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Mathematical Modelling of Semiconductor Devices and circuits: A Review

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Abstract:
The objective of the article is to prepare a lucid mathematical model of circuits containing semiconductor devices (MOSFET & BJT) to analyse the circuits with much of ease. The mathematical modelling, using partitioning method of matrix, can be used to analyse the complicated circuits of amplifiers.
Key Words: Mathematical Modelling, BJT, MOSFET, Op amp, Floating Admittance Matrix.

1. Introduction:

Mathematical modelling should form the basis of any educational system, especially, the engineering education, because it indicates the proper place with suitable tools to achieve the desired result for a set of given input variables. Thus, it plays very big role in the engineering education considering the views of all stake holders to improve the quality of the outcome. It is needless to say that stake holders are students, guardians, faculty positions, management groups, and very importantly; the industrialist as employers and others. These variables are properly set in the mathematical model to achieve the desired outcome. If the desired result is not achieved, a relook in the mathematical modelling with fine tuning of increasing or decreasing any one or more variables, is done to achieve the set goal. Thus, the variables of mathematical modelling should be tuned such that the set goal should be achieved very easily and neatly.

There are numerous methods of mathematical modelling available in the literature based on equivalent circuit approach [1-9]. Chirlian [1] suggested very general approach; wherein any three terminal devices can fit in it. Mitra [5], Gray [7], and Millman [3,9] have provided more particular equivalent circuit approaches for BJTs and MOSFETs. One has to select the proper mathematical modelling scheme in a given constraint to achieve the best result with ease. As an instance, the transfer function of linear, time-invariant, differential equation system is best suited for the Laplace transform method. The nullor [10-11] and admittance matrix [12] methods have been used in the symbolic form extensively in the past. We have suggested an elegant mathematical modelling approach for both active devices and passive circuits and components, called the floating admittance matrix model. As the word spelt floating, it does not have any reference terminal in the analysis and design of any circuit whether active or passive or mixed of the active devices and passive components. The floating admittance matrix (FAM) model has been developed for the BJT or MOSFET to demonstrate the beauty of the method over other conventional techniques. The outcome of the developed mathematical models has been tested on any amplifier configuration that corroborates the result obtained in the available literature and over rides in the simplicity.

The floating admittance matrix model presented here is so simple that even a pure mathematician without the knowledge of electronic devices, can work and analyse all transfer functions of any circuits, provided the parameters of devices are known to him. The analysis and design of any circuit using floating admittance matrix model is based on pure mathematical maneuvering of matrices. The transfer functions are expressed as ratio of minors with proper signs, called cofactors of first and or second order. The mathematical modelling using FAM approach provides a leverage to the designer to adjust their style of design comfortably.

The conventional approach to mathematical model of the actives devices such as BJT and FET/MOSFET [1-9] uses its equivalent circuit as per the requirement of (a) either large signal or small signal models, (b) low frequency or high frequency modes so and so forth. For cascaded or cascoded connections of many devices (BJTs and MOSFETs) or the combinations of both BJTs and MOSFETs, in any circuit, the conventional method of equivalent circuit approach becomes very cumbersome. All

types of transfers functions such as voltage gain, current gain, input resistance (impedance), output resistance (impedance), and power gains of any complicated circuit are obtained very easily based on the matrix partitioning method using the floating admittance matrix approach. This FAM technique very well satisfies the superposition theorem. The computer can very well be used for complicated networks, because the method uses only cofactors of the developed FAM.

The analysis becomes lucid and corroborates the transfer functions obtained in literatures. These transfer functions depend only on the cofactors of the FAM of any circuit; active and or passive or a combination of both. The outstanding merit of the floating admittance matrix is that it can be written by inspection for simple circuits.

2. Concept of nullor:

The circuit symbols of the active devices 'nullator' and 'norator' [5] are shown in Fig. 1. It is evident from Fig. 1 that the nullator [10-11] is a two-pole network with no current and voltage, whereas norator is also a two-pole network without any restriction on its voltage and current.

The nullor is an active two port network consisting of nullator and norator. The circuit symbol of the nullor in the most basic form is depicted in Fig. 2. Thus, the nullator is connected as the input port of the nullor whereas the norator forms its output port.

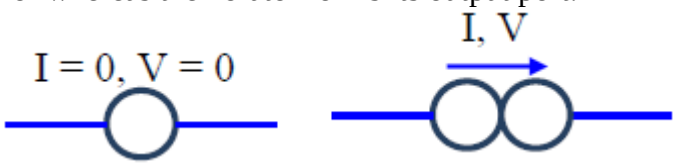

Fig. 1 Symbolic form of nullator and norator

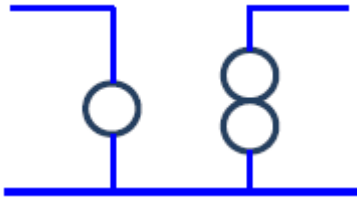

Fig. 2 Two port symbol of nullor

Fig. 3 shows the approximate equivalent of ideal active devices such as BJT, MOSFET, and Op amp in the form of nullor. The use of the symbolic form of nullor is almost extinct from the analysis of active network in the current scenario. The use of symbolic form of BJT, MOSFET, and Op amp in place of the nullor, as per the circuit requirement, is the current trends in the circuit analysis.

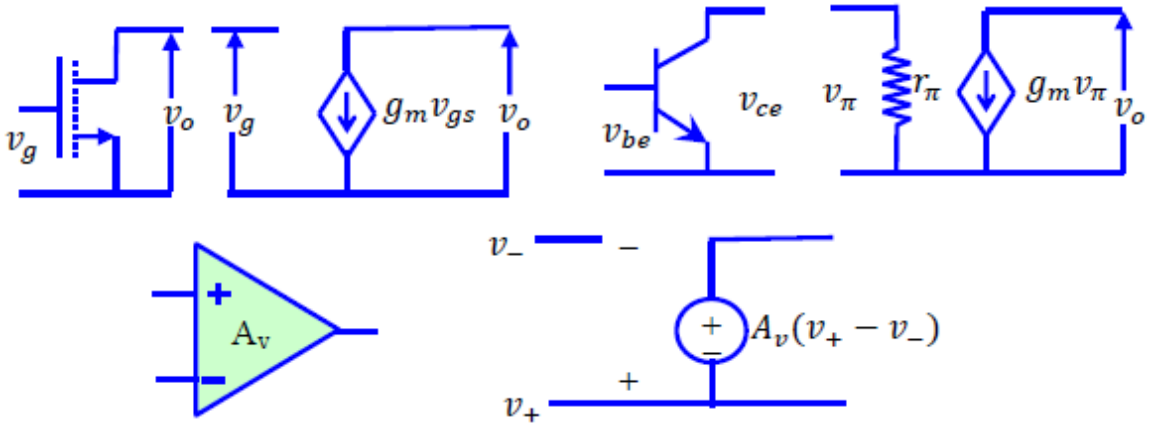

Fig. 3 practical symbols nullor

Replacing each of the non-ideal Op amp or transistor as finite gain voltage controlled current source (VCCS) is tantamount of task. This is possible only in the case, when the active element falls in the category of the metal oxide field effect transistor (MOSFET) or a transconductance Op amp. On the contrary, when an active device is supposed to function as voltage control voltage source (VCVS), or current controlled current source (CCCS), or a transresistance type of device, then Op amp and BJT are particularly suitable active devices.

The nullor based analysis and synthesis use ideal Op amp and BJT/ MOSFET as the dependent source and impedance (resistor) converter. The scaling technique of the elements of a matrix and its movement within itself is permissible provided the port equivalence is maintained.

The single four-terminal floating nullor in the form of a current conveyer can realize a single resistor controlled sinusoidal oscillator [12]. The frequency of oscillation and the condition of oscillation are independently controlled by separate resistors. The major advantage of this oscillator lies in the use of only one four terminals floating nullor in contrast to the others using more numbers of four terminal floating nullor. The model permits topological method of solution of the network.

A network analysis problem can be solved unequivocally [11], if an unambiguous relationship can be established between currents and voltages of the two-poles forming the network. The nullator, in turn, represents two restrictions, namely, the insertion of a nullator into a real circuit makes the analysis problem redundant and the number of the possible independent Kirchhoff equations is being increased by one; while the number of relationships of voltages and currents are increased by two. The insertion of a narrator into the circuit adds another independent Kirchhoff equation leaving the number of restrictions for voltages and currents unchanged. Accordingly, the insertion of a narrator makes the problem indefinite. For an equal number of inserted nullators and narrators, the network calculation problem can be solved.

Though a number of circuits are available in the literature for capacitance multiplier, but the scheme presented [14] uses only one differential ideal Op amp to realize the frequency dependent resistance in the form of capacitance multiplier. The scheme facilitates the on-demand realization of both negative as well as positive values of resistances. The value of realized capacitance is proportional to the gain of the Op amp. Higher the value of gain of the Op amp, higher will be the capacitance multiplication factor.

The ideal transistor and the ideal Op amp represent an active nullor as the small signal circuit element. For the gain of the nullor tending towards infinity, results into dependent source of any of the four possible types i.e. voltage-controlled current-source, current-controlled voltage-source, voltage-controlled voltage-source, and current-controlled current-source. The input admittance and output impedance for a VCVS should strictly be zero.

The book [15] stresses fundamental theory of circuit analysis for professional applications with the reinforcement by using frequent examples. The book defines and demonstrate the schematic symbol and corresponding notation of (a) resistors, (b) capacitors, (c) inductors, (d) voltage sources, and (e) current sources etc. very well. The circuit schematic symbols for (a) voltage-controlled voltage-source (VCVS), (b) current-controlled voltage-source (CCVS), (c) voltage-controlled current-source (VCCS), and (d) current-controlled current-source (CCCS) are also available along with many examples in the book. The circuit symbol for a voltage mode operational amplifier, first-order linear model of the Op amp, a voltage amplifier as the gain element, equivalent circuit of the voltage amplifier are available [15] as symbolic models.

Any value of input and output resistances (impedances) of both positive and negative magnitude can be realized, has been demonstrated[14].

3. Concept of floating admittance

A very simple circuit of an admittance Y connected between two voltage sources V_1 and V_2 is shown in Fig. 4 without any reference point.

Fig. 4 Series connected Y

These floating terminal voltages V_1 and V_2 results in the currents I_1 and I_2. The relationships between current I_1 and I_2 and voltages V_1 and V_2 are expressed in the 2x2 matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y & -Y \\ -Y & Y \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (1)$$

The coefficient matrix in Eq. (1), is called the floating admittance matrix. To have better sense of floating admittance matrix method of modelling [13-14, 16-20], let us have a practical circuit called

bridge-T attenuation [21]. The bridge-T attenuator is very versatile circuit used in communication network as shown in Fig. 5.

The coefficient floating admittance matrix of Fig. 5 can be written by inspection very easily by inspection in the form of 4x4 matrix as;

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ G_1+G_3 & -G_1 & -G_3 & 0 & 1 \\ -G_1 & 2G_1+G_2 & -G_1 & -G_2 & 2 \\ -G_3 & -G_1 & G_1+G_3+G_0 & -G_0 & 3 \\ 0 & -G_2 & -G_0 & G_2+G_0 & 4 \end{bmatrix} \quad (2)$$

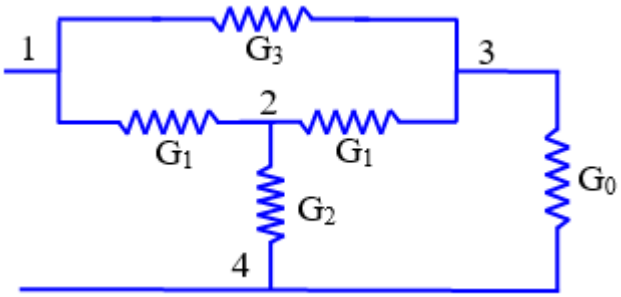

Fig. 5 Bridge-T passive attenuator

The input resistance [13, 16-18] of the bridge-T between its terminals 1 & 4 in Fig. 5 is expressed as;

$$R_{in} = R_{i(14)} = \frac{|Y_{14}^{14}|}{|Y_4^4|}_{g_s=0} \quad (3)$$

$$|Y_{14}^{14}| = (2G_1+G_2)(G_1+G_3+G_0) - G_1^2$$

For $G_1^2 = G_0^2 = G_2G_3$

$$|Y_{14}^{14}| = \frac{2G_0}{G_2}(G_0^2 + 2G_0G_2 + G_2^2) = \frac{2G_0}{G_2}(G_0+G_2)^2$$

$$|Y_4^4| = \frac{2G_0^2}{G_2}(G_0+G_2)^2$$

$$R_{in} = R_{i(14)} = \frac{\frac{2G_0}{G_2}(G_0+G_2)^2}{\frac{2G_0^2}{G_2}(G_0+G_2)^2} = R_0 \quad (4)$$

Equation (4) indicates that the input and output ports of Fig. 5 are matched under the condition $G_1^2 = G_0^2 = G_2G_3$.

The voltage transfer function between terminals 3 and 4 and 1 and 4 of the bridge-T network in Fig. 5 is expressed as;

$$A_v|_{14}^{34} = sgn(3-4)sgn(1-4)(-1)^{12}\frac{|Y_{34}^{14}|}{|Y_{14}^{14}|} = \frac{|Y_{34}^{14}|}{|Y_{14}^{14}|} \quad (5)$$

$$|Y_{34}^{14}| = \frac{2G_0^2}{G_2}(G_0+G_2)$$

$$A_v|_{14}^{34} = \frac{v_{34}}{v_{14}} = \frac{\frac{2G_0^2}{G_2}(G_0+G_2)}{\frac{2G_0}{G_2}(G_0+G_2)^2} = \frac{G_0}{G_0+G_2} = \frac{R_2}{R_2+R_0} \quad (6)$$

The propagation constant (N) [21] of the bridge-T network in Fig. 5 is described as;

$$e^{\gamma} = \frac{v_{14}}{v_{34}} = 1 + \frac{R_0}{R_2} = N, \frac{R_0}{R_2} = N-1 \quad (7)$$

$$R_2 = \frac{R_0}{N-1},\ R_3 = \frac{R_0^2}{R_2} = (N-1)R_0, R_2 = R_0 \quad (8)$$

For a given value of propagation constant (N) and the normal value of characteristic impedance ($R_0 = 600\ \Omega$), the other components of the bridge-T attenuator can be designed very easily using FAM approach.

Hence, we conclude that the design of bridge-T attenuator becomes very simple using floating admittance matrix approach in comparison to the conventional method.

4. CONCLUSION:

In principle, the proposed technique can be used to model any circuit, whether active or passive or the combination of both. The proposed "floating admittance matrix" is an elegant approach to mathematical modelling of active and passive devices and circuits. The specific advantage of the floating admittance matrix approach lies in the fact that the algebraic sum of all elements of any row or of any column yielding zero provides first check that the process of analysis and design is in the correct direction of any circuit. Once the floating admittance matrix of the active device is known, the rest of the floating admittance matrix of the complete circuit containing passive components, can be written by inspection without any difficulty. The rigorous equivalent circuit analysis with more active devices in any circuit can be avoided using this technique.

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