6	11	16	21	26
	4	-19	-15	-11	-7	-3	1	5	9	13	17	21
	3	-14	-11	-8	-5	-2	1	4	7	10	13	16
	2	-9	-7	-5	-3	-1	1	3	5	7	9	11
Y[1]	1	-4	-3	-2	-1	0	1	2	3	4	5	6
Y[0]	0	1	1	1	1	1	1	1	1	1	1	1
Y[-1]	-1	6	5	4	3	2	1	0	-1	-2	-3	-4
	-2	11	9	7	5	3	1	-1	-3	-5	-7	-9
	-3	16	13	10	7	4	1	-2	-5	-8	-11	-14
	-4	21	17	13	9	5	1	-3	-7	-11	-15	-19
	-5	26	21	16	11	6	1	-4	-9	-14	-19	-24
	-6	31	25	19	13	7	1	-5	-11	-17	-23	-29
	-7	36	29	22	15	8	1	-6	-13	-20	-27	-34
	-8	41	33	25	17	9	1	-7	-15	-23	-31	-39
	-9	46	37	28	19	10	1	-8	-17	-26	-35	-44
	-10	51	41	31	21	11	1	-9	-19	-29	-39	-49

a		0	0	0	0	0	0	0	0	0	0	0
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		2	2	2	2	2	2	2	2	2	2	2
	10	-48	-38	-28	-18	-8	2	12	22	32	42	52
	9	-43	-34	-25	-16	-7	2	11	20	29	38	47
	8	-38	-30	-22	-14	-6	2	10	18	26	34	42
	7	-33	-26	-19	-12	-5	2	9	16	23	30	37
	6	-28	-22	-16	-10	-4	2	8	14	20	26	32
	5	-23	-18	-13	-8	-3	2	7	12	17	22	27
	4	-18	-14	-10	-6	-2	2	6	10	14	18	22
	3	-13	-10	-7	-4	-1	2	5	8	11	14	17
	2	-8	-6	-4	-2	0	2	4	6	8	10	12
Y[1]	1	-3	-2	-1	0	1	2	3	4	5	6	7
Y[0]	0	2	2	2	2	2	2	2	2	2	2	2
Y[-1]	-1	7	6	5	4	3	2	1	0	-1	-2	-3
	-2	12	10	8	6	4	2	0	-2	-4	-6	-8
	-3	17	14	11	8	5	2	-1	-4	-7	-10	-13
	-4	22	18	14	10	6	2	-2	-6	-10	-14	-18
	-5	27	22	17	12	7	2	-3	-8	-13	-18	-23
	-6	32	26	20	14	8	2	-4	-10	-16	-22	-28
	-7	37	30	23	16	9	2	-5	-12	-19	-26	-33
	-8	42	34	26	18	10	2	-6	-14	-22	-30	-38
	-9	47	38	29	20	11	2	-7	-16	-25	-34	-43
	-10	52	42	32	22	12	2	-8	-18	-28	-38	-48

Figure 1. Respectively $HS[1,1,1]$ and $HS[2,2,2]$.

Now, see in the figure below $HS[0,0,0] + 17 = HS[17,17,17]$ and $HS[0,0,0] + 41 = HS[41,41,41]$:

	a	0	0	0	0	0	0	0	0	0	0	0
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	17	17	17	17	17	17	17	17	17	17	17
	10	-33	-23	-13	-3	7	17	27	37	47	57	67
	9	-28	-19	-10	-1	8	17	26	35	44	53	62
	8	-23	-15	-7	1	9	17	25	33	41	49	57
	7	-18	-11	-4	3	10	17	24	31	38	45	52
	6	-13	-7	-1	5	11	17	23	29	35	41	47
	5	-8	-3	2	7	12	17	22	27	32	37	42
	4	-3	1	5	9	13	17	21	25	29	33	37
	3	2	5	8	11	14	17	20	23	26	29	32
	2	7	9	11	13	15	17	19	21	23	25	27
Y[1]	1	12	13	14	15	16	17	18	19	20	21	22
Y[0]	0	17	17	17	17	17	17	17	17	17	17	17
Y[-1]	-1	22	21	20	19	18	17	16	15	14	13	12
	-2	27	25	23	21	19	17	15	13	11	9	7
	-3	32	29	26	23	20	17	14	11	8	5	2
	-4	37	33	29	25	21	17	13	9	5	1	-3
	-5	42	37	32	27	22	17	12	7	2	-3	-8
	-6	47	41	35	29	23	17	11	5	-1	-7	-13
	-7	52	45	38	31	24	17	10	3	-4	-11	-18
	-8	57	49	41	33	25	17	9	1	-7	-15	-23
	-9	62	53	44	35	26	17	8	-1	-10	-19	-28
	-10	67	57	47	37	27	17	7	-3	-13	-23	-33

	a	0	0	0	0	0	0	0	0	0	0	0
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	41	41	41	41	41	41	41	41	41	41	41
	10	-9	1	11	21	31	41	51	61	71	81	91
	9	-4	5	14	23	32	41	50	59	68	77	86
	8	1	9	17	25	33	41	49	57	65	73	81
	7	6	13	20	27	34	41	48	55	62	69	76
	6	11	17	23	29	35	41	47	53	59	65	71
	5	16	21	26	31	36	41	46	51	56	61	66
	4	21	25	29	33	37	41	45	49	53	57	61
	3	26	29	32	35	38	41	44	47	50	53	56
	2	31	33	35	37	39	41	43	45	47	49	51
Y[1]	1	36	37	38	39	40	41	42	43	44	45	46
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	46	45	44	43	42	41	40	39	38	37	36
	-2	51	49	47	45	43	41	39	37	35	33	31
	-3	56	53	50	47	44	41	38	35	32	29	26
	-4	61	57	53	49	45	41	37	33	29	25	21
	-5	66	61	56	51	46	41	36	31	26	21	16
	-6	71	65	59	53	47	41	35	29	23	17	11
	-7	76	69	62	55	48	41	34	27	20	13	6
	-8	81	73	65	57	49	41	33	25	17	9	1
	-9	86	77	68	59	50	41	32	23	14	5	-4
	-10	91	81	71	61	51	41	31	21	11	1	-9

Figure 1. Respectively $HS[17,17,17]$ and $HS[41,41,41]$.

10.1 Example for $a = 1$

Now, see in the figure below $HS[17,17,17]$ and $HS[41,41,41]$ rotated CCW one step. See in the columns ± 1 the two largest of Euler's Lucky numbers A007635 Primes of form $n^2 \pm n + 17$ and A005846 Primes of the form $n^2 \pm n + 41$.

	a	1	1	1	1	1	1	1	1	1	1	1
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	17	17	17	17	17	17	17	17	17	17	17
	10	67	77	87	97	107	117	127	137	147	157	167
	9	53	62	71	80	89	98	107	116	125	134	143
	8	41	49	57	65	73	81	89	97	105	113	121
	7	31	38	45	52	59	66	73	80	87	94	101
	6	23	29	35	41	47	53	59	65	71	77	83
	5	17	22	27	32	37	42	47	52	57	62	67
	4	13	17	21	25	29	33	37	41	45	49	53
	3	11	14	17	20	23	26	29	32	35	38	41
	2	11	13	15	17	19	21	23	25	27	29	31
Y[1]	1	13	14	15	16	17	18	19	20	21	22	23
Y[0]	0	17	17	17	17	17	17	17	17	17	17	17
Y[-1]	-1	23	22	21	20	19	18	17	16	15	14	13
	-2	31	29	27	25	23	21	19	17	15	13	11
	-3	41	38	35	32	29	26	23	20	17	14	11
	-4	53	49	45	41	37	33	29	25	21	17	13
	-5	67	62	57	52	47	42	37	32	27	22	17
	-6	83	77	71	65	59	53	47	41	35	29	23
	-7	101	94	87	80	73	66	59	52	45	38	31
	-8	121	113	105	97	89	81	73	65	57	49	41
	-9	143	134	125	116	107	98	89	80	71	62	53
	-10	167	157	147	137	127	117	107	97	87	77	67

	a	1	1	1	1	1	1	1	1	1	1	1
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	41	41	41	41	41	41	41	41	41	41	41
	10	91	101	111	121	131	141	151	161	171	181	191
	9	77	86	95	104	113	122	131	140	149	158	167
	8	65	73	81	89	97	105	113	121	129	137	145
	7	55	62	69	76	83	90	97	104	111	118	125
	6	47	53	59	65	71	77	83	89	95	101	107
	5	41	46	51	56	61	66	71	76	81	86	91
	4	37	41	45	49	53	57	61	65	69	73	77
	3	35	38	41	44	47	50	53	56	59	62	65
	2	35	37	39	41	43	45	47	49	51	53	55
Y[1]	1	37	38	39	40	41	42	43	44	45	46	47
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	47	46	45	44	43	42	41	40	39	38	37
	-2	55	53	51	49	47	45	43	41	39	37	35
	-3	65	62	59	56	53	50	47	44	41	38	35
	-4	77	73	69	65	61	57	53	49	45	41	37
	-5	91	86	81	76	71	66	61	56	51	46	41
	-6	107	101	95	89	83	77	71	65	59	53	47
	-7	125	118	111	104	97	90	83	76	69	62	55
	-8	145	137	129	121	113	105	97	89	81	73	65
	-9	167	158	149	140	131	122	113	104	95	86	77
	-10	191	181	171	161	151	141	131	121	111	101	91

Figure 1. Respectively $HS[18,17,18]$ and $HS[42,41,42]$.

Taking all these examples together, note how the density of odd Prime numbers in each quadrant decreases as we increase the absolute values of the elements that form each Hyperboctys.

10.1 Example for $a = 4$

Now, see in the figure below the FMT rotated 4 steps CCW resulting in $HS[4,0,4]$. Then, see $HS[4,0,4] + 41 = HS[45,41,45]$.

In the columns ± 2 appear the sequence A279241 $4n^2 \pm 2n + 41$.

	a	4	4	4	4	4	4	4	4	4	4	4
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	0	0	0	0	0	0	0	0	0	0	0
	10	350	360	370	380	390	400	410	420	430	440	450
	9	279	288	297	306	315	324	333	342	351	360	369
	8	216	224	232	240	248	256	264	272	280	288	296
	7	161	168	175	182	189	196	203	210	217	224	231
	6	114	120	126	132	138	144	150	156	162	168	174
	5	75	80	85	90	95	100	105	110	115	120	125
	4	44	48	52	56	60	64	68	72	76	80	84
	3	21	24	27	30	33	36	39	42	45	48	51
	2	6	8	10	12	14	16	18	20	22	24	26
Y[1]	1	-1	0	1	2	3	4	5	6	7	8	9
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	9	8	7	6	5	4	3	2	1	0	-1
	-2	26	24	22	20	18	16	14	12	10	8	6
	-3	51	48	45	42	39	36	33	30	27	24	21
	-4	84	80	76	72	68	64	60	56	52	48	44
	-5	125	120	115	110	105	100	95	90	85	80	75
	-6	174	168	162	156	150	144	138	132	126	120	114
	-7	231	224	217	210	203	196	189	182	175	168	161
	-8	296	288	280	272	264	256	248	240	232	224	216
	-9	369	360	351	342	333	324	315	306	297	288	279
	-10	450	440	430	420	410	400	390	380	370	360	350

	a	4	4	4	4	4	4	4	4	4	4	4
	b	-5	-4	-3	-2	-1	0	1	2	3	4	5
	c	41	41	41	41	41	41	41	41	41	41	41
	10	391	401	411	421	431	441	451	461	471	481	491
	9	320	329	338	347	356	365	374	383	392	401	410
	8	257	265	273	281	289	297	305	313	321	329	337
	7	202	209	216	223	230	237	244	251	258	265	272
	6	155	161	167	173	179	185	191	197	203	209	215
	5	116	121	126	131	136	141	146	151	156	161	166
	4	85	89	93	97	101	105	109	113	117	121	125
	3	62	65	68	71	74	77	80	83	86	89	92
	2	47	49	51	53	55	57	59	61	63	65	67
Y[1]	1	40	41	42	43	44	45	46	47	48	49	50
Y[0]	0	41	41	41	41	41	41	41	41	41	41	41
Y[-1]	-1	50	49	48	47	46	45	44	43	42	41	40
	-2	67	65	63	61	59	57	55	53	51	49	47
	-3	92	89	86	83	80	77	74	71	68	65	62
	-4	125	121	117	113	109	105	101	97	93	89	85
	-5	166	161	156	151	146	141	136	131	126	121	116
	-6	215	209	203	197	191	185	179	173	167	161	155
	-7	272	265	258	251	244	237	230	223	216	209	202
	-8	337	329	321	313	305	297	289	281	273	265	257
	-9	410	401	392	383	374	365	356	347	338	329	320
	-10	491	481	471	461	451	441	431	421	411	401	391

Figure 1. Respectively $HS[4,0,4]$ and $HS[45,41,45]$.

In the columns ± 2 appear the sequence A279241 $4n^2 \pm 2n + 41$.

11 Where are the Paraboctys?

Suppose this Hyperboctys infinite sequence $HS[1,0,1] + n$.
See the 6 initial consecutive structures in the range $0 \leq n \leq 5$.

$$HS[1,0,1] + 1 = HS[2,1,2]$$
$$HS[2,1,2] + 1 = HS[3,2,3]$$
$$HS[3,2,3] + 1 = HS[4,3,4]$$
$$HS[4,3,4] + 1 = HS[5,4,5]$$
$$HS[5,4,5] + 1 = HS[6,5,6]$$
$$\ldots$$
$$HS[f, g, h] + 1 = HS[f + 1, g + 1, h + 1]$$

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		0	0	0	0	0	0	0	0	0	0	0
	10	50	60	70	80	90	100	110	120	130	140	150
	9	36	45	54	63	72	81	90	99	108	117	126
	8	24	32	40	48	56	64	72	80	88	96	104
	7	14	21	28	35	42	49	56	63	70	77	84
	6	6	12	18	24	30	36	42	48	54	60	66
	5	0	5	10	15	20	25	30	35	40	45	50
	4	-4	0	4	8	12	16	20	24	28	32	36
	3	-6	-3	0	3	6	9	12	15	18	21	24
	2	-6	-4	-2	0	2	4	6	8	10	12	14
Y[1]	1	-4	-3	-2	-1	0	1	2	3	4	5	6
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	6	5	4	3	2	1	0	-1	-2	-3	-4
	-2	14	12	10	8	6	4	2	0	-2	-4	-6
	-3	24	21	18	15	12	9	6	3	0	-3	-6
	-4	36	32	28	24	20	16	12	8	4	0	-4
	-5	50	45	40	35	30	25	20	15	10	5	0
	-6	66	60	54	48	42	36	30	24	18	12	6
	-7	84	77	70	63	56	49	42	35	28	21	14
	-8	104	96	88	80	72	64	56	48	40	32	24
	-9	126	117	108	99	90	81	72	63	54	45	36
	-10	150	140	130	120	110	100	90	80	70	60	50

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		1	1	1	1	1	1	1	1	1	1	1
	10	51	61	71	81	91	101	111	121	131	141	151
	9	37	46	55	64	73	82	91	100	109	118	127
	8	25	33	41	49	57	65	73	81	89	97	105
	7	15	22	29	36	43	50	57	64	71	78	85
	6	7	13	19	25	31	37	43	49	55	61	67
	5	1	6	11	16	21	26	31	36	41	46	51
	4	-3	1	5	9	13	17	21	25	29	33	37
	3	-5	-2	1	4	7	10	13	16	19	22	25
	2	-5	-3	-1	1	3	5	7	9	11	13	15
Y[1]	1	-3	-2	-1	0	1	2	3	4	5	6	7
Y[0]	0	1	1	1	1	1	1	1	1	1	1	1
Y[-1]	-1	7	6	5	4	3	2	1	0	-1	-2	-3
	-2	15	13	11	9	7	5	3	1	-1	-3	-5
	-3	25	22	19	16	13	10	7	4	1	-2	-5
	-4	37	33	29	25	21	17	13	9	5	1	-3
	-5	51	46	41	36	31	26	21	16	11	6	1
	-6	67	61	55	49	43	37	31	25	19	13	7
	-7	85	78	71	64	57	50	43	36	29	22	15
	-8	105	97	89	81	73	65	57	49	41	33	25
	-9	127	118	109	100	91	82	73	64	55	46	37
	-10	151	141	131	121	111	101	91	81	71	61	51

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		2	2	2	2	2	2	2	2	2	2	2
	10	52	62	72	82	92	102	112	122	132	142	152
	9	38	47	56	65	74	83	92	101	110	119	128
	8	26	34	42	50	58	66	74	82	90	98	106
	7	16	23	30	37	44	51	58	65	72	79	86
	6	8	14	20	26	32	38	44	50	56	62	68
	5	2	7	12	17	22	27	32	37	42	47	52
	4	-2	2	6	10	14	18	22	26	30	34	38
	3	-4	-1	2	5	8	11	14	17	20	23	26
	2	-4	-2	0	2	4	6	8	10	12	14	16
Y[1]	1	-2	-1	0	1	2	3	4	5	6	7	8
Y[0]	0	2	2	2	2	2	2	2	2	2	2	2
Y[-1]	-1	8	7	6	5	4	3	2	1	0	-1	-2
	-2	16	14	12	10	8	6	4	2	0	-2	-4
	-3	26	23	20	17	14	11	8	5	2	-1	-4
	-4	38	34	30	26	22	18	14	10	6	2	-2
	-5	52	47	42	37	32	27	22	17	12	7	2
	-6	68	62	56	50	44	38	32	26	20	14	8
	-7	86	79	72	65	58	51	44	37	30	23	16
	-8	106	98	90	82	74	66	58	50	42	34	26
	-9	128	119	110	101	92	83	74	65	56	47	38
	-10	152	142	132	122	112	102	92	82	72	62	52

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		3	3	3	3	3	3	3	3	3	3	3
	10	53	63	73	83	93	103	113	123	133	143	153
	9	39	48	57	66	75	84	93	102	111	120	129
	8	27	35	43	51	59	67	75	83	91	99	107
	7	17	24	31	38	45	52	59	66	73	80	87
	6	9	15	21	27	33	39	45	51	57	63	69
	5	3	8	13	18	23	28	33	38	43	48	53
	4	-1	3	7	11	15	19	23	27	31	35	39
	3	-3	0	3	6	9	12	15	18	21	24	27
	2	-3	-1	1	3	5	7	9	11	13	15	17
Y[1]	1	-1	0	1	2	3	4	5	6	7	8	9
Y[0]	0	3	3	3	3	3	3	3	3	3	3	3
Y[-1]	-1	9	8	7	6	5	4	3	2	1	0	-1
	-2	17	15	13	11	9	7	5	3	1	-1	-3
	-3	27	24	21	18	15	12	9	6	3	0	-3
	-4	39	35	31	27	23	19	15	11	7	3	-1
	-5	53	48	43	38	33	28	23	18	13	8	3
	-6	69	63	57	51	45	39	33	27	21	15	9
	-7	87	80	73	66	59	52	45	38	31	24	17
	-8	107	99	91	83	75	67	59	51	43	35	27
	-9	129	120	111	102	93	84	75	66	57	48	39
	-10	153	143	133	123	113	103	93	83	73	63	53

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		4	4	4	4	4	4	4	4	4	4	4
	10	54	64	74	84	94	104	114	124	134	144	154
	9	40	49	58	67	76	85	94	103	112	121	130
	8	28	36	44	52	60	68	76	84	92	100	108
	7	18	25	32	39	46	53	60	67	74	81	88
	6	10	16	22	28	34	40	46	52	58	64	70
	5	4	9	14	19	24	29	34	39	44	49	54
	4	0	4	8	12	16	20	24	28	32	36	40
	3	-2	1	4	7	10	13	16	19	22	25	28
	2	-2	0	2	4	6	8	10	12	14	16	18
Y[1]	1	0	1	2	3	4	5	6	7	8	9	10
Y[0]	0	4	4	4	4	4	4	4	4	4	4	4
Y[-1]	-1	10	9	8	7	6	5	4	3	2	1	0
	-2	18	16	14	12	10	8	6	4	2	0	-2
	-3	28	25	22	19	16	13	10	7	4	1	-2
	-4	40	36	32	28	24	20	16	12	8	4	0
	-5	54	49	44	39	34	29	24	19	14	9	4
	-6	70	64	58	52	46	40	34	28	22	16	10
	-7	88	81	74	67	60	53	46	39	32	25	18
	-8	108	100	92	84	76	68	60	52	44	36	28
	-9	130	121	112	103	94	85	76	67	58	49	40
	-10	154	144	134	124	114	104	94	84	74	64	54

a		1	1	1	1	1	1	1	1	1	1	1
b		-5	-4	-3	-2	-1	0	1	2	3	4	5
c		5	5	5	5	5	5	5	5	5	5	5
	10	55	65	75	85	95	105	115	125	135	145	155
	9	41	50	59	68	77	86	95	104	113	122	131
	8	29	37	45	53	61	69	77	85	93	101	109
	7	19	26	33	40	47	54	61	68	75	82	89
	6	11	17	23	29	35	41	47	53	59	65	71
	5	5	10	15	20	25	30	35	40	45	50	55
	4	1	5	9	13	17	21	25	29	33	37	41
	3	-1	2	5	8	11	14	17	20	23	26	29
	2	-1	1	3	5	7	9	11	13	15	17	19
Y[1]	1	1	2	3	4	5	6	7	8	9	10	11
Y[0]	0	5	5	5	5	5	5	5	5	5	5	5
Y[-1]	-1	11	10	9	8	7	6	5	4	3	2	1
	-2	19	17	15	13	11	9	7	5	3	1	-1
	-3	29	26	23	20	17	14	11	8	5	2	-1
	-4	41	37	33	29	25	21	17	13	9	5	1
	-5	55	50	45	40	35	30	25	20	15	10	5
	-6	71	65	59	53	47	41	35	29	23	17	11
	-7	89	82	75	68	61	54	47	40	33	26	19
	-8	109	101	93	85	77	69	61	53	45	37	29
	-9	131	122	113	104	95	86	77	68	59	50	41
	-10	155	145	135	125	115	105	95	85	75	65	55

Figure 1. sequence $(HS[1,0,1], HS[2,1,2], HS[3,2,3], HS[4,3,4], HS[5,4,5], HS[6,5,6])$.

If we sequentially place all Hyperboctys on top of each other where each is parallel to the XY plane, then each section parallel to the YZ plane will be a Paraboctys. Paraboctys and hyperboctys are structures perpendicular to each other.

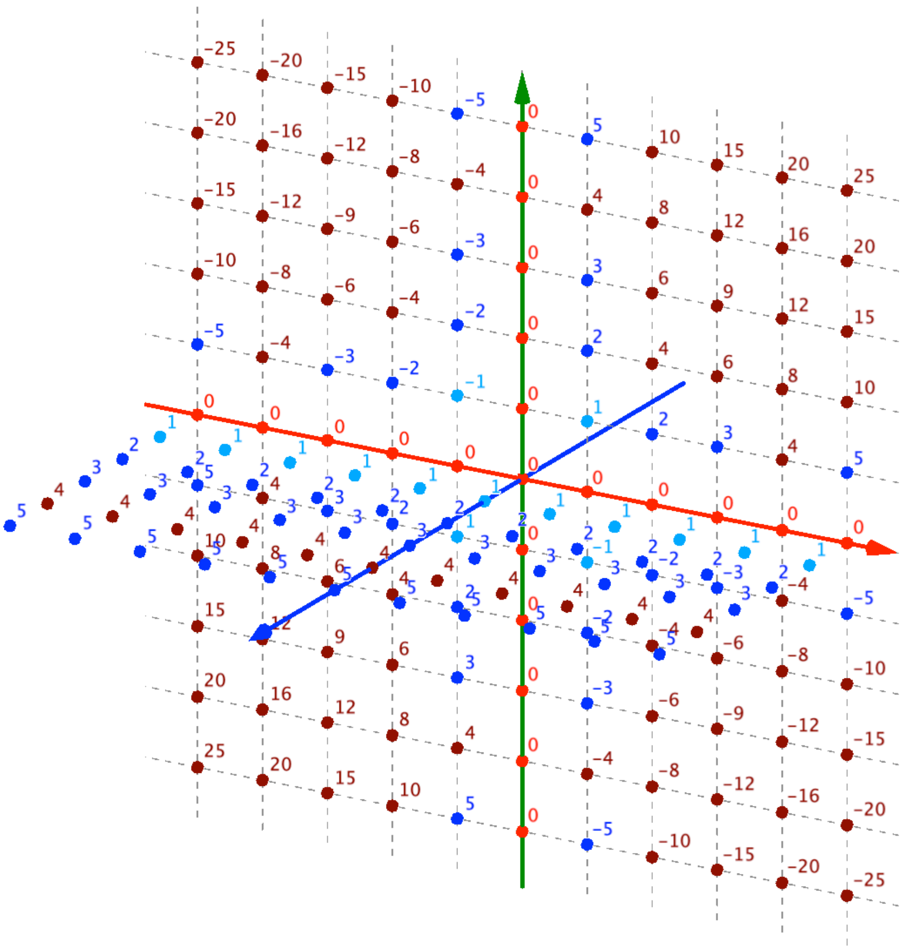

Figure 1. Idea in 3D of the sequence
$(HS[1,0,1], HS[2,1,2], HS[3,2,3], HS[4,3,4], HS[5,4,5], HS[6,5,6])$.

12 General view of all Hyperboctys

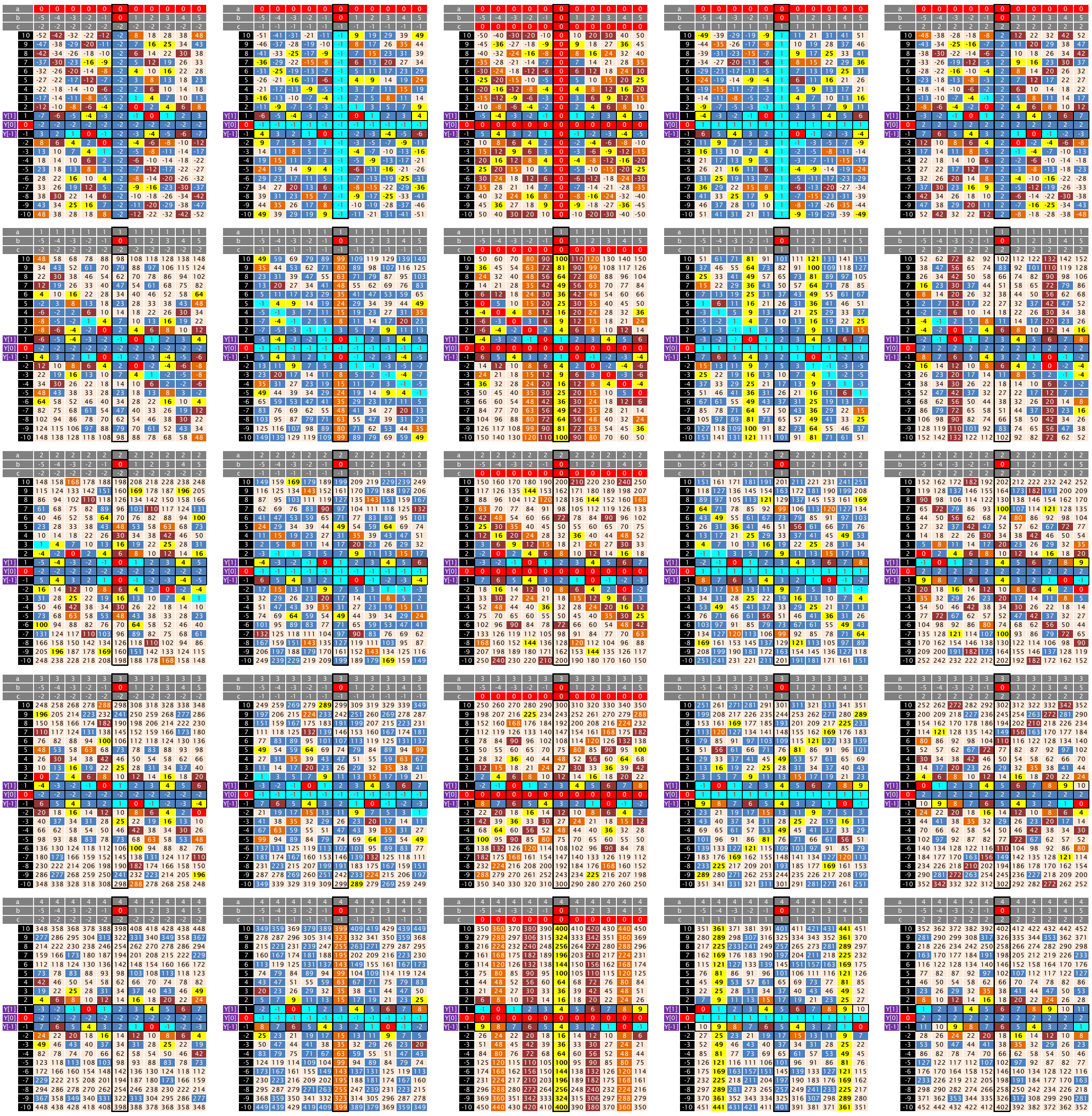

Figure 1. Overview of all possible hyperboctys generation in positive and negative sides.

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References

[1]*The On-Line Encyclopedia of Integer Sequences*, available online at http://oeis.org.

[2]Set of the simplest polynomial equations, inflection point equations, and the two possible recurrence equations, up to degree 6

[3]Offset in Quadratics

[4]Quadratics Classification

[5]The Hyperbolic Sieve of Primes and Products xy