



## Green Supply Chain Management Model with Reliability and Time Dependent Demand

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September 25, 2022

# Green Supply Chain Management Model With Reliability And Time Dependent Demand

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## Abstract

*In the communications of green supply chain a creation must go throughout numerous phases from development of supply and renovation of raw materials to storage space of finish supplies. In this paper we introduce the green supply chain model for single manufacturer and supplier with dependable manufacturing process for constant deteriorating items. It deals with time dependent demand. For the reduction of carbon emission many countries execute many policies. Main motive of execution of these policies is to earn maximum profit along with environmentology. We also consider carbon emission reduction under reliability effect. Shortages are allowed in this paper. The purpose of this study is to expand supply chain models that reduce the total cost with carbon tax. The aim of this paper is to provide a reliable production model with carbon emission cost. The testament of a model is demonstrated by a numerical illustration with the help of sensitivity analysis with the help of Mathematica 12.0 software. The disparities of different parameters on optimum result are being explore through graphical representation and further discussed in detail.*

**Keywords:** Green Supply Chain, Deteriorating item, Carbon Emission Cost, Time Dependent Demand, Carbon tax

## 1. INTRODUCTION

Over the few past decades, most of the studies that included industrial inventory scenarios assumed a stable production pace. Simply while demand can be predict with full guarantee is this theory suitable. However, in the actual world, with a repeatedly varying and impulsive business environment, order for manufactured goods does not stay steady over time. As a result, variations in the order to reduce pointless expenditures necessitate flexibility in the production pace. If the items are of deteriorating nature, particular curiosity is essential to reduce the losses. The blow of deterioration cannot be unobserved. The losses of inventory due to deterioration raise the total cost and minor the total profit. Various models with stable and Weibull rates of deterioration were also confer by (Singh et al. 2013). Recently, (Khanna et al. 2017) confer deteriorating objects with defective objects and selling price dependent demand, shortages back-ordering under acknowledgment investment.

Moreover, (Gautam et al. 2020) and (Rini et al. 2021) urbanized an inventory representations with uneven innovation rates and carbon emission. (Kamna et al. 2017) confer the Sustainable inventory policy for defective manufacture system with energy convention. To boost profit, many industries have effort and urbanized conservation technologies to avoid economic failure. Special industries need assorted maintenance methodologies to diminish deterioration in goods. Deteriorating goods like fruits need refrigeration and diverse dairy goods like milk and curd demand for cooling tanks. In addition, assured goods require air-tight storeroom to keep away from decomposition. The present studies relate goods like fruits, vegetables, and dairy products with a definite existence. Due to capable supervision of deteriorating objects, frequent researchers considered the benefits of deal in conservation procedure (Priyamvada R, et al. 2021) and (Priyamvada G, et al. 2021). Renewable energy has a bang on all three stakes (Economic, environmental, and social factors) in the subsequent method: industries apply conventional energy at a high cost for financial growth. As a cost efficient resolution, renewable

energy consequently create up a segment of an production's largely energy utilize (Sarkar 2020) and (Sarkar 2021). Single-item, multi source with carbon-constrain batch size models were introduced by (Absi et al. 2013) and (Absi et al. 2016). Important phenomenon such as overexert of natural assets, growth of ecological effluence, and global warming have fascinated nationwide government's concentration.

Governmental interfering signify a important function in budding cleaner equipment and generate a green supply chain to attain eco-friendly invention (Safarzadeh et al., 2020) and (Yang et al., 2020). Ecological Protection Agency persuades purchasers to obtain green goods; and executes an online organization to evaluate every manufactured goods carbon emission. India's government has started a variety of economic curriculums to sustain green invention, together with the "Unnat Jyoti by Affordable LEDs for All" (UJALA) scheme to inspire consumers to purchase LED lights. In South Asia, the adolescent generations have an elevated ecological attentiveness level. About 80% of consumers considered goods ecological features in the Middle East, and a lesser amount about half of them prioritize procuring eco-friendly goods (Li et al., 2021). Due to the increase in ecological awareness of clients and governments' interferences, several industries have come into developed green products like energy-saving lamps, fusion automobiles, and reusable shopping bags (Liu et al., 2020a) and (Liu et al., 2020b). Like Adidas, companies apply green technologies such as MMVEA and Eco-Grip to detract baneful stuffs developed in the industrialized method to reduce the unfavorable inventions affects on the environment. In India, Coca-Cola reuses waste bottles that vendor assemble (Heydari et al., 2020a).

Nowadays, different forms of industry are budding and as an effect, new manufactured goods are introduced in the marketplace every day. The core cause for this is the rising population and the goods they utilize. Therefore, a large range of goods are budding based on modern plans. In addition, several companies propose a large collection of goods for a single use, screening various advantages, for example, at present there are a huge number of citizens who utilize petrol and diesel for bikes, cars and trucks, whereas there are a lot of industries who sold those vehicles. So, the advantage is equal, but the number of companies that manufacturing is increasing. Therefore, the energy presented from renewable supply is inadequate and, thus, the stage of consumption of energy offered from non-renewable resources happens. But when energy is produced from such non-renewable resources, injurious discharges of carbon are emitted. Most of the companies release the largest part of dangerous carbon emissions, which are injurious to the environment and direct to climate change. In each production, the pretended manufactured goods annoyed lots of places and several citizens and ultimately reaches the customer, which is known as supply chain. The supply chain affords a connection among supply chain players (i.e. supplier, third party logistics, manufacturer and end buyer/retailer), in which inventory management acting a small part and is an essential element of the supply chain.

In latest years, various researchers acknowledged the significance of carbon emissions in supply chain management, and considered the outcomes of carbon emission policies in supply chain operations decisions. (Song et al. 2012) integrated the carbon emission issues into a single-cycle newsboy model, evaluated and manipulates of different carbon emission plan on project order policies. (Choi 2013) logically study the bang of the carbon footprint tax on two-tiered style supply chain systems, and additional explored the importance of the carbon footprint tax on style supply chain management under a clean extensive pricing agreement and a reduction money agreement. Carbon emission limitations pretense fresh challenges to project and create project procedure executive more difficult. The drop of carbon emissions has also motivate enterprises to review prepare judgment in the supply chain. While the project are strictly linked to the further projects within one supply chain (for example, a car manufacturer relies deeply on the dealer of steel, glass, rubber and plastic), these projects in the same supply chain are concurrently pretentious by the carbon emissions convention. In U.K., a study in 2008 showed that more than two-third citizens will offer partiality to acquire the goods from the firms who take a dynamic fraction in carbon emission lessening. In the meantime, as a plan to encourage enterprises to recover ecological presentation, carbon emissions restraint has burly externality and public awareness features, and cannot produce straight financial profit to the enterprises.

National governments are inclined to relay some carbon emissions plans to inspire the enterprises to vigorously apply the policy of carbon emissions reduce when maximizing the enterprises' earnings. A

carbon tax is a cost-efficient governmental strategy for the lessening of carbon emissions and is extremely suggested by many experts (Zhang and Baranzini, 2004), (Oreskes, 2011) and (Li et al, 2017). Some European countries, such as the Netherlands, Sweden, Finland, and Norway have executed carbon taxes for years (Baranzini et al., 2000). The Chinese government will be commanding a carbon tax because of rising stress to diminish emissions (Deng et al., 2015). Even though various of studies focused on governmental carbon emissions policy or policies in supply chain operations decision models, the factors connected to carbon emissions policies were indulgence as exogenous variables, and the governments were not occupied in the decision making processes of the models.

## 2. ASSUMPTIONS

The subsequent suppositions are taken to expand mathematical formulation:

- Demand for the item is known and time dependent.
- Shortages are allowed and partially backlogged.
- Deterioration rate is constant.
- System involves single producer and single supplier.
- The investment made in technology to rate carbon emissions is represented by  $m(l)$  where  $0 < m(l) < 1$ , and  $m(l)$  is increasing the work of  $L$ .
- The investment ratio between supplier and producer to reduce carbon emissions is the capital investment ratio  $(1-\gamma)$  and  $\gamma$ .
- Lead time is zero.
- Backlogging rate is  $e^{-\delta t}$ .

## 3. NOMENCLATURES

The subsequent nomenclatures are used to expand the mathematical formulation:

**TABLE -1**

$Su_p$	Setup cost for producer
$h_p$	Holding cost for producer
$d_p$	Deterioration cost for producer
$\theta_1$	Deterioration rate for producer
$A$	Reliability rate to produce good items
$l$	Reduction of carbon emission
$\gamma$	Investment ratio between producer and supplier
$\alpha$	Amount paid for each unit of carbon emission
$P$	Production rate per unit time
$S_p$	Shortage cost for producer
$L_p$	Lost sale cost for producer
$\bar{S}u_p$	Setup cost for producer under carbon emission
$\bar{h}_p$	Holding cost for producer under carbon emission

$\bar{d}_p$	Deterioration cost for producer under carbon emission
$\theta_2$	Deterioration rate for supplier
$\bar{S}_p$	Shortage cost for producer under carbon emission
$\bar{L}_p$	Lost sale cost for producer under carbon emission
$O_s$	Ordering cost for supplier
$h_s$	Holding cost for supplier
$d_s$	Deterioration cost for supplier
$\bar{O}_s$	Ordering cost for supplier under carbon emission
$\bar{h}_s$	Holding cost for supplier under carbon emission
$\bar{d}_s$	Deterioration cost for supplier under carbon emission
$t_1$	Production period
$t_2$	Non-Production period
$t_3$	Shortage period
$T$	Total length of time cycle

#### 4. PROBLEM DESCRIPTION

##### ❖ Mathematical Formulation For Producer

In the producer's inventory system the total time  $T$  is divided into four intervals such as  $[0, t_1]$ ,  $[t_1, t_2]$ ,  $[t_2, t_3]$  and  $[t_3, T]$  where  $[0, t_1]$  is production period,  $[t_1, t_2]$  is non-production period,  $[t_2, t_3]$  is shortage period and at time period  $[t_3, T]$  all the backlogged shortages are cleared. The mathematical formulation for the producer is given by

$$I'_{p_1}(t) + \theta_1 I'_{p_1}(t) = AP - (a + bt), \quad 0 \leq t \leq t_1 \quad (1)$$

$$I'_{p_2}(t) + \theta_1 I'_{p_2}(t) = -(a + bt), \quad t_1 \leq t \leq t_2 \quad (2)$$

$$I'_{p_3}(t) = -B(a + bt), \quad t_2 \leq t \leq t_3 \quad (3)$$

$$I'_{p_4}(t) = AP - (a + bt), \quad t_3 \leq t \leq T \quad (4)$$

With boundary conditions

$$I_{p_1}(t) = 0; \quad t = 0$$

$$I_{p_1}(t) = Q; \quad t = t_1$$

$$I_{p_2}(t) = 0; \quad t = t_2$$

$$I_{p_3}(t) = 0; \quad t = t_2$$

$$I_{p_4}(t) = 0; \quad t = T$$

Solutions of these differential equations are given below:

$$I_{p_1}(t) = \left[ \left( \frac{AP}{\theta_1} - \frac{a}{\theta_1} + \frac{b}{\theta_1^2} \right) [1 - e^{-\theta_1 t}] - \frac{bt}{\theta_1} \right], \quad (5)$$

$$I_{p_2}(t) = \left[ \left( -\frac{a}{\theta_1} + \frac{b}{\theta_1^2} \right) [1 - e^{\theta_1(t_2-t)}] + \frac{bt_2 e^{\theta_1(t_2-t)}}{\theta_1} - \frac{bt}{\theta_1} \right], \quad (6)$$

$$I_{p_3}(t) = \left[ \left( \frac{a}{\delta} - \frac{b}{\delta^2} \right) [e^{-\delta t} - e^{-\delta t_2}] - \frac{bt_2 e^{-\delta t_2}}{\delta} + \frac{bte^{-\delta t}}{\delta} \right], \quad (7)$$

$$I_{p_4}(t) = \left[ (AP - a)(t - T) - \frac{b}{2}(t^2 - T^2) \right], \quad (8)$$

At,  $I_{p_1}(t) = Q$ ;  $t = t_1$ , from equation (6) we get,

$$Q = \left[ \left( -\frac{a}{\theta_1} + \frac{b}{\theta_1^2} \right) [1 - e^{\theta_1(t_2-t_1)}] + \frac{bt_2 e^{\theta_1(t_2-t_1)}}{\theta_1} - \frac{bt_1}{\theta_1} \right], \quad (9)$$

Producer's total cost depends upon the following factors:

#### a) Setup Cost:

The setup cost for the machines prior to produce or purchase from external suppliers. This cost includes loading, unloading, transportation costs etc.

$$SuC_p = S_{u_p} \quad (10)$$

#### b) Holding Cost:

Holding cost includes the cost of storing and managing the inventory such as warehousing, transporting, cost of maintaining stores, installation, insurances and deterioration etc.

$$\begin{aligned} HC_p &= h_p \left[ \int_0^{t_1} I_{p_1}(t) dt + \int_{t_1}^{t_2} I_{p_2}(t) dt \right] \\ HC_p &= h_p \left[ \frac{APt_1^2}{2} + \frac{at_2^2}{2} + \frac{bt_2^3}{2} - at_1 t_2 - bt_1 t_2^2 + \frac{bt_1^2 t_2}{2} \right] \end{aligned} \quad (11)$$

#### c) Deterioration Cost:

Deterioration cost is the cost in which the products such as food items, medicines, fruits and blood etc. are deteriorate. The biggest problem for any supply manager is to maintain the deteriorating items.

$$\begin{aligned} DC_p &= d_p \theta_1 \left[ \int_0^{t_1} I_{p_1}(t) dt + \int_{t_1}^{t_2} I_{p_2}(t) dt \right] \\ DC_p &= d_p \theta_1 \left[ \frac{APt_1^2}{2} + \frac{at_2^2}{2} + \frac{bt_2^3}{2} - at_1 t_2 - bt_1 t_2^2 + \frac{bt_1^2 t_2}{2} \right] \end{aligned} \quad (12)$$

#### d) Shortage Cost:

Shortage Cost is the cost when demand is exceeds from the available inventory for an item. In this demand and goodwill may be lost.

$$\begin{aligned} SC_p &= -S_p \left[ \int_{t_2}^{t_3} I_{p_3}(t) dt + \int_{t_3}^T I_{p_4}(t) dt \right] \\ SC_p &= -S_p \left[ (2t_3 T - T^2) \left( \frac{AP}{2} - \frac{a}{2} \right) + (2t_3 t_2 - t_2^2) \left( \frac{a}{2} - \frac{b}{\delta} \right) + t_3^2 \left( \frac{b}{\delta} - \frac{AP}{2} \right) + \right. \\ &\quad \left. \frac{b}{6} (T^3 - t_3^3 - 4t_2^3 + 6t_3 t_2^2 - 3t_3 T^2) \right] \end{aligned} \quad (13)$$

#### e) Lost Sale Cost:

Lost sale is occurring when the items are out of stock.

$$\begin{aligned} LS_p &= L_p \left[ \int_{t_2}^{t_3} (1 - e^{-\delta t})(a + bt) dt \right] \\ LS_p &= L_p \left[ \frac{a\delta}{2} (t_3^2 - t_2^2) + \frac{b\delta}{3} (t_3^3 - t_2^3) \right] \end{aligned} \quad (14)$$

Therefore total average cost of the producer is

$$\text{Total Cost (TC}_p) = \frac{1}{T} [\text{SuC}_p + \text{HC}_p + \text{DC}_p + \text{SC}_p + \text{LS}_p] \quad (15)$$

The technology investment for the reduction of carbon emission is (l). The ratio of producer investment to a total of  $0 \leq \gamma \leq 1$  reduces carbon emission per unit for the producer indicated the investment by  $\gamma l$ . So the producer's total cost is

$$T_{\text{CEp}} = \frac{1}{T} [\text{SuC}_p + \text{HC}_p + \text{DC}_p + \text{SC}_p + \text{LS}_p] + \gamma l \quad (16)$$

As the reduction rate of carbon emission  $m(l)$ , the total amount for carbon emission produced by the manufacturer per unit time is

$$\overline{\text{TA}_{\text{CEp}}} = \frac{1-m(l)}{T} [\overline{\text{SuC}_p} + \overline{\text{HC}_p} + \overline{\text{DC}_p} + \overline{\text{SC}_p} + \overline{\text{LS}_p}] \quad (17)$$

For each of the unit emitted due to carbon tax regulation  $\alpha$  amount is paid for the total. So that total with carbon tax is

$$\Pi_{\text{Cp}} = T_{\text{CEp}} + \alpha \overline{\text{TA}_{\text{CEp}}} \quad (18)$$

### ❖ Mathematical Formulation For Supplier

During the interval  $[0, T]$ , the inventory stage decreases due to deterioration as well as order. The supplier's inventory system can be characterize by the linear differential equation

$$\dot{I}_s(t) + \theta_2 I_s(t) = -(a + bt), \quad 0 \leq t \leq T \quad (19)$$

Using the boundary conditions

$$I_s(t) = 0; \quad t = T$$

The solution of above differential equation is

$$I_s(t) = \left[ \left( -\frac{a}{\theta_2} + \frac{b}{\theta_2^2} \right) [1 - e^{\theta_2(T-t)}] + \frac{bTe^{\theta_2(T-t)}}{\theta_2} - \frac{bt}{\theta_2} \right], \quad (20)$$

Supplier's total cost depends upon the following factors:

#### a) Ordering Cost:

It is the cost of placing an order to an outsider. Ordering cost includes details like order quantities etc.

$$\text{OC}_s = O_s \quad (21)$$

#### b) Holding Cost:

It is the cost associated with holding one unit of inventory for one unit of time. Holding cost varies with level of inventory and occasionally with the length of time an item is held

$$\begin{aligned} \text{HC}_s &= \sum_{i=1}^n h_s \left[ \int_{(i-1)T}^{iT} I_s(t) dt \right] \\ \text{HC}_s &= \sum_{i=1}^n h_s \left[ \frac{aT}{2} (i-1) - \frac{aT^2}{2} (i^2 - 2) + \frac{bT^3}{2} \right] \end{aligned} \quad (22)$$

#### c) Deterioration Cost:

Deterioration is define as damage, decay, change, loss in unique rate and price in a product that results in the falling usefulness from the inventive one

$$\text{DC}_s = \sum_{i=1}^n \theta_2 d_s \left[ \int_{(i-1)T}^{iT} I_s(t) dt \right]$$

$$DC_s = \sum_{i=1}^n \theta_2 d_s \left[ \frac{aT}{2} (i-1) - \frac{aT^2}{2} (i^2 - 2) + \frac{bT^3}{2} \right] \quad (23)$$

Therefore total average cost of the supplier is

$$\text{Total Cost (TC}_s) = \frac{1}{T} [OC_s + HC_s + DC_s] \quad (24)$$

The supplier's total technical savings in carbon emission reduction is (1). This is because of the corporation between the producer and the supplier, where the supplier ratio is  $1-\gamma$ . The total carbon emission per unit time by the supplier are reduced by  $(1-\gamma)l$ .

$$T_{CEs} = \frac{1}{T} [OC_s + HC_s + DC_s] + (1-\gamma)l \quad (25)$$

The total amount for carbon emission created per unit time is

$$\overline{TA}_{CEs} = \frac{1-m(l)}{T} [\overline{OC}_s + \overline{HC}_s + \overline{DC}_s] \quad (26)$$

Total  $\alpha$  total is compensated for each unit of carbon emissions that go all along with carbon tax regulation. Therefore, the supplier's total cost along with carbon tax is

$$\Pi_{Cs} = T_{CEs} + \alpha \overline{TA}_{CEs} \quad (27)$$

Total cost for included model is

$$\Pi = \Pi_{Cp} + \Pi_{Cs} \quad (28)$$

## 5. OPTIMAL SOLUTION

The main aim of the current study is to reduce the total cost of the integrated model by optimizing the cycle time. We can solve it by using software (WOLFRAM MATHEMATICA 12.0). According to the present model, there are four independent variables in the total cost  $t_1$ ,  $t_2$ ,  $t_3$  and  $T$ . To improve the total cost and to observe the value of all the independent parameters, the following steps are pursued.

## 6. ALGORITHMS

Total profit becomes a function of four independent variables  $t_1$ ,  $t_2$ ,  $t_3$  and  $T$ . This is the objective function, hence optimal solution is

$$\frac{\partial \Pi}{\partial t_1} = 0, \frac{\partial \Pi}{\partial t_2} = 0, \frac{\partial \Pi}{\partial t_3} = 0 \text{ and } \frac{\partial \Pi}{\partial T} = 0 \quad (29)$$

Obtained values of  $t_1$ ,  $t_2$ ,  $t_3$  and  $T$  satisfy the following conditions

$$\frac{\partial^2 \Pi}{\partial t_1^2} > 0, \frac{\partial^2 \Pi}{\partial t_2^2} > 0, \frac{\partial^2 \Pi}{\partial t_3^2} > 0 \text{ and } \frac{\partial^2 \Pi}{\partial T^2} > 0 \quad (30)$$

## 7. NUMERICAL EXEMPLIFICATION

On the bases of above study, following values of various parameters are considered:

$$\begin{aligned} Su_p &= 40, h_p = 0.3, d_p = 0.1, \theta_1 = 0.9, A = 50, P = 30, a = 50, b = 0.05, S_p = 4, \\ \delta &= 0.05, L_p = 5, \gamma = 0.5, \alpha = 0.2, \theta_2 = 0.8, O_s = 5, n = 2, h_s = 0.05, d_s = 0.5, \\ m &= 0.005, l = 0.5, \overline{Su}_p = 2, \overline{h}_p = 5, \overline{d}_p = 0.5, \overline{S}_p = 0.5, \overline{L}_p = 0.5, \overline{O}_s = 5, \overline{h}_s = 0.9, \\ \overline{d}_s &= 2 \end{aligned}$$

And the result of the above problem is bare minimum total cost  $\Pi = \$19.6303$  and optimal values are  $t_1 = 0.06040$  days,  $t_2 = 1.81523$  days,  $t_3 = 2.33604$  days and  $T = 2.35908$  days.



## 8. PERCEPTIVITY ANALYSIS

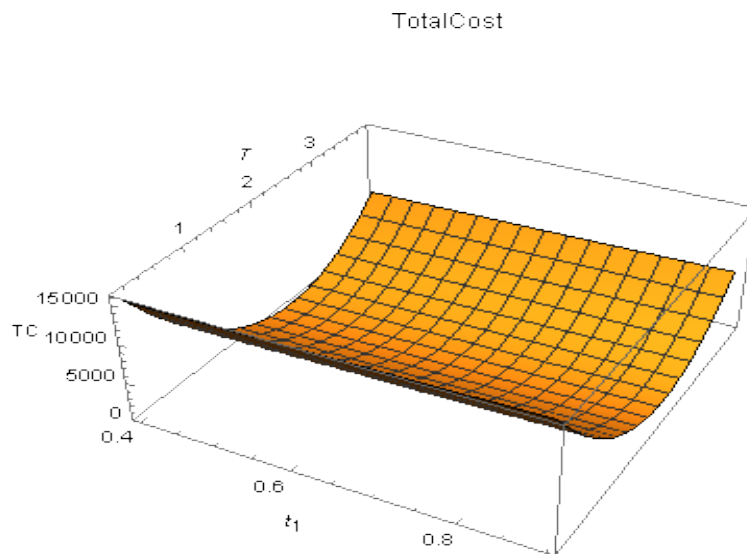
The effect of changes in different parameters like Reliability, Demand rate, holding cost, lost sale cost etc. on the value of optimal time and total cost. Sensitivity analysis is performed by changing each parameter at -10%, -5%, 5%, 10%.

**Table 2: Sensitivity Analysis of Optimal Solution w.r.t. time and total cost**

Parameters	(%) Changes	Total Cost	$t_1$	$t_2$	$t_3$	T
<b>A</b>	-10	18.9841	0.06879	1.86056	2.39181	2.41805
	-5	19.3282	0.06432	1.83640	2.36209	2.38662
	5	19.8977	0.05693	1.79652	2.31302	2.33474
	10	20.1362	0.05384	1.77987	2.29254	2.31308
<b><math>h_p</math></b>	-10	16.9851	0.06701	2.01423	2.57733	2.60237
	-5	18.3764	0.06352	1.90915	2.44990	2.47388
	5	20.7671	0.05759	1.73067	2.23358	2.25578
	10	21.8031	0.05504	1.65407	2.14081	2.16224
<b><math>d_p</math></b>	-10	18.8932	0.06223	1.87.36	2.40286	2.42645
	-5	19.2673	0.06130	1.84235	2.36891	2.39222
	5	19.9828	0.05953	1.78895	2.30419	2.32697
	10	20.3253	0.05868	1.76347	2.27332	2.29584
<b><math>\theta_1</math></b>	-10	18.1119	0.06418	1.92906	2.47403	2.49821
	-5	18.8941	0.06223	1.87028	2.40277	2.42636
	5	20.3245	0.05068	1.76353	2.27339	2.29592
	10	20.9803	0.05707	1.71487	2.21444	2.23648
<b>P</b>	-10	18.9841	0.06878	1.86056	2.39181	2.41805
	-5	19.3282	0.06432	1.8364	2.36209	2.38662
	5	19.8977	0.05693	1.79652	2.31302	2.33474
	10	20.1362	0.05384	1.77987	2.29254	2.31308
<b>a</b>	-10	22.6111	0.05387	1.79905	2.31994	2.34049
	-5	21.1356	0.05710	1.8065	2.3271	2.34888
	5	18.0947	0.06377	1.8251	2.34654	2.37087
	10	16.5281	0.06721	1.83598	2.35842	2.38406
<b>b</b>	-10	19.8793	0.05972	1.79463	2.30733	2.33012
	-5	19.7559	0.06006	1.80481	2.32152	2.34443
	5	19.5025	0.06607	1.82959	2.35089	2.37407
	10	19.3723	0.06111	1.83682	2.3661	2.38941
<b><math>S_p</math></b>	-10	16.3347	0.06643	1.99668	2.63154	2.65979
	-5	18.1454	0.06309	1.89645	2.46849	2.49387
	5	20.871	0.05816	1.748	2.22617	2.24728
	10	21.9236	0.05628	1.69139	2.1335	2.15297
<b><math>\delta</math></b>	-10	17.7398	0.06439	1.93539	2.50815	2.53262
	-5	18.7275	0.06229	1.87238	2.41796	2.44168
	5	20.4597	0.05867	1.76308	2.26119	2.28361
	10	21.2251	0.05708	1.71525	2.19244	2.2143
<b><math>L_p</math></b>	-10	15.1015	0.07017	2.10931	2.75213	2.77861
	-5	16.0414	0.06810	2.04778	2.66465	2.69041
	5	17.7367	0.06443	1.93674	2.50842	2.53288
	10	18.5041	0.06279	1.88725	2.4383	2.46219

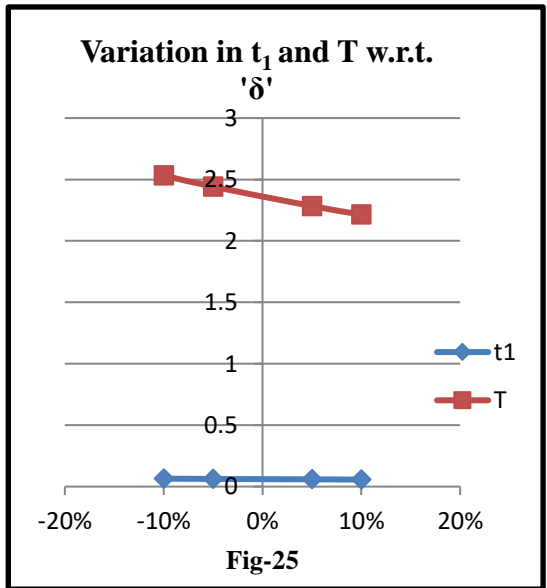
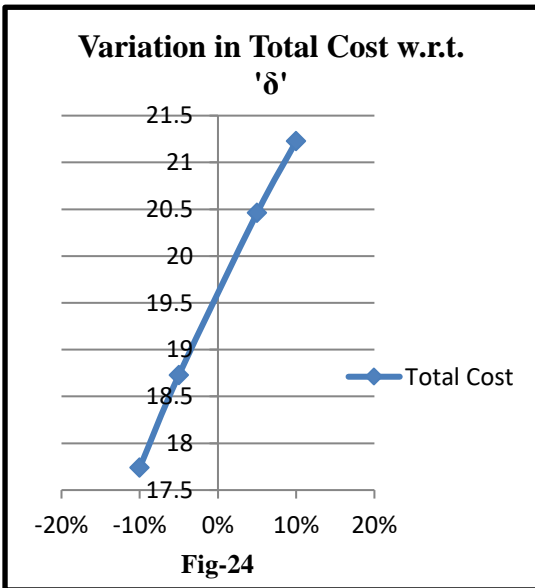
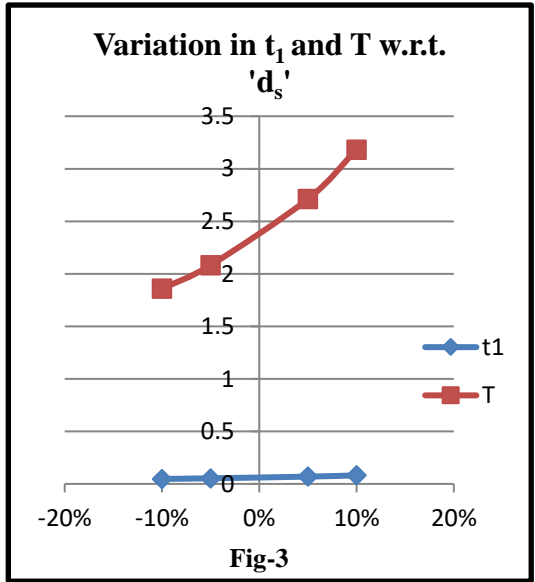
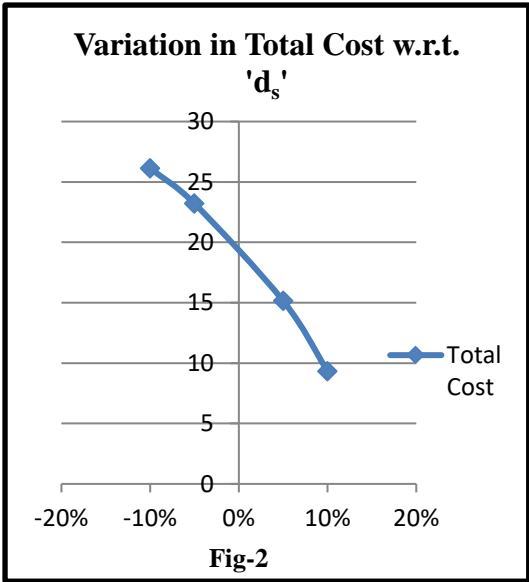
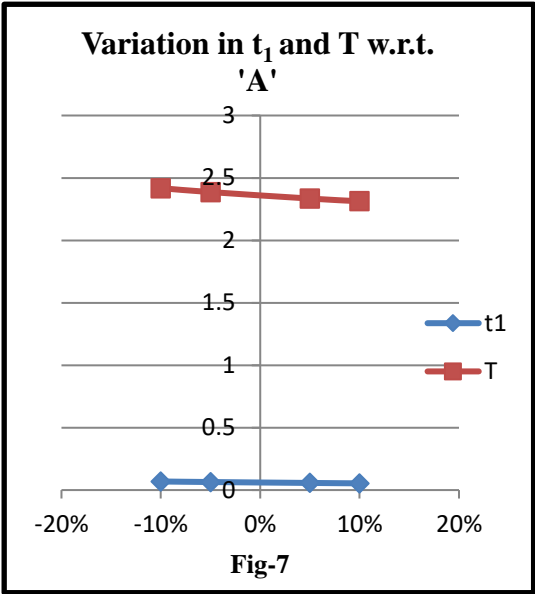
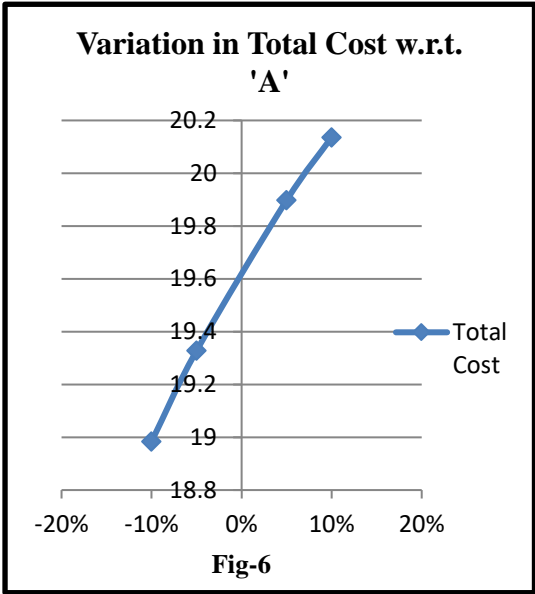
$\alpha$	-10	20.2438	0.06336	1.90438	2.40351	2.42592
	-5	19.9840	0.06172	1.85516	2.36443	2.38712
	5	19.1921	0.05933	1.78299	2.31659	2.34005
	10	18.6762	0.05847	1.75722	2.30479	2.32872
$\theta_2$	-10	29.8435	0.04034	1.21161	1.55913	1.57452
	-5	25.5853	0.04866	1.46187	1.88122	1.89979
	5	10.6534	0.07867	2.36556	3.04446	3.07446
	10	0.43504	0.10097	3.03807	3.91028	3.94876
$O_s$	-10	16.4176	0.06620	1.98993	2.58366	2.60875
	-5	16.6676	0.06620	1.98993	2.58366	2.60875
	5	17.1676	0.06620	1.98993	2.58366	2.60875
	10	17.4176	0.06620	1.98993	2.58366	2.60875
$h_s$	-10	20.5971	0.05848	1.75741	2.26161	2.28392
	-5	20.1204	0.05943	1.78589	2.29828	2.32095
	5	19.1263	0.06140	1.84546	2.37495	2.39838
	10	18.6078	0.06244	1.87663	2.41507	2.43889
$d_s$	-10	26.1329	0.04758	1.42964	1.83973	1.85789
	-5	23.2065	0.05332	1.60231	2.06198	2.08232
	5	15.1505	0.06941	2.08642	2.68512	2.71116
	10	9.3431	0.08142	2.4485	3.15123	3.18228

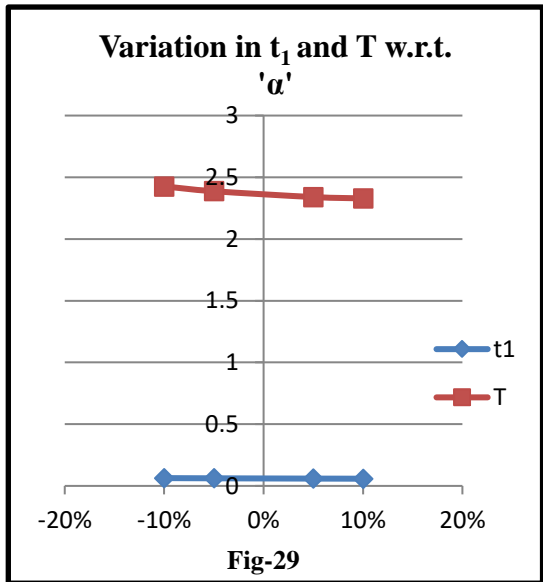
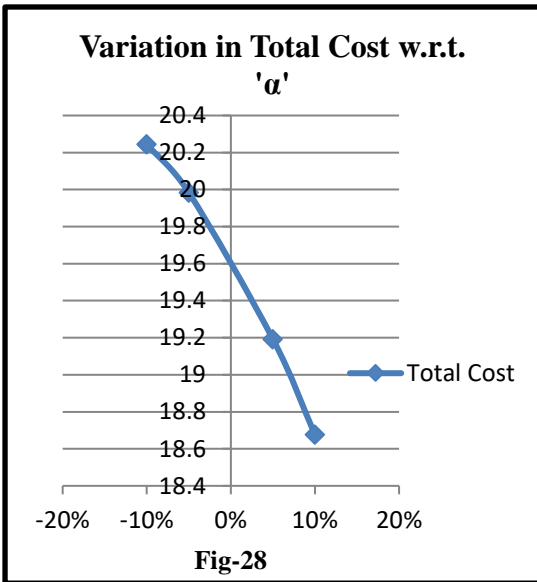
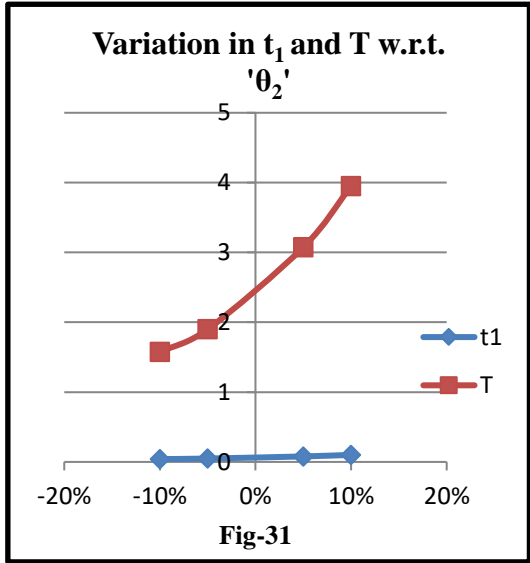
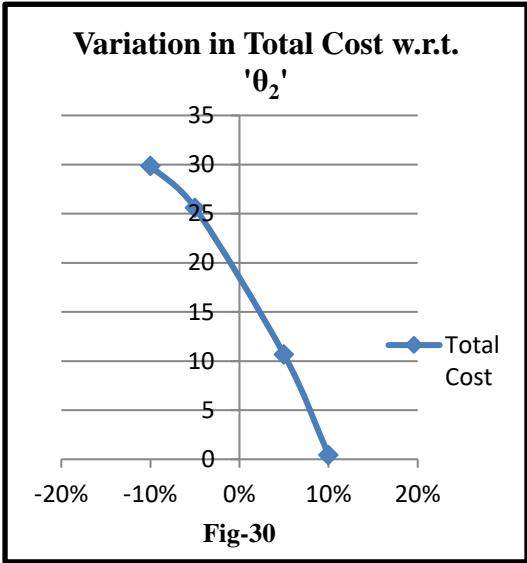
Fig.1: Convexity of Total Cost w.r.t. T and  $t_1$



## 9. OBSERVATION

- It is examined that when the value of reliability 'A', holding cost ' $h_p$ ', deterioration cost ' $d_p$ ', deterioration rate ' $\theta_1$ ', production rate 'P', shortage cost ' $S_p$ ', backlogging rate ' $\delta$ ' and lost sale cost ' $L_p$ ' is increases then total cost is also increases and value of  $t_1$ ,  $t_2$ ,  $t_3$  and T are decreasing and vice-versa.
- It is observed that when the values of setup cost ' $S_{up}$ ' is increase and decrease the value of total cost is increases and decreases respectively and the value of  $t_1$ ,  $t_2$ ,  $t_3$  and T stays same.
- It is seen that when the value of demand rates 'a' and 'b' is increases then the value of total cost is decreases and when it is decreases the value of total cost is increases on the other hand the value of  $t_1$ ,  $t_2$ ,  $t_3$  and T are decreases and increases when the value of demand rates are increases and decreases respectively.





- It is observed that when the value of ' $\alpha$ ', is increases and decreases then the value of total cost,  $t_1$ ,  $t_2$ ,  $t_3$  and T are decreases and increases respectively.
- It is examined that when the value of ' $\theta_2$ ', ' $h_s$ ' and ' $d_s$ ' is increases the value of total cost is decreases and when the value of  $\theta_2$ ,  $h_s$ , and  $d_s$  decreases then the value of total cost is increases, on the other hand the value of  $t_1$ ,  $t_2$ ,  $t_3$  and T is increases on increasing the value of  $\theta_2$ ,  $h_s$ , and  $d_s$  and vice-versa.
- It is seen that when the value of ordering cost ' $O_s$ ', is increases or decreases the value of total cost is always decreases and the values of  $t_1$ ,  $t_2$ ,  $t_3$  and T are remain constant.
- It is observed that when the value of reduction rate ' $l$ ' is increases and decreases the value of total cost,  $t_1$ ,  $t_2$ ,  $t_3$  and T are increases and decreases respectively.

## 10. CONCLUSION

In this model we include effort on a green inventory model with shortages and reliability. We have also discussed about carbon emission and carbon trade tax. We enclose the mathematical illustration, and the deviation of the promising parameters that involved in this model. It has been originate that if reliability rate is raise than the total cost is also increases. It is seen that when the value of require rates 'a' and 'b' is raise then the value of total cost is decreases and when it is reduce the rate of total cost is increases. When the reduction rate is increases or decreases then the total cost is also increases or decreases respectively. The model can be extensive with diverse kind of reduction technology for carbon emission and carbon trade tax also.

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