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Abstract:
Liquid level control has been a continuous challenge in a considerable number of industrial processes, especially when one has to simultaneously satisfy multiple control performance specifications. This paper presents a comparative study between Particle Swarm Optimization and Gray Wolf Optimization as tuning methodologies for a flatness-based controller, which has proved to be a useful tool to provide efficient solutions to tracking control tasks. Numerical simulations of a nonlinear coupled tanks system were performed to demonstrate the potential of using such bio-inspired algorithms as tuning methods for nonlinear controllers.

Keywords: Liquid Level Control, Differential Flatness Theory, Particle Swarm Optimization, Gray Wolf Optimization, Nonlinear Coupled Tanks System.

1. INTRODUCTION

Liquid level control is commonly found in a broad range of industrial and commercial applications: oil and chemical processing, waste management, water purification systems, and nuclear power generation plants. Open-loop controllers and proportional-integral-derivative (PID) controllers are widely used in industrial liquid-level control applications with moderate performance requirements (Åström et al., 2002; Yang, 2020). The main limitations to the use of these control strategies are mainly due to (i) the presence of nonlinear dynamics inherent in the liquid level systems and (ii) system parameter variations (Joseph et al., 2022).

One way to address these issues is through the use of traditional nonlinear control techniques such as sliding mode control (Ardjal et al., 2022), back-stepping control (Pan et al., 2005), model predictive control (Scheurenberg et al., 2022), and fuzzy logic controller (Mien, 2017). Alternatively, differential flatness theory (Fliess et al., 1992; Rigatos, 2015) has received considerable attention from some researchers in the last decade to provide efficient solutions to the nonlinear liquid level control problem (Michaud and Robert, 2010; Huang and Sira-Ramírez, 2015; Minh and Tan, 2021). However, the controller gain tuning problem still presents significant theoretical challenges in order to handle multiple control design requirements.

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To overcome the tuning problem, some bio-inspired algorithms have emerged over time. Among them, two algorithms that have shown good results are Gray Wolf Optimization (GWO) (Das et al., 2015) and Particle Swarm Optimization (PSO) (Bouallègue et al., 2012). GWO adopts the hierarchical nature of gray wolves and lists the best solution as alpha, followed by beta and delta in descending order. Additionally, its hunting technique of tracking, encircling, and attacking are also modeled mathematically to find the best-optimized solution (Hatta et al., 2019). PSO is inspired by the information circulation and social behavior observed in bird flocks and fish schools. Such inspirations translate that the particles are characterized not only by their position but also by their speed (Marini and Walczak, 2015). These two bio-inspired algorithms were already used in different engineering fields: modeling bones numerically (Sen et al., 2023), mineral exploration (Essa and Munschy, 2019), improvement of energy efficiency (Djerioui et al., 2019), and prediction of global solar radiation Tao et al. (2021).

In this paper, we propose to use both GWO and PSO as tools to tune the gains of the flatness-based controller for a nonlinear coupled tanks system, where the first is cylindrical and the second is conical. Besides ensuring the convergence of the system output to the desired setpoint, the main idea is to implement these algorithms to determine the controller gains that simultaneously satisfy multiple control design objectives, such as mitigating overshoot while reducing the rise time for the output of the system.

2. COUPLED TANKS SYSTEM

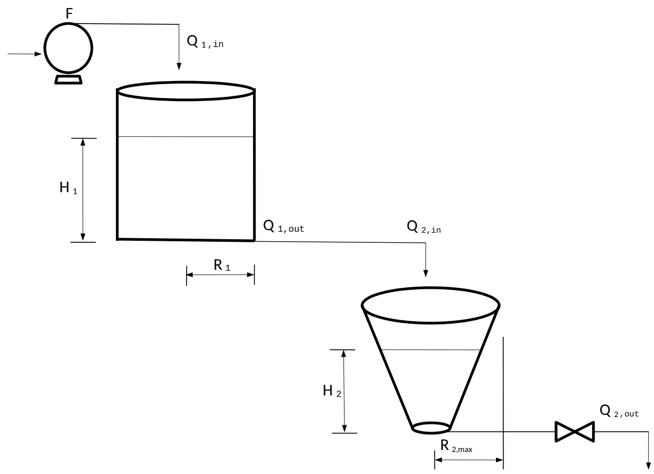

Figure 1. Schematic diagram of a coupled tanks system.

Coupled tanks represent an important plant model to be studied as it acts as the basic liquid level control problem occurring in process industries and it is a well-known benchmark problem for testing and analyzing the tracking performance of designed controllers. The mathematical model of the nonlinear coupled tanks system considered in this paper consists of three main components in cascade (see Figure 1): a centrifugal pump, a cylindrical tank, and a conical tank.

The cylindrical tank has a constant circular cross-sectional area $A_{1,in}$ with a radius of R_1, a maximum height $H_{1,max}$ and is fed by the centrifugal pump at a flow rate $Q_{1,in}$ given according to the following affinity law (Takacs, 2017):

$$Q_{1,in} = Q_{nom} \frac{F}{F_{nom}} \tag{1}$$

with F being the power supply frequency of the pump motor in Hz, and the pair (Q_{nom}, F_{nom}) representing nominal flow rate and frequency of the pump, respectively, whose values are normally provided by the pump manufacturers.

From Bernoulli's equation for steady, non-viscous, not compressible flow, the flow rate exiting the tank $Q_{1,out}$ can be expressed by:

$$Q_{1,out} = \rho A_{1,out} \sqrt{2gH_1} \tag{2}$$

where ρ, g and H_1 represent the fluid density, the gravitational constant, and the water level in tank 1, respectively. In addition, the cross-sectional area of the pipe from tank 1 to tank 2 is given by $A_{1,out}$.

The mass balance principle of the liquid level in tank 1, which is defined as the tank's inflow and outflow difference, can be written as the following first-order differential equation:

$$\rho \dot{H}_1 A_{1,in} = Q_{1,in} - Q_{1,out} \tag{3}$$

Substituting (1) and (2) in (3), we arrive at the differential equation of the cylindrical tank:

$$\dot{H}_1 = C_1 F - C_2 \sqrt{H_1} \tag{4}$$

where the constant parameters C_1 and C_2 are calculated as follows:

$$C_1 = \frac{Q_{nom}}{F_{nom} \rho A_{1,in}} \tag{5}$$

$$C_2 = \frac{A_{1,out}\sqrt{2g}}{A_{1,in}} \tag{6}$$

For the conical tank, its cross-sectional area $A_{2,in}$ is a function that varies according to its water level H_2, which is represented mathematically as:

$$A_{2,in} = \pi \left(\frac{H_2 R_{2,max}}{H_{2,max}} \right)^2 \tag{7}$$

where $R_{2,max}$ and $H_{2,max}$ represent the maximum radius and height of tank 2, respectively.

In a similar way to the centrifugal tank, the mass balance principle of the liquid level in a conical tank can be written as the following first-order differential equation:

$$\rho \dot{H}_2 A_{2,in} = Q_{2,in} - Q_{2,out} \tag{8}$$

with the input flow rate $Q_{2,in} = Q_{1,out}$ and the flow rate exiting the tank $Q_{2,out}$ given by:

$$Q_{2,out} = \rho A_{2,out} \sqrt{2gH_2} \tag{9}$$

where $A_{2,out}$ is the outlet area of tank 2.

Substituting (2), (7) and (9) in (8), we arrive at the differential equation of the conical tank:

$$\dot{H}_2 = C_3 \frac{\sqrt{H_1}}{H_2^2} - C_4 \frac{1}{H_2^{3/2}} \tag{10}$$

where the constant parameters C_3 and C_4 are calculated as follows:

$$C_3 = \frac{A_{1,out}\sqrt{2g}}{\pi} \left(\frac{H_{2,max}}{R_{2,max}} \right)^2 \tag{11}$$

$$C_4 = \frac{A_{2,out}\sqrt{2g}}{\pi} \left(\frac{H_{2,max}}{R_{2,max}} \right)^2 \tag{12}$$

3. DESIGN OF FLATNESS-BASED CONTROLLER

Consider the input-affine nonlinear MIMO system in the state space model of the form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u = f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_i &= h_i(x), \qquad i = 1, \cdots, m \end{aligned} \tag{13}$$

where $x \in \mathbb{R}^n$, u_i, $y_i \in \mathbb{R}$, f and g_1, $\cdots, g_m$ are smooth vector fields, and h_i are smooth functions.

Roughly speaking, a dynamic system is called differentially flat if there exists a vector $y_z \in \mathbb{R}^m$ as below:

$$y_z = \phi_z(x, u, \dot{u}, \cdots, u^{(l)}), \tag{14}$$

with the state and input expressions parameterized as follows:

$$x = \phi_x(y_z, \dot{y}_z, \cdots, y_z^{(q)}) \tag{15}$$

$$u = \phi_u(y_z, \dot{y}_z, \cdots, y_z^{(q)}, y_z^{(q+1)}) \tag{16}$$

where ϕ_z, ϕ_x and ϕ_u are smooth vector functions. The vector y_z indicates the flat outputs of the system with l and q being finite numbers. In the particular case that y_z are exclusively functions of the state vector, we say that the system is x-flat.

Defining now $(x_1, x_2, x_3, x_4)^T = (H_1, \dot{H}_1, H_2, \dot{H}_2)^T$, the nonlinear coupled tanks system can be represented by the following continuous state-space representation from (4) and (10):

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= C_1 F - C_2\sqrt{x_1} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= C_3\sqrt{x_1} - C_4 \dfrac{1}{x_3^{1.5}} \end{cases}, \qquad y = x_3 \tag{17}$$

The aforementioned model is found to be differentially flat with the flat output represented by the water level of the conical tank (i.e., $y_z = x_3$). As a result, all system variables can be written as differential functions of y_z:

$$x_1 = \frac{y_z(C_4 + y_z^{1.5}\dot{y}_z)^2}{C_3^2} \tag{18}$$

$$x_2 = \dot{x}_1 \tag{19}$$

$$x_3 = y_z \tag{20}$$

$$x_4 = \dot{y}_z \tag{21}$$

$$F = \frac{(\dot{x}_1 + \sqrt{x_1}C_2)}{C_1} \tag{22}$$

By manipulating (22), it follows from the differential flatness property of (17) that the system can be transformed into a linear system in the Brunovsky canonical form as follows:

$$\ddot{y}_z = \lambda_f(y_z,\dot{y}_z) + \lambda_g(y_z,\dot{y}_z)F = \upsilon \tag{23}$$

where λ_f and λ_g are smooth functions, and υ represents the new control variable.

For the previous description, a suitable feedback control law can be defined as follows:

$$\upsilon = \ddot{y}_z^* - k_1\left(\dot{y}_z - \dot{y}_z^*\right) - k_0\left(y_z - y_z^*\right) \tag{24}$$

where the desired water level y_z^* and its time derivatives $\dot{y}_z^*$ and $\ddot{y}_z^*$ are assumed to be known, and the controller gains k_i are chosen such that $p(s) = s^2 + k_1 s + k_0 s$ is a Routh-Hurwitz polynomial.

Therefore, from (23), the control input that is actually exerted on (17) is given by:

$$F = \left[\upsilon - \lambda_f(y_z,\dot{y}_z)\right]\lambda_g^{-1}(y_z,\dot{y}_z) \tag{25}$$

4. BIO-INSPIRED ALGORITHMS

4.1 Gray Wolf Optimizer

As introduced in Mirjalili et al. (2014), Gray Wolf Optimizer is a population-based meta-heuristic optimization algorithm inspired by the social behavior of gray wolves and has been successfully applied to a wide range of optimization problems (Faris et al., 2018; Negi et al., 2021).

The basic idea behind the GWO algorithm is to simulate the hunting behavior of gray wolves in order to find the optimal solution to an optimization problem. In a pack of gray wolves, there are typically four types of wolves: alpha, beta, delta, and omega. The alpha wolf is the pack's leader and is responsible for making most of the decisions. The beta wolf is the second-in-command and helps the alpha make decisions. The delta and omega wolves are lower-ranking members of the pack and typically follow the alpha and beta.

The encircling behavior, in a mathematical representation, of a prey during the hunt can be represented by the following equations:

$$\vec{X}(t+1) = \vec{X_p} - \vec{A}\cdot\left|\vec{C}\cdot\vec{X_p}(t) - \vec{X}(t)\right| \tag{26}$$

where t represents the current iteration, $\vec{A}$ and $\vec{C}$ are coefficient vectors, $\vec{X_p}$ and $\vec{X}$ indicate the position vector of a prey and a gray wolf, respectively. The vectors $\vec{A}$ and $\vec{C}$ are given by:

$$\vec{A} = 2\vec{a}\cdot\vec{r_1} - \vec{a} \tag{27}$$

$$\vec{C} = 2\vec{r_2} \tag{28}$$

with the components of $\vec{a}$ decreasing linearly from 2 to 0 during the iterations. In addition, the vectors $\vec{r_1}$ and $\vec{r_2}$ are randomly generated in the range of 0 to 1.

When the gray wolves are encircling their prey, it is assumed that the alpha (the best candidate solution), beta, and delta have superior knowledge of the potential location of prey. Thus, the first three best solutions obtained are stored and the remaining search agents, are required to update their positions based on the position of the best search agent. The following equations are proposed to represent this behavior.

$$\vec{X_1} = \vec{X_\alpha} - \vec{A_1}\cdot\left|\vec{C_1}\cdot\vec{X_\alpha} - \vec{X}\right| \tag{29}$$

$$\vec{X_2} = \vec{X_\beta} - \vec{A_2}\cdot\left|\vec{C_2}\cdot\vec{X_\beta} - \vec{X}\right| \tag{30}$$

$$\vec{X_3} = \vec{X_\delta} - \vec{A_3}\cdot\left|\vec{C_3}\cdot\vec{X_\delta} - \vec{X}\right| \tag{31}$$

The fitness of each wolf is calculated based on its position (which is the vector of variables that are being optimized). The algorithm updates the wolves' position accordingly with the best values of the fitness function. The algorithm terminates when a stopping criterion is met (e.g., a maximum number of iterations is reached or the fitness of the best wolf is below a certain threshold). Lastly, the best value found by the algorithm is given by:

$$\vec{X}(t+1) = \frac{\vec{X_1} + \vec{X_2} + \vec{X_3}}{3} \tag{32}$$

4.2 Particle Swarm Optimization

Particle swarm optimization is a population-based stochastic algorithm motivated by the intelligent collective behavior of some animals and evolutionary theories, such as flocks of birds and schools of fish, (Eberhart and James Kennedy, 2010; Wang et al., 2018). These authors suggested the PSO algorithm as a way to optimize continuous non-linear functions. Another PSO method advantage is its computational efficiency and cost-effectiveness, as it does not require complex operations (Eberhart and James Kennedy, 1999).

In the PSO approach, each candidate solution is called a "particle" and represents a point in a N-dimensional space with N being the number of parameters to be optimized. The population of S candidate solutions constitutes the swarm, which can be represented by:

$$X = \{x_1, x_2, \cdots, x_S\} \tag{33}$$

Considering the N-dimensional position for the i^{th} particle of the swarm described by the vector x_i:

$$x_i = [x_{i1}, x_{i2}, \cdots, x_{iD}] \tag{34}$$

In searching for the problem's optimal solution, the new position for the i^{th} particle is evaluated at each iteration based on the following equation of motion (Marini and Walczak, 2015):

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{35}$$

where t and $t+1$ indicate two successive iterations used in the algorithm and v_{ij} represents the velocity vector for the i^{th} particle.

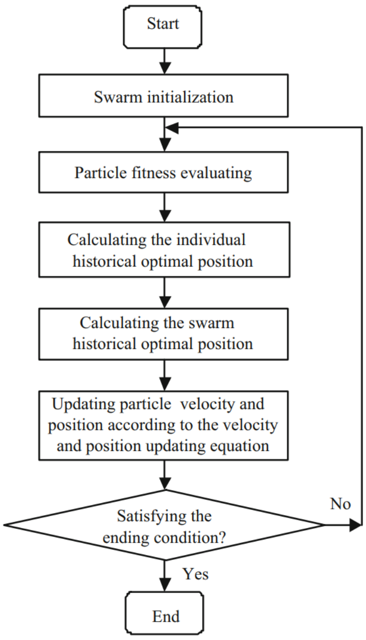

Figure 2. Particle Swarm Optimization algorithm flowchart, exposed in Wang et al. (2018).

The velocity vectors govern the way particles move across the search space and can be defined for the i^{th} particle as:

$$v_i(t+1) = w(t)v_i(t) + c_1U_1\left(p_{best,i} - x_i(t)\right) + c_2U_2\left(g_{best} - x_i(t)\right) \quad (36)$$

where the pair $\left(p_{best,i}, g_{best}\right)$ represents the coordinates of the best solution obtained so far by the ith particle and the overall best solution obtained by the swarm, respectively. The constant parameters c_1 and c_2 are called cognitive and social coefficients, respectively, and modulate the magnitude of the steps taken by the particle in the direction of its personal best and global best. The random factors U_1 and U_2 are two diagonal matrices of uniformly distributed random numbers in range $[0,1]$, so that both the influence of c_1 and c_2 has a stochastic influence on updating the ith particle's velocity. The inertia factor w play the role of balancing the global and local searches.

Similar to the GWO method, the PSO iterative process is repeated until a stopping criterion is met, as can be seen in Figure 2.

4.3 Fitness Function

The fitness function is a mathematical function used to evaluate the solution's quality or the candidate solution inside the problem search space. The process selects a candidate solution as the input, producing a value as the output representing the solution's quality or fitness. The algorithm goal is to find the solution with the lowest fitness value, being the best solution to the problem.

The fitness function proposed in this paper is based on the Integral of Time Squared Error (ITSE) index for preventing the overshooting of H_1 and H_2 (Carrasco and Salgado, 2009):

$$J = \int_{t_0}^{t_f} \left|\alpha_1\left(H_1(\tau) - H_1^*(\tau)\right) + \alpha_2\left(H_2(\tau) - H_2^*(\tau)\right)\right|^2 \tau\, d\tau \quad (37)$$

where the weights α_1 and α_2 can be used to prioritize the settling time of H_2 over H_1 or vice-versa.

Lastly, for both GWO and PSO, the aforementioned fitness function is used to find the flatness-based controller gains k_0 and k_1. The results are compared and discussed in Section 5.

5. NUMERICAL SIMULATIONS

In this section, a numerical study was conducted for the nonlinear coupled tanks system with parameters listed in Table 1 to illustrate the effectiveness of GWO and PSO as tuning methods for the flatness-based controller proposed in Section 3.

In the simulation tests, the control signals saturate at a frequency of 10 Hz and 90 Hz. The initial liquid level in all tanks is assumed to be 0.25 m. The desired output setpoint h_2^* is designed to be 0.75 m, which results in $h_1^* = 0.42$ m and $F^* = 20.30$ Hz. The simulation time lasts for 25 s with a sampling time of 0.01 s.

For both optimization methods, the following parameters were chosen from the authors' experience: the number of iterations and the size of the population are equal to 100 and 30, respectively; the weights $\alpha_1 = 1.5$ and $\alpha_2 = 3.0$ in order to prioritize the settling time of H_2 over H_1; the search space was restricted to $[0.04, 0.81]$ for k_0 and $[0.40, 1.80]$ for k_1. In addition, the remaining PSO parameter values were defined as c_1 and c_2 are both 2.05; w was gradually decreased from 0.9 towards 0.2.

The controller gains found that minimizing the fitness function proposed in (37) using each tuning method can be seen in Table 2. Simulation results for the closed-loop system are illustrated in Figure 3.

As can be seen in the figure, the control scheme successfully drives all variables to their desired setpoints in finite time. We also observe that the level of the cylindrical tank did not reach its maximum value, and the level of the conical tank did not exhibit overshoot with a settling time of approximately 5 seconds for PSO and 7 seconds for GWO.

Lastly, the convergences of the best fitness evaluation for both optimization algorithms are plotted in Figure 4 with respect to their number of iterations used. After the 34^{th} iteration, the GWO achieved a minimum value for the fitness function and remained stable. In the second one, the same behavior can be observed for the PSO, but from the 65^{th} iteration.

Table 1. Coupled tanks system parameters.

Parameter	Value
Q_{nom}	16.67 Kg/s
F_{nom}	60 Hz
ρ	998.23 Kg/m^3
g	9.81 m/s^2
$H_{1,max}$	1.0 m
$H_{2,max}$	1.0 m
$R_{2,max}$	0.176 m
$A_{1,in}$	0.0146 m^2
$A_{1,out}$	0.0020 m^2
$A_{2,out}$	0.0015 m^2

Table 2. Controller gains tuned by the GWO and PSO algorithms.

Controller gain	GWO	PSO
k_0	0.5258	1.6488
k_1	1.7882	1.4767

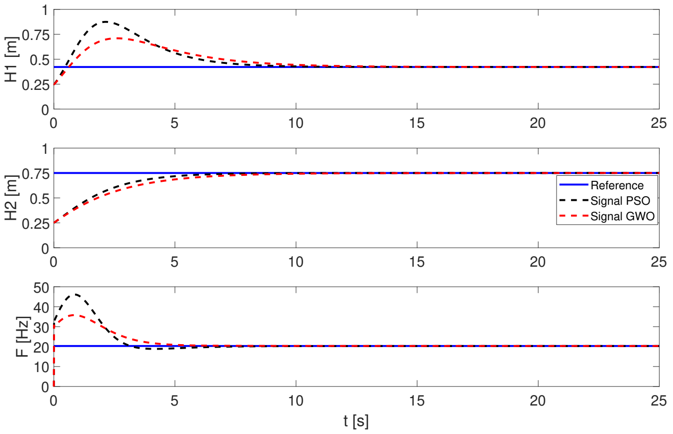

Figure 3. Response of the system with gains tuned by the GWO and PSO algorithm.

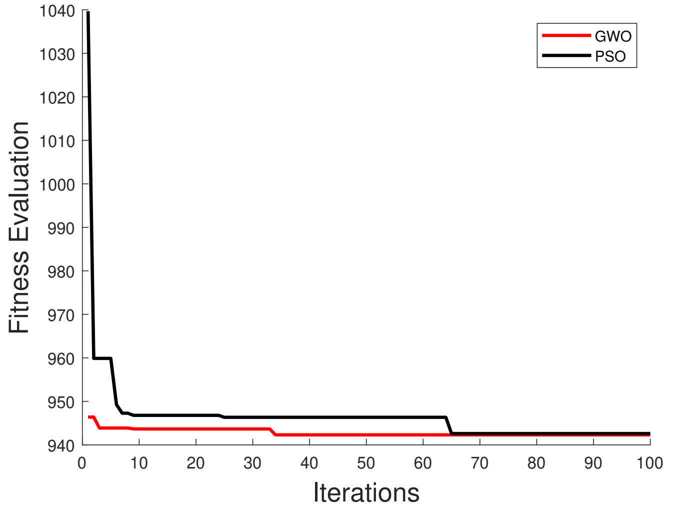

Figure 4. GWO and PSO convergence of the best fitness evaluation in all iterations.

6. CONCLUSION

In this paper, we have investigated the use of two bio-inspired algorithms as tuning methods for a flatness-based controller. It has been shown through simulation findings that both techniques ensure that multiple control design objectives are met simultaneously in order to solve a liquid level control problem for a nonlinear coupled tanks system. Although both methods achieved the control requirements, the GWO managed to find a better result for the fitness function with fewer iterations than the PSO. In contrast, the PSO converged to the H_2 reference slightly faster than GWO.

Further analyses will include the possibility of dealing with differentially flat nonlinear systems with multiple-input multiple-output whose flat output vector is not a measurable variable. Future works will be concerned with extending our research to be applied to cases where the system plant for the control design is uncertain, but the simulation model is close to reality. For example, the oil reservoir forecasting problem without model calibration using a recent perspective based on the sequential model aggregation technique.

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