Induced Gravity in the Space of Momentum

Vitaly Kuyukov

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sfubms@gmail.com

General relativity is an excellent model of gravity, but the theory is classical, not described by a quantum theory. There are many models of quantum gravity, they can be divided into background and independent from the background of the theory.

In this paper, we consider the canonical approach to quantum gravity. But a completely different geometry and the concept of space is used.

Instead of real space, consider the geometry of the fields in the momentum space. For this, the coordinates are considered by operators who are set in the space of momentum.

$$\{x_\beta, k_\beta\} = i$$

$$\delta x \, \delta k = 1$$

Note that this vector the momentum is similar to the metric space. Thus, this definition is an independent about the background. Next, can use a differential geometry to analyze the space of this momentum.

Consider the metric of space the 4-momentum for vacuum

$$dk^2 = g_{\alpha\beta} dk_\alpha dk_\beta$$

Metric has spatial components, that is, it describes the three-dimensional geometry of this the space.

Consider the definition of volume the space of momentum for fluctuation the vacuum

$$dV_k = dk_x dk_y dk_z$$

If there is time, we use a 4-dimensional volume for this geometry in the form of

$$d\Omega = dt \, dV_k$$

The concept of tensor the density the energy is defined as the energy matter ratio to the volume of momentum the vacuum.

$$dM \, c^2 = T_{00} \, dV_k$$

$$T = g^{\alpha\beta} \, T_{\alpha\beta}$$

These definitions need to form a quantum geometry.

We make certain steps, consider combinations of fundamental constants, we find the desired combination that corresponds this the density of energy

$$b = Gh/c^2 = c \, L^2$$

Quantum geometry is described by a wave function, which is considered on the space metric.

We formulate a hypothesis. It is possible that there is a Schrödinger equation for this geometry, then a quantum evolution and description of the metric of the space momentum will be considered in the form of

$$\delta\Psi/\delta\Omega = - b \, \delta^2\Psi/\delta g^2$$

As can be seen, the evolution of the wave function occurs along the specified 4-dimensional volume and depends on the state of the metric of the field space.

In general, this equation has only an mathematum definition. It can describe the quantum dynamics of the field metric in terms of wave function and evolution in general form.

However, the optimism is exist, perhaps this structure has a geometric formulation of the quantum field theory in form the equation Wheeler-de Witt.

It is convenient to consider the fluctuation metric in the form fluctuation in the space of momentum for vacuum

$$r \, \delta\kappa = 1$$

$$\delta g = -G \, M \, |k - k_0|$$

Free vacuum has yet has a free the space of momentum in the form the state the Euclidean geometry.

For further study of this model, it is necessary to consider the differential analysis in the form of Riemann geometry

$$\Delta = \delta^{\beta}\delta_{\beta} = \delta^2/\delta k^2$$

$$R_{\alpha\beta} \sim \Delta g_{\alpha\beta}$$

Here, differential operations occur in the space of momentum.

Special attention should be done for the square of the curvature

$$R^2 = R_{\alpha\beta}R^{\alpha\beta} - \delta_{\alpha}\delta_{\beta}R^{\alpha\beta}$$

For such the field geometry, can find a general action. In determining and normalizing the constant, the action has a physical meaning only for a square of curvature

$$dS = \sigma \, R^2 \, d\Omega$$

$$\sigma = c^4/16\pi G$$

Thus, a mathematum description of the metric and curvature of the space of momentum in the form of a variational method is obtained.

This allows to find a scalar tensor matter as a scalar quadratic curvature in this model

$$R^2 = 8\pi G \, T$$

It is interesting to note that this definition is almost equivalent to the equation Einstein for ordinary real space.

In our model, a geometry the space of momentum for vacuum is used, which is independent of the background, in this sense the real space is a secondary concept.

It can be noted that the curvature of the space vacuum is proportional to the surface area. Perhaps our model can describe the statistical behavior of the field geometry. For instance information for horizon the black hole

$$\Phi = F/4L^2$$

$$\Phi = \ln p$$

This indicates that the quantum field theory can be formulated by geometrical form, in particular on the surface.

In this case, only a small part of this model was considered. In the next paper, we will make a deeper review and consequences for geometry, gravity, and even problem the time. In general, further research will show which geometric shape of the world will be expressed in the general model.

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