Indeterminism in Quantum Mechanics: Beyond and/or Within Causation

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Abstract. *The problem of indeterminism in quantum mechanics usually being considered as a generalization determinism of classical mechanics and physics for the case of discrete (quantum) changes is interpreted as an only mathematical problem referring to the relation of a set of independent choices to a well-ordered series therefore regulated by the equivalence of the axiom of choice and the well-ordering "theorem". The former corresponds to quantum indeterminism, and the latter, to classical determinism. No other premises (besides the above only mathematical equivalence) are necessary to explain how the probabilistic causation of quantum mechanics refers to the unambiguous determinism of classical physics. The same equivalence underlies the mathematical formalism of quantum mechanics. It merged the well-ordered components of the vectors of Heisenberg's matrix mechanics and the non-ordered members of the wave functions of Schrödinger's undulatory mechanics. The mathematical condition of that merging is just the equivalence of the axiom of choice and the well-ordering theorem implying in turn Max Born's probabilistic interpretation of quantum mechanics. Particularly, energy conservation is justified differently than classical physics. It is due to the equivalence at issue rather than to the principle of least action. One may involve two forms of energy conservation corresponding whether to the smooth changes of classical physics or to the discrete changes of quantum mechanics. Further both kinds of changes can be equated to each other under the unified energy conservation as well as the conditions for the violation of energy conservation to be investigated therefore directing to a certain generalization of energy conservation.*

Key words: *causation, choice and well ordering, determinism, Hilbert space of quantum mechanics, indeterminism, probabilistic causation*

Prehistory, background, and context:

Indeterminism is one of both most striking and most fundamental features of quantum mechanics therefore challenging or generalizing even the idea of exact and experimental science. Any single result of quantum measurement is fundamentally random. The smooth laws of classical physics describing the apparatus and its readings can be unified with the discrete quantum changes of any quantum entities only at this cost.

Though any single measurement is fundamentally random, any statistical ensemble being big enough obeys classical causation. Thus one can suggest that the randomness of quantum mechanics is only seeming, and in fact, it is only generalized referring to the whole rather than to a single element or its measurement.

That kind of randomness (if that was the case) might remember the "randomness" of classical statistical thermodynamics. Its "randomness" is actually seeming as far as the variable quantities of any single element of the statistical ensemble are conditioned deterministically and causally by the laws of classical mechanics, which any element of those obeys. The randomness is seeming or due to the fact that the choice of a single element is random. Anyway, exact hidden variables absolutely determining the motion of any element exist, and they are only omitted in the statistical thermodynamic description since it refers to the ordinal of the statistical ensemble at issue rather than to any certain well-ordering belonging to the class of equivalence definitive for the ordinal meant in statistical thermodynamics.

One might coin the term "incomplete" as to any statistical and thermodynamic description in an absolutely rigorous meaning: if the ordinal be complemented by the relevant information determining just one well-ordering among the class of that ordinal. That information can be supplied by classical mechanics always. However, it does not make sense to the thermodynamic behavior of the whole depending only on the ordinal rather than to which exact well-ordering belonging to the ordinal happens.

Metaphor and analogy from the classical statistical thermodynamics to quantum mechanics is well-known as the conjecture of hidden variables in quantum mechanics and supported by scientist as Albert Einstein who did not accept the indeterminism of quantum mechanics as contradicting science at all. However many experiments confirm that the hypothesis of "hidden variables" is false. Quantum mechanics (as far as its mathematical formalism of the separable complex Hilbert space is relevant) does not admit those "hidden variables" in principle:

In fact, their availability contradicts the equivalence of the smooth description in terms and readings of the apparatus, on the one hand, and the discrete description of any quantum entities, on the other hands. In other words, the objectivity of quantum mechanics as an experimental science contradicts any "hidden variables", and accordingly, implies quantum indeterminism.

Even, the absence of hidden variables in quantum mechanics, and respectively, its indeterminism are rooted much deeper, in the foundations of mathematics, properly in the equivalence of the axiom of choice and the well-ordering "theorem". Thus, a model of quantum indeterminism in terms only of set theory is possible.

The link between the "completeness of quantum mechanics" (versus Einstein, Podolsky, and Rosen's "incompleteness of quantum mechanics") and that equivalence in set theory can be traced as follows:

The equivalence equates a non-ordered ensemble of possible choices of any single element of an infinite set in virtue of the axiom of choice, on the one hand, and the well-ordering of all elements of the set, on the other hand. One can interpret the former as a coherent quantum state, and the latter, as the well-ordered series of its measurements. Thus, that equivalency is interpretable as the absence of hidden variables and implies it. As far as quantum indeterminism is identifiable as the "completeness of quantum mechanics" (i.e. the absence of hidden variables), the unity of choice and well-ordering implies quantum indeterminism as well.

The thesis is:

Quantum mechanics involves a special kind of scientific and even mathematical and rigorous explanation, which is an extension of causality equating determinism to indeterminism in quantum mechanics. The most investigations of causation in quantum mechanics state only about the probabilistic causality in quantum mechanics as an extension of causal determinism of classical physics, but nothing about the special way for the equivalence between the classical and probabilistic causation in quantum mechanics to be established and proved.

The proof is divided into two independent parts:

(1) The indeterminism in quantum mechanics is beyond the classical determinism

(2) The indeterminism in quantum mechanics is partly equivalent to that in classical physics

The latter viewpoint (i.e. that of quantum mechanics discussing quantum indeterminism as a partial equivalent of classical causation) is rather fruitful: It generated in final analysis a new theory, that of quantum information, studying the experimental phenomena of entanglement.

A sketch of the proof:

Preliminary notes: That proof needs a rigorous formalization of the concepts of 'determinism' and 'quantum indeterminism'. The same terms will be used below for the formal definitions while 'causation' and 'causality' both 'classical' and 'quantum' will be not defined allowing the free plurality of uses. The 'determinism' should be defined both in terms of set theory and logic and in those of physics and time.

The same is valid to the 'quantum indeterminism'.

'Determinism' is any choice of a well-ordered explanation representing both a well-ordered set and a well-founded series: a "chain" of premises and sequences with a beginning. There is always a one-to-one mapping between the well-ordered set, the well-founded series and a physical sequence of causes and effects, which are also well-ordered for the "arrow" of time. Consequently, the choice of a well-ordering is the core of the definition though that well-ordering can be interpreted as a series of generalizing explanations as a chain of premises and sequences as well as a temporal "arrow" of causes and effects.

'Quantum indeterminism' is any collection, which does not allow any well-ordering in principle.

Analogically it can be interpreted equally well as a set or as a kind of explanation or as a coherent state in quantum mechanics. The theorems about the absence of hidden variables in quantum mechanics (Neumann 1932; Kochen, Specker 1968) imply the absence of any well-ordering in any coherent state in any quantum system before measurement. Consequently, the availability/ absence of well-ordering is the demarcation correspondingly between 'determinism' and 'quantum indeterminism'.

(1) The same coherent state (at issue in the definition of 'quantum indeterminism) is transformed after measurement into an equivalent statistical ensemble by the mediation of a well ordering in time as follows:

a) The coherent state is measured giving a determine, but random value of the measured quantity at a moment of time

b) The coherent state is measured again giving another (in principle), determine, but random value of the same quantity at a following moment of time

c) ... again and again the same, theoretically to infinity...

d) Thus the coherent state is transformed by a (theoretically infinite) series of measurements into an equivalent series well-ordered in time

e) The coherent state is represented as the mix of all measured values after a probability is assigned to each value as the ratio (or as the limit of the series of ratios) of the cases of this value to all cases

f) That mix equivalent to the coherent state is a statistical ensemble as the entire result of the measurement of this quantity corresponding to this coherent state

This scheme cannot be described within the 'determinism' formalized above as one and the same cause causes a plurality of effects, each of which is with a different probability. The interpretation of quantum indeterminism as probabilistic causation describes it as that generalization of classical causation, after which a set of possible values can be caused by a probability exactly determined to each of them.

However, a few additional conclusions can be made on the above scheme:

(2) A necessary condition for the coherent state unorderable in principle and its equivalent (d) well-ordered in time to be equated is the well-ordering theorem equivalent to the axiom of choice: There is no way for a statistical ensemble to be obtained from any coherent state otherwise than by the axiom of choice. This implies a well-ordering to be chosen in the description of the coherent state though only the "pure" existence of a well-ordering can be state for it is obtained just by the axiom of choice. Even more, the theorems about the absence of hidden variables imply no constructive way for that way-ordering to be built. In other words, the statistical ensemble is equivalent to a forever unknown well-ordering of the coherent state and thus 'quantum indeterminism' should be equated to 'determinism' always existing for any coherent state, but forever unknown as a principle.

The relation between the statistical ensemble and the single and unknown well-ordering is the relation between an ordinal defined correspondingly in Cantor (1897) – Russell (Russell, Whitehead any edition) and Neumann (1923). The Cantor – Russell definition is admissible as the ordinals are small: "ω" is an enough limit. The ordinal defined in Neumann should be interpreted as a representative of 'determinism' for any statistical ensemble corresponding one-to-one to an ordinal defined in Cantor – Russell. However, this representative exists only "purely" for it is a mapping of a coherent state necessarily requiring the axiom of choice.

The main conclusion:

Quantum indeterminism is rooted very deeply, in the foundations of mathematics, especially, in the equivalence of the well-ordering "theorem" and the axiom of choice. That ground in the tradition and history of physics is well-known as the unambiguous link of symmetries and groups, which share the same most fundamental link of an ordinal and the corresponding class of equivalence of well-orderings therefore shared by any quantum state relevant to a certain continuous (smooth) state of the measuring apparatus.

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